

Scientific and Scholarly Contributions of Distinguished Research & Emeritus Professor A K Md Ehsanes Saleh

Shahjahan Khan

Department of Mathematics and Computing
Australian Centre for Sustainable Catchments
University of Southern Queensland
Toowoomba, Queensland, AUSTRALIA
Email: *khans@usq.edu.au*

Abstract

This invited paper highlights most of the original and outstanding research and scholarly contributions of Professor A K Md Ehsanes Saleh, Carleton University, Canada. It covers his significant and diverse scientific accomplishments in many different areas of Statistical sciences. Attempts are made to include his individual as well as joint work with his PhD students, leading research associates, and international collaborators along with some historical perspectives. His exceptional achievements have been recognised by all leading professional Statistics bodies of the world in terms of honouring him with Fellowship, Membership, and Gold Medals.

Keywords and phrases: Order statistics; nonparametric methods; normal, Student-t and elliptic distributions; asymptotic theory; preliminary test and shrinkage estimation; decision theory; quasi-empirical Bayes estimation; regression quantiles; meta analysis; time series and measurement error models.

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1 Introduction

Professor Saleh has made seminal contributions in diverse areas of Statistics namely, nonparametric statistics; order statistics; robust (L-, M-, and R-) estimation; preliminary test and Stein-type shrinkage estimation; empirical Bayes estimation; multivariate analysis; time series analysis; regression quantiles; decision theory, and in

the development of asymptotic theory in many of these areas. He has also worked and applied his proposed methods in various popular Statistical models such as simple, multiple and multivariate regression, one-way analysis of variance, multivariate analysis of variance, parallelism, time series, meta-analysis and measurement error models. He worked with the above models with independent and correlated normal errors, as well as non-normal errors, such as, the Student-t, and elliptically symmetric contoured errors/responses. His latest book, Saleh (2006), captures most of his work in the area of nonparametric and parametric methods for preliminary test and shrinkage estimations, and highlights his *quasi-empirical Bayes* (QEB) method of estimation for practitioner.

He is arguably the most successful PhD student of the Department of Statistical and Actuarial Sciences, University of Western Ontario, Canada. His outstanding research and professional contributions led to many leading Statisticians to nominate him for various international awards and honours including the prestigious Islamic Countries Society of Statistical Sciences (ISOSS) Gold Medal and the rank of Honorary Member of the Statistical Society of Canada.

Saleh has made outstanding contributions in diverse areas of Statistical research. The forthcoming Sections provide a comprehensive description of his significant achievements in some sort of chronological order along with the leading collaborators. Obviously this short paper does not include all his scientific and scholarly contributions, so the interested readers may refer to his curriculum vitae for a complete list of his publications.

A rough topic-wise account of Saleh's scientific contributions are provided in the forthcoming Sections.

2 Order Statistics

The first two notable scientific contributions of Professor Saleh are in the area of order statistics. His papers on *Asymptotic Optimum Quantiles for the Estimation of Parameters of Negative Exponential Distribution* and *Estimation of the Parameters of the Exponential Distribution Based on Optimum Order Statistics in Censored Samples* appeared in two different issues of a top Statistics journal, the *Annals of Mathematical Statistics*, in 1966. The first paper was a part of his Masters thesis and the second one was from his PhD dissertation. The first paper was the outcome of the problem posed by H L Harter in the above journal in 1961 challenging for a mathematical proof for the uniqueness of the optimal quantiles when estimating the

scale parameter of the exponential distribution based on a few quantiles, and the second paper was in response to a referee's comment on the first paper to deal with the problem of censored samples.

In the theory of estimation it is well known that when all the observations in a sample are available, it is sometimes possible to find estimators that are the most efficient linear combinations of a given number of order statistics. In many practical situations, people encounter censored samples, and it is desired to obtain linear estimators based on a few optimal order statistics from such a sample. The paper of Saleh (1966) concerns the determination of the optimal set of order statistics for a given integer k (where k is much less than the number of observations in the censored sample), in estimating the parameters of the exponential distribution in complete and censored samples. The study is based on the asymptotic theory and under Type II censoring scheme.

Another equally significant paper of Saleh is on *Estimating Quantiles of Exponential Distribution* which appeared in "Statistics and Related Topics", North Holland Publishers in 1981. This paper opened the door for the asymptotic theory of BLUE based on a few order statistics in estimating quantile functions. As a matter of fact, a collection entitled "Contributions to Order Statistics" by A K Md Ehsanes Saleh, Carleton University, contains 30 papers in a sequence of collaborative work with K M Hassanein, Mir M Ali, and D Umbach, based on the results of the first 3 papers of Saleh for large samples and the Annals paper of Kulldorff (1964) for finite samples.

Saleh (1967) introduced the idea of simplified estimation for 'middle censored samples' based on a few order statistics in a paper in Technometrics and another paper in the Naval Research Logistics with Mohammad Ahsanullah. Saleh (1972) supervised the Masters thesis of M A Rahim entitled "Estimation of the parameters of double exponential distribution based on k optimum order statistics from middle censored samples". Several papers were written on this topic. In 1975, he initiated the study of the exact moments of order statistics from truncated exponential distributions with Christine Scott and D B Junkins.

Saleh et al. (1983) produced a general paper on "Estimating the quantile function of a location-scale family of distributions based on a few order statistics". Finally Saleh et al. (1994) introduced the general theory on optimum quantiles for regression models based on "Regression Quantile" theory in an unpublished paper entitled "L-Estimation of the parameters in a linear model based on a few selected Regression Quantiles", and its application in a paper entitled "Joint estimation and test of hypothesis of regression and scale parameters in a simple linear model with Cauchy

errors based on a few Regression Quantiles” (1994) by Saleh, K M Hassanein and E F Brown.

3 Nonparametric methods

Professor Saleh embarked on a research program in the area of nonparametric statistics beginning with the paper entitled “Asymptotically Most Powerful Rank Tests of Regression Parameters from Grouped Data” in 1969 and initiated collaboration with M S Srivastava in their first paper “On a Class of Nonparametric Estimates of Regression Parameters” in 1970 while both were research Fellow at the Summer Research Institute at the Queens University, Kingston, Canada. He wrote a very significant paper entitled “Hodges-Lehmann Estimate of Location Parameter in Censored Samples” which is an extension of Hodges-Lehmann Estimate in censored samples and appeared in the Annals of the Institute of Statistical Mathematics in 1976. In subsequent years he worked on the problem of “Preliminary Test R-estimation of Regression Parameters” which is a broader extension of Han and Bancroft’s paper of 1968. His first seminal paper on this topic is “Nonparametric Estimation of Location Parameter after a Preliminary Test on Regression” which appeared in the Annals of Statistics in 1978. Immediately, there was an extension of this paper to “Nonparametric Estimation of Location Parameter after a Preliminary Test on Regression in the Multivariate Case” which appeared in the Journal of Multivariate Analysis in 1979. These two papers were the beginning of decades of collaboration with Pranab K Sen of North Carolina University.

The introduction of preliminary test estimation and test in nonparametric set up opened the door for rich and strong research, and these two papers were at the core of many further research. In 1978, Bradley Efron invited Saleh to Stanford University for the Summer and Fall terms. There he met Charles Stein and discussed the nonparametric approach to preliminary test estimation. He encouraged Saleh to pursue the nonparametric approach to Stein-type shrinkage estimation. There after Saleh produced another sequence of more than 40 significant research articles in collaboration with P K Sen on simple linear model, parallelism model, multivariate location model, general multivariate linear model of varied type.

His other contributions on preliminary test method includes the joint work with Sen in 1982 on Nonparametric test of location after a preliminary test on regression. The multivariate version of this problem was published in 1983. These two papers were the nonparametric base for further development.

Saleh and Sen proposed nonparametric shrinkage estimation in a parallelism problem in *Sankhya* (1985). Sen and Saleh (1985) proposed the shrinkage R-, and M-estimation of regression parameters in the *Annals of Statistics* (1985 & 1987) respectively. They studied shrinkage R-estimation in 1986, published in the *Communications in Statistics: Theory and Methods*, and shrinkage L-estimation in a 1987 paper in the journal of *Computational Statistics and Data Analysis*. In 1989 Saleh and T Shiraishi proposed some R- and M-estimators of regression parameters under certain restrictions. A robust version of Stein's estimator for multivariate location parameter was given by Jana Jureckova and Saleh (1990).

4 Decision Theory

Saleh was invited to the Pacific Area Statistical Conference, held in Tokyo in 1986. While in Japan, M Sugiura of the University of Tsukuba, invited him to visit his Department for 2 days. Sugiura introduced two bright fresh PhD scholars, T Kubokawa (Decision Theoretic Statistics) and T Shiraishi (Nonparametric Statistics) to Saleh and asked him to get them in Canada to train them abroad with collaborative publications. At that time Saleh was working with a problem related to Pitman nearness with respect to preliminary test estimators with Sen. He was attracted to the problem and the work of Kubokawa. Soon their first collaborative paper entitled "The Stien paradox in the sense of the Pitman closeness" (1989) was published in the *Annals of Statistics* with Sen. Shiraishi visited Saleh at the Carleton University in 1988 and collaborated on the paper entitled "On some R- and M-Estimators of regression parameters under uncertain restrictions" which was published in the *Journal of Japan Statistical Society* in 1989. The next paper on "Robust estimation for the parameters of multiple-design multivariate linear models under general restriction" was published in the journal of *Nonparametric Statistics* in 1993. This Japanese collaboration led to 11 collaborative research papers with T Kubokawa, T Shiraishi, S Makita, T Honda, Y Konno and M Ushijma. These papers are documented in a collection of papers entitled "Contributions to Decision Theoretic Statistics" (2005) by A K Md Ehsanes Saleh, Carleton University, Canada. In this collaboration, Christian Robert of Ceremade - Universit Paris-Dauphine got involved when he was a regular visitor of Saleh. His first paper with Saleh was the empirical Bayes solution to parallelism problem, and around the same time both Kubokawa and Robert collaborated on the empirical Bayes solution to Covariance matrix estimation under entropy loss, estimation of noncentrality parameters and robust estimation of

common regression coefficients under spherical symmetry.

The two significant papers on variance and covariance estimation by Saleh with Kubokawa are “Empirical Bayes Estimation of the covariance matrix of a normal distribution with unknown mean under entropy loss” in (1992) and “On improved positive Estimators of variance components” in (1993).

Saleh has supervised many PhD theses on parametric theory of preliminary test and shrinkage estimation. These include (i) “Estimation strategy under uncertain prior information” (1987) by Syed E Ahmed, (ii) “Interface of preliminary test approach and empirical Bayes approach to shrinkage estimation” (1990) by Abdunnabi M Ali, (iii) “Preliminary test approach estimation: Regression model with spherically symmetric errors” (1995) by S M T Tabatabaey, (iv) “Some Aspect of Statistical Inference of Logistic Regression parameters” (1996) by M A Matin, (v) “Preliminary test approach to shrinkage estimation of the parameters in a variety of statistical models” (1997) by Javed Chowdhury, (vi) “On some Contributions to Measurement Error Models” (2003) by H M Kim, and (vi) “Estimation of parameters in an elliptically contoured distributions” (2008) by Mohammed Arashi.

M S Srivastava and Saleh (2006) proposed *estimation of the mean vector of a multivariate normal distribution* under sub-space restriction.

Saleh has attracted many scholars to collaborate with him because of his solid grasp on the problem, and superb staistical/mathetical knowledge to complete the project with a successful outcome.

5 Time Series Models

Saleh was invited to the American Mathematical Society Conference (1989) held at the Ball State University, USA. He presented the freshly published paper on Stein estimation for AR(p)-model with Guassian error. Saleh’s research on time series model came to the focus on this presentation and Hira L Koul got interested in the shrinkage estimator. They wrote their first paper on R-estimation for AR(p) model when Saleh was invited by Koul in 1991, the paper was published in 1993. When Saleh was the Eugene Lukacs Professor at the Bowling Green University in 1992, he organized a conference on Regression Quantiles to understand the research progress in the problem. When Koul invited him again to the Michigan State University he suggested to study the regression quantile problem on AR(p) model.

Saleh’s on going collaboration with Koul produced the next fundamental paper on “Auto-Regressive Quantiles and Related rank score Process” (1995) published

in the *Annals of Statistics*. This paper has made basic impact on the time series problems. Included in the series is the PhD thesis entitled “Some contribution to L-estimation in Linear Regression and auto-regression models (1995) by S Alimoradi. This thesis produced two strong papers which appeared in the *Journal of Statistical Research* and in the “*Handbook of Statistics*” edited by N Balakrishnan and published by North Holland.

Saleh was invited to the 7th European Statistics Meeting in 1974 held in the Prague. He presented the paper entitled “Behrens-Fisher Problem for the Exponential Distribution” which was a deviation from the commonly used Normal theory. He was mostly interested to talk to Professor J Hajek but he died few days before the meeting. Later in 1983, Jana Jureckova invited Saleh to the Meeting of European Statisticians in Palermo, Sicily. In the following year, Jureckova visited Carleton to collaborate on regression quantiles/Stein estimation with Saleh. This visit and several subsequent visits, resulted in several papers on the robustified Stein-type estimation, regression quantiles and improved L-estimation for the linear models in the later years.

6 Improved estimation - uncertain non-sample prior information

Along side developing improved estimators under the nonparametric framework, Saleh contributed significantly in the area for many different parametric models. His discoveries confirmed that the preliminary test and shrinkage estimators are performing better than the classical estimators under conditions such as quadratic risk. Saleh published a large number of articles in this area on a varieties of models with different types of error/response distributions.

Mohammad Ahsanullah and Saleh (1972) proposed preliminary test estimator for the intercept after testing on the slope of a simple regression model. V K Rohatgi and Saleh (1988) provided estimators of the parameters for the location-scale family of distributions in complete and Type II censored samples.

Stein-type shrinkage estimator for a multivariate mean vector was discussed in Ahmed and Saleh (1988) for the two-sample problem. In 1990 the authors considered the improved estimation for the multivariate normal model. Improved estimation in the contingency table was proposed by Saleh, Sen and Gupta (1989) in the *Journal of American Statistical Association*.

When B M G Kibria was a graduate student at the Carleton University, Saleh

introduced him to Ridge Regression and produced the joint paper entitled “Performance of some new preliminary test ridge regression estimators and their properties”(1993). Since then they published a series of collaborative papers in this area.

Saleh (1992) proposed shrinkage estimation of the parameters of an autoregressive Gaussian process. Kubokawa, Robert and Saleh (1993) suggested a method for the estimation of non-centrality parameters. Saleh (1999) proposed estimation of regression coefficients in exponential regression model with censored observations. S Khan and Saleh (2002) discussed the pre-test and Stein-type shrinkage estimation of the mean for the univariate normal model. C Robert and Saleh proposed re-centered confidence sets in 2002.

Saleh in his (2002) paper with Zahirul Hoque and S Khan proposed improved estimators for the slope parameter of liner regression model in the presence of uncertain non-sample prior information published in the Journal of Statistical Research. Their work on the estimation of intercept parameter appeared in the Statistical Papers in 2005. Bashir U Khan and Saleh (2006) published a paper on *Estimation of regression parameters: A two-sample problem*. They suggested improved estimation for the parallelism problem in 2007 in the Journal of Applied Probability and Statistics.

7 Improved estimation for non-normal models

The customary use of the normal model for the errors/responses is under serious question even when the population distribution is symmetric but have heavier tails than the normal distribution. Also, the normal model fails to incorporate dependent but uncorrelated responses. In such cases the multivariate Student-t distribution provides an appropriate model for the population. Elliptically symmetric contoured distribution is a more general class of distributions that encompass normal and Student-t distributions as special cases. Saleh conducted extensive research using the Student-t, elliptically symmetric contoured and other non-normal distributions.

Saleh was an external expert examiner of the PhD thesis of Shahjahan Khan from the University of Western Ontario, Canada, on “Prediction distributions of multilinear models with multivariate Student-t errors” in 1992. Saleh became interested to conduct research in the area of improved estimation under the Student-t models. In his first visit to Australia, Saleh visited the University of Southern Queensland for two weeks in 1994 to initiate joint work with S Khan. Their first joint paper on “Preliminary test estimators of the mean for sampling from some Student-t populations”

was published in 1995 in the Journal of Statistical Research. S Khan was invited to visit Saleh at Carleton University in 1996 and 2006 to continue the collaboration. They have produced 10 papers on various preliminary test and Stein-type shrinkage estimation for a varieties of models including the simple, multiple and multivariate regression models and parallelism model. Two of S Khan's PhD students, Z Hoque and R M Yunus are also working in this area with inspiration and guidance from Saleh.

Kubokawa, Robert and Saleh (1991) published a paper on *Robust estimation of common regression coefficients under spherically symmetry* in the Annals of Institute of Statistical Mathematics. Improved estimation on maximum likelihood estimator of coefficient matrix in a growth curve model was proposed in Kubokawa, Morita and Saleh (1992). Another paper on robust estimation for the parameter of multiple-design multivariate linear model under general restriction was published in 1993 with T Shiraishi in the Journal of Nonparametric Statistics.

In 1995 Saleh published a paper on *Preliminary test estimators of the mean for sampling from some Student-t populations* with S Khan. Their second publication in the area on *Shrinkage pre-test estimator of the intercept parameter for a regression model with multivariate Student-t errors* appeared in the Biometrical Journal in 1997. Khan and Saleh (1998) compared the performances of estimators of means based on p samples from multivariate Student-t populations.

A joint paper of Saleh with Baki Billah on *Conflict between pre-test estimators induced by three large sample tests under a regression model with Student-t error* appeared in The Statistician in 1999. Kibria and Saleh (2004) discussed the same problem under the ridge regression scheme.

Mohammed Arashi, S M M Tabatabaey and Saleh proposed improved estimators of parameters of parallelism model with elliptically distributed errors in *Metrika* in 2008.

8 Test of hypothesis

Saleh's first work on test of hypothesis was published in 1967 on *Asymptotic most powerful rank test of regression parameters from grouped data* in the Journal of Statistical Research. J P Dionne and Saleh (1977) provided a generalization of the Savage test.

Testing after pre-test: Testing of a parameter of a model is a common task in statistical inference. Like, the preliminary test estimation, improved tests can

be defined for the test of a parameter in the presence of uncertain non-sample prior information on the value of another parameter. Combining both the sample data and the non-sample prior information, one may define preliminary test based improved test. The power function as well as the size and power of the improved test are derived and analysed. Performance of the proposed test is compared with the commonly used test based on the sample data alone.

Saleh worked in this area under the nonparametric set up. References include Sen and Saleh (1982) for simple regression model and Saleh and Sen (1983) for multivariate regression model. They also provided the asymptotic properties of the test statistic for testing after pretest in the same year. In 1984 they proposed test of homogeneity of the intercepts after preliminary test on the slopes for the parallelism model. Saleh and Sen (1985) developed the asymptotic theory of testing hypothesis for location and scale parameters based on a few selected order statistics in the paper “Asymptotic relative efficiency of some joint-test for location and scale parameters based on a few order statistics”.

Saleh, Ali and Umbach (1983) used selected quantiles for tests of significance. The proposed tests of significance for the exponential distribution based on selected quantiles in 1985. In 1984 Saleh proposed locally most powerful rank test for two sample problem with M L Puri.

9 Quasi-Empirical Bayes estimation

In his book *Theory of Preliminary Test and Stein-Type Estimation with Applications* (Wiley 2006) Saleh coins the term *quasi-empirical Bayes* (QEB) estimation method for the preliminary test and Stein-type shrinkage estimation. On page 157 of the book he describes the QEB method and argues that both the methods produce the same estimators.

Saleh initiated the empirical Bayes estimation with Malay Ghosh of Florida University, and P K Sen in 1989 with their paper on *Empirical Bayes subset estimation in regression models* in *Statistics and Decision*. He employed the same method with Abdunanbi Ali in 1991 for regression model in the journal of Japan Statistical Society. In the same year his paper with C Robert appeared in the *Annales D’Economie et de Statistique* on point and interval estimations in parallelism model.

In a 2006 paper Saleh with K S Hassanein, R Hassanein, and Kim applied the QEB methodology for improving meta-analysis in the *Journal of Biopharmaceutical Statistics*. In 2007 he has published another paper in this area on *Quasi-Empirical*

Bayes Methods of estimation in ARMA(p,q) models with vague prior information on MA(q) with M Ould Haye.

10 Other works

Saleh worked in other areas of Statistics publishing a large number of high quality papers. M Rahman and Saleh provided an *explicit form of the distribution of Behrens-Fisher statistic* in 1975 in the Journal of Royal Statistical Society.

While visiting Bowling Green State University in 1992-93, Saleh collaborated with Arjun K Gupta on Categorical Data. As a result two collaborative papers of importance were published, namely, “Improved Estimation in a contingency Table: Independence structure” (1989) in the Journal of the American Statistical Association. and “Estimation of Multinomial Probabilities under a model constraint” (1996) in the Journal of Multivariate Analysis.

Meta-analysis: This is a statistical method to combine data from several independent studies conducted using randomised control trails for making inferences. Analyses are done for relative risks and odd ratios for binary data, and weighted mean difference for ratio/scale data. Saleh published a paper on “Quasi-empirical Bayes Methodology for Improving Meta-Analysis” with K M Hassanein, R Hassanein and H M Kim in the Journal of Biopharmaceutical Statistics in 2006.

Professor Saleh wrote scientific and scholarly articles on eminent Statisticians such as Q M Hossain in 1993, C R Rao in 1995, D A S Fraser in 1996 (jointly with M S Haq), I P Fellegi in 1997, and B Efron in 1999.

Recently Saleh has published several papers in the area of measurement error models. H M Kim and Saleh proposed *preliminary test prediction of population total under regression model with measurement error* in 2002. They published a second paper on preliminary test estimator of the parameters of simple regression model with measurement error in *Metrika* in 2003. Their paper on improved estimation for regression model with measurement error appeared in the Journal of Multivariate Analysis in 2005.

Saleh also worked on the area of uncertain non-sample prior information using *coefficient of distrust* to reflect the level of non-confidence on the prior information. He also suggested the *maximin* method to determine an optimal level of significance for the preliminary test estimator to guarantee minimization of maximum risk.

Very lately Saleh has started a research project on “Nonparametric methods of test of hypothesis and estimation for the parameters of Measurement error models”

with J Jureckova.

11 Concluding Remarks

The seminal and outstanding contributions of Saleh in the statistical research have been well recorded in his various research publications in details. His remarkable scholarship in the subject has been reflected in his mentoring role for many post-doctoral fellows, and brilliant supervision of 12 PhD students and many other post-graduate students. He is still very active in joint research with some of the leading Statisticians of the world as well as his former students, and is expected to continue producing more superior quality results in the years to come. Here we have included only a subset of his research papers to highlight the major research contributions of Saleh. As such, it does not include all the papers that he has produced/published, nor does it list all the papers that his students/collaborators have published with him as coauthors. For a partial list of Professor Saleh's publications readers may refer to his updated curriculum vitae.

I feel deeply honoured for being invited to write this scholarly article reflecting the landmark achievements of a Statistical giant of our time. I believe this paper will continue to serve as a summary and historical record of the superb accomplishments of Saleh for all interested research workers in various fields of Statistics and related disciplines in the years to come.

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List of Publications of Professor A K Md Ehsanes Saleh