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MESHLESS RADIAL BASIS FUNCTION METHOD FOR UNSTEADY INCOMPRESSIBLE VISCOUS FLOWS

A dissertation submitted by

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Dedication

To the memory of my grandparents & my father,

and to my mother, my wife & sons.

Notes to Readers

To facilitate the reading of this thesis, a number of files are included on the attached CD to provide colour presentation as well as animation of some numerical results in this thesis. The contents of the CD include:

1. thesis.pdf: An electronic version of this thesis with colour figures;
2. chap4-test4-1b.avi: An animation showing the numerical simulation of a bubble that moves and deforms in a shear flow (Test 4, Chapter 4);
3. chap4-test5-4b.avi: An animation showing the numerical simulation of four bubbles that move, deform and merge together in a shear flow (Test 5, Chapter 4);
4. chap5-test3-vc.avi: An animation showing the evolution of the velocity along the mid-vertical and horizontal lines in a lid-driven cavity flow (Test 3, Chapter 5);
5. chap5-test3-stream.avi: An animation showing the evolution of the streamfunction in a lid-driven cavity flow (Test 3, Chapter 5);
6. chap5-test3-vort.avi: An animation showing the evolution of the vorticity in a lid-driven cavity flow (Test 3, Chapter 5);
7. chap6-test1-br.avi: An animation showing the numerical simulation of two bubbles rising upward in an interfacial flow (Test 1, Chapter 6).

Abstract

This thesis reports the development of new meshless schemes for solving time-dependent partial differential equations (PDEs) and for the numerical simulation of some typical unsteady incompressible viscous flows.

The new numerical schemes are based on the Indirect/Integrated Radial Basis Function Network (IRBFN) method which is fully meshless as no element-type mesh is required. The IRBFN method has been successfully applied to solve time-independent elliptic PDEs, some steady fluid flows and recently unsteady Navier-Stokes equations in streamfunction-vorticiy formulation using simple time integration methods (e.g. first-order backward Euler method). The main objective of the present research is to devise and implement meshless numerical schemes for unsteady problems in computational fluid dynamics where not only the accuracy but also the efficiency and stability of the numerical schemes are of primary concerns. In addition, the effects of different parameters of the IRBFN method on the accuracy, stability and efficiency of the proposed numerical schemes are extensively studied in this research.

As the first step in extending the IRBFN method to various types of time-dependent PDEs, two numerical schemes combining the IRBFN method with high-order time stepping algorithms are developed for solving parabolic, hyperbolic, and advection-diffusion equations. Sensitivity analysis of the method to point density, time-step size and shape parameter are extensively performed to study the influence of these parameters to the overall accuracy of the method.

A further extension of the IRBFN method for incompressible fluid flows with moving interfaces, especially for passive transport problems is accomplished in this research with a novel meshless approach in which the level set method is coupled with the the IRBFN method for capturing moving interfaces in an ambient fluid flow without any explicit computation of the actual front location.

Another contribution of this research is the development of two new meshless schemes based on the IRBFN method for the numerical simulation of unsteady incompressible viscous flows governed by the Navier-Stokes equations. In the new schemes, the splitting approach is used to deal with the momentum equation and the incompressibility constraint in a segregated manner. Numerical experiments on the new schemes in terms of accuracy and stability are performed for verification purposes.

Finally, a novel meshless hybrid scheme is developed in this research to numerically simulate interfacial flows in which the motion and deformation of the interface between the two immiscible fluids are fully captured. Unlike the passive transport problems mentioned above where the influence of the moving interface on the surrounding fluid is ignored, the interfacial flows are studied here with the surface tension taken into account. As a result, a two-way interaction between the moving interface and the ambient flow is fully investigated.

All numerical schemes developed in this research are verified through a wide range of transient problems including different kinds of time-dependent PDEs, typical passive transport problems and interfacial flows as well as unsteady incompressible viscous flows governed by Navier-Stokes equations.

Certification of Dissertation

I certify that the idea, experimental work, results and analyses, software and conclusions reported in this dissertation are entirely my own effort, except where otherwise acknowledged. I also certify that the work is original and has not been previously submitted for any other award.

Lan Mai-Cao, Candidate	Date

ENDORSEMENT

Prof. Thanh Tran-Cong, Principal supervisor	Date
Dr. Ruth Mossad, Associate supervisor	Date

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Papers Resulting from the Research

Book Chapters

1. N. MAI-DUY, L. MAI-CAO and T. TRAN-CONG (2006) "A new meshless RBF-based method for unsteady fluid flow analysis", Advances in Meshless Methods, Chapter XI, J. Sladek and V. Sladek (Eds), Tech Science Press, ISBN: 0-9717880-2-2, pp241-262.

Journal Papers

1. L.MAI-CAO and T.TRAN-CONG (2008) "A Meshless Approach to Capturing Moving Interfaces in Passive Transport Problems", CMES: Computer Modeling in Engineering and Sciences, 31(3), 157-188.

2. N. MAI-DUY, L. MAI-CAO and T. TRAN-CONG (2007) "Computation of transient viscous flows using indirect radial basis function networks". CMES: Computer Modeling in Engineering and Sciences, 18(1), 59-77.

3. L. MAI-CAO and T. TRAN-CONG (2005) "A meshless IRBFN-based method for transient problems", CMES: Computer Modeling in Engineering and Sciences, 7(2), 149-171.

4. L. MAI-CAO and T. TRAN-CONG (2008) "Meshless projection schemes for unsteady incompressible Navier-Stokes equations", in preparation.

5. L. MAI-CAO and T. TRAN-CONG (2008) "Meshless approach to interfacial flows", in preparation.

Conference Papers

1. L. MAI-CAO and T. TRAN-CONG (2006) "A meshless level-set scheme for interfacial flows", in Nguyen Quoc Son and Nguyen Dung (Eds), International Conference on Non-linear Analysis & Engineering Mechanics Today, Ho Chi Minh City, Vietnam, 11-14 December 2006, CD paper No 26.

2. L. MAI-CAO and T. TRAN-CONG (2005) "An IRBFN-based scheme for moving interfaces in an incompressible viscous flow", Keynote Lecture, in "Advances in Computational & Experimental Engineering and Sciences", SM Sivakumar, A Meher Prasad, B Dattaguru, S Narayanan, AM Rajendrean, SN Atluri (Eds), pp473-479.

3. L. MAI-CAO and T. TRAN-CONG (2004) "Element-Free Simulation for Non-Newtonian Flows", Keynote Lecture in Advances in Computational and Experimental Engineering and Sciences, S.N. Atluri and A.J.B. Tadeu (Ed), Tech Science Press, pp1332-1338.

4. L. MAI-CAO and T. TRAN-CONG (2003) "A meshless level set approach to interface capturing", Keynote Lecture, International Conference on Computational & Experimental Engineering and Sciences, July 24-29, Corfu, Greece, CD Chapter 17.

5. L. MAI-CAO and T. TRAN-CONG (2003) "Solving time-dependent PDEs with a meshless IRBFN-based method", Invited paper, International Workshop on Meshfree Methods 2003, July 21-23, Lisbon, Portugal, pp119-124.

Contents

Acronyms & Abbreviations

AB-CN	Adams Bashforth-Crank Nicolson
BDF	Backward Differentiation Formula
BEM	Boundary Element Method
DRBFN	Direct Radial Basis Function Network
FDM	Finite Difference Method
FEM	Finite Element Method
FVM	Finite Volume Method
GMRES	Generalized Minimal Residual Method
IPC	Incremental Pressure Correction
IPCPP	Incremental Pressure Correction with Pressure Prediction
IRBFN	Indirect/Integrated Radial Basis Function Network
MQ	MultiQuadric
NSE	Navier-Stokes Equation
PDE	Partial Differential Equation
RBF	Radial Basis Function
RK	Runge-Kutta
SL	Semi-Lagrangian
SVD	Singular Value Decomposition
TCN	Taylor-Crank-Nicolson
TE	Taylor-Euler
TPS	Thin Plate Splines

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