Mode-Matching Analysis of a Shielded Rectangular Dielectric-Rod Waveguide

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Abstract—Rectangular cross-sectional dielectric waveguides are widely used at millimeter wavelengths. In addition, shielded dielectric resonators having a square cross section are often used as filter elements; however, there is almost no information available on the effect of the shield. Rectangular or square dielectric waveguide is notoriously difficult to analyze because of the singular behavior of the fields at the corners. Most published analyses are for materials with a low dielectric constant, and do not include the effects of a shield. This paper describes a numerically efficient mode-matching method for the analysis of shielded dielectric-rod waveguide, which is applicable to both low and high dielectric-constant materials. The effect of the shield on the propagation behavior is studied. The shield dimensions may be selected such that the shield has a negligible effect so that results can be compared with free-space data. The results are verified by comparison with several sets of published data, and have been confirmed by measurement for a nominal $\varepsilon_r$ of 37.4.

Index Terms—Dielectric resonators, dielectric waveguides, electromagnetic propagation in nonhomogeneous media, mode-matching methods, shielding.

I. INTRODUCTION

Dielectric waveguides are an attractive alternative to metal waveguides at millimeter-wave frequencies due to their lower propagation loss, lower cost, and easier fabrication [1]. Rectangular dielectric waveguides form a large proportion of these and have uses in integrated optics and millimeter-wave integrated circuits and transmission lines. However, there has always been difficulty obtaining accurate propagation coefficients for the various modes on these structures. There is no closed-form solution to the problem [1] and the methods used either rely on approximations, as in the procedure originated by Marcatili [2] and improved by Knox and Toulios [3], or are numerical in nature. The main numerical techniques range from the circular harmonic analysis of Goell [4], finite-element [5] and finite-difference [6] procedures, to mode matching.

Mode-matching methods have been applied to the dielectric image line by Solbach and Wolff [7], and to the homogeneous inverted strip guide by Mittra et al. [8]. The latter used a similar procedure to [7], with the mode-matching techniques developed by Mittra and Lee [9]. In a very comprehensive paper, Strube and Arndt [10] have applied the method of Solbach and Wolff to the shielded dielectric image line. The first part of their paper used this procedure, together with the inclusion of an extra electric wall, to analyze propagation on infinite shielded image guide. As well as propagating and evanescent modes, complex modes and backward waves were identified and thoroughly investigated. Complex modes can only exist in pairs having complex conjugate propagation coefficients, and couple so that the total power flow is always reactive. A backward wave is one in which the power flows in the opposite direction to the wavefronts. The second part of their paper finds the scattering matrix of a transition from shielded dielectric image guide to rectangular waveguide, by matching the tangential fields at the interface. A comparison of measured and calculated $S_{11}$ results were used to verify the method. The results obtained by Strube and Arndt correspond to those modes that can exist in the dielectric-rod waveguide shown in Fig. 1 when the $x$-axis is an electric wall. However, these do not include some of the dominant modes, for which the $x$-axis is a magnetic wall. To obtain the full set of modes for this waveguide, it is necessary to consider all four types of symmetry.

An analysis of complex and backward waves in an inhomogeneously filled waveguide has been carried out by Omar and Schunemann [11]. A method to predict the presence of complex modes in inhomogeneous lossless dielectric waveguide can be found in [12].

An alternative mode-matching (boundary element) method for the shielded dielectric-rod waveguide, incorporating dyadic Green’s functions, was developed by Collin [13] and Collin and Kisienki [14].

A problem with numerical solutions is that they can suffer from slow convergence due to divergence of the electric field at the corners of the dielectric where the refractive index changes abruptly [15]. However, for most purposes, sufficient accuracy can still be obtained for a relatively small number of basis functions.

In a typical situation, the permittivity of the dielectric $\varepsilon_{r2}$ will be higher than the surrounding medium $\varepsilon_{r1}$ (usually air) so that the electromagnetic fields will be concentrated in the dielectric line, and the proportion outside it will decay away exponentially.
The ability of a high-permittivity material to contain and concentrate the fields, together with the availability of high-Q temperature stable materials, has led to the development of the dielectric resonator as a filter element. In filter applications, the dielectric resonators are often enclosed in metallic shields or cavities to prevent unwanted coupling, as shown in Fig. 1.

Cavity filters incorporating dielectric resonators are widely used in mobile base stations and other demanding applications. Traditionally, many of these have used cylindrical resonators. Designers have sought to reduce the size of these filters by using multiple mode cavities. This has led to increased interest in resonators that have a square cross section, and also in cubical resonators. Dielectric filter cavities may be analyzed using the methods developed by Zaki and Atia. The propagation characteristics of an infinite cylindrical waveguide containing a dielectric rod were first established. A cylindrical cavity was then modeled as a length of this guiding structure, terminated in short lengths of empty waveguide [16]. In a later paper, this was extended to cylindrical dielectric resonators in rectangular waveguide and cavities [17]. This paper represents the first step in a similar study of the shielded square section dielectric resonator.

II. ANALYSIS USING THE MODE-MATCHING METHOD

An advantage of a mode-matching method is that it has relatively good processing speed due to its semianalytical nature. It also allows visualization of the fields in the structure by solving for the unknown coefficients of the basis function equations. Another advantage is that it can be used with reasonably high values of permittivity [10] show results as high as $\varepsilon_r = 50$). The other numerical methods cited, with the exception of the finite-difference method of Schweig and Bridges [6], have only been applied to relatively low values ($\varepsilon_r \approx 2.5$).

Due to the symmetrical nature of the shielded dielectric waveguide, only one-quarter of the structure needs to be analyzed. Fig. 2 shows how the cross section is divided into three regions. Regions $I$ and $II_1$ surround the dielectric rod and are filled with a medium of permittivity $\varepsilon_{r1}$, which, in this paper, will be considered to be air ($\varepsilon_{r1} = 1$). Region $II_2$ is the dielectric rod with permittivity $\varepsilon_{r2}$. The outer shield will be considered as a perfect electric conductor (electric wall). The bottom and left-hand-side (LHS) symmetry planes, coincident with the $x$- and $y$-axes, may be either electric or magnetic walls. The selection of wall types will determine the types of symmetry that can exist in the structure.

In this paper, as in [8] and [10], a modification of the mode-matching method of Solbach and Wolff [7] will be used so that the effect of the proximity of the shield to the dielectric can be ascertained. However, to provide calculation of all modes possible in this structure, additional basis functions to cater for the full range of symmetries (see Section II-A) have had to be provided. This variation will be called the modified Solbach and Wolff (MSW) method through the remainder of this paper.

A. Basis Functions

The modes that can propagate in a shielded rectangular dielectric-rod waveguide are all hybrid modes, i.e., they always have field variation along either the horizontal or vertical dielectric-rod boundaries and, thus, have both electric- and magnetic-field components in the longitudinal direction [18]. In each of the regions in Fig. 2, the field patterns for these modes can be built up from superpositions of appropriate basis functions, which are transverse magnetic or transverse electric with respect to the $y$-direction. These will be designated TM$^0$ (electric) and TE$^0$ (magnetic), respectively, and are indicated by subscripts $e$ and $h$. The cross section has two axes of symmetry, which means there are four possible symmetries. In this paper, these will be classified according to the behavior of the $H_z$-field component, following Schweig and Bridges [6]. For example, superscript $eo$ will indicate that $H_z$ is an even function of $x$ and an odd function of $y$.

It is most efficient to derive the basis function fields from vector potentials. From Balanis [19], the magnetic vector potential for a TM$^0$ wave propagating in the $z$-direction in a nonmagnetic region with rectangular boundaries is of the form

$$A_y(x,y,z) = \begin{bmatrix} C_1 \cos(\beta_3x) + D_1 \sin(\beta_3x) \\ C_2 \cos(\beta_3y) + D_2 \sin(\beta_3y) \end{bmatrix} A_3 e^{-j\gamma z}. \quad (1)$$

The longitudinal-field components can then be obtained from

$$E_z(x,y,z) = -j \frac{1}{\omega \mu_0 \varepsilon} \frac{\partial A_y}{\partial y} \quad (2)$$
$$H_z(x,y,z) = \frac{1}{\mu_0} \frac{\partial A_y}{\partial x}. \quad (3)$$

From these expressions, it can be seen that $E_z$ and $H_z$ will have opposite types of symmetry. The longitudinal electric field will be as follows:

$$E_z(x,y,z) = j \frac{\beta_3 \gamma}{\omega \mu_0 \varepsilon} \begin{bmatrix} C_1 \cos(\beta_3x) + D_1 \sin(\beta_3x) \\ C_2 \cos(\beta_3y) + D_2 \sin(\beta_3y) \end{bmatrix} A_3 e^{-j\gamma z}. \quad (4)$$

Using the boundary conditions imposed by the shield, appropriate equations for $E_z$ can be selected in each region and for
each symmetry. For the case of $H_z$, an even function of $x$ and odd function of $y$ (or $z$), these are as follows:

$$E_{zne}^{(I)\alpha} = j \frac{A_{zne}^{(I)\alpha}}{\omega H_o E_o} \sin \left( \beta_{zne}^{(I)\alpha} (a_2 - x) \right) \cos \left( \beta_{yne}^{(I)\alpha} y \right) e^{-y z}$$

$$E_{zne}^{(II)\alpha} = j \frac{B_{zne}^{(II)\alpha}}{\omega H_o E_o} \sin \left( \beta_{zne}^{(II)\alpha} x \right) \times \sin \left( \beta_{yne}^{(II)\alpha} (b_2 - y) \right) e^{-y z}$$

and from this

$$\frac{B_{zne}^{(II)\alpha}}{B_{zne}^{(I)\alpha}} = \frac{\beta_{yne}^{(II)\alpha} \cos \left( \beta_{yne}^{(II)\alpha} b_2 \right)}{\beta_{yne}^{(I)\alpha} \sin \left( \beta_{yne}^{(II)\alpha} (b_2 - b_1) \right)}$$

and also

$$H_{zne}^{(II)\alpha} = H_{zne}^{(I)\alpha}.$$  

Substituting (10) into (11) gives the transcendental equation

$$\frac{1}{\beta_{yne}^{(II)\alpha}} \cot \left( \beta_{yne}^{(II)\alpha} (b_2 - b_1) \right) = \frac{\varepsilon_{r2}}{\beta_{yne}^{(I)\alpha}} \tan \left( \beta_{yne}^{(II)\alpha} b_1 \right)$$

The wavenumbers can then be obtained from numerical solutions of (12) after substitution of the relation

$$\beta_{yne}^{(II)\alpha} \beta_{yne}^{(II)\alpha} = \beta_{yne}^{(I)\alpha} \beta_{yne}^{(I)\alpha} + \frac{\varepsilon_{r2} (\varepsilon_{r2} - 1)}{\beta_{yne}^{(I)\alpha} \beta_{yne}^{(I)\alpha}}$$

which is derived using $\beta_{zne}^{(II)\alpha} = \beta_{zne}^{(II)\alpha}$ and the region II equations of (6).

Similarly, for the $TE_y$ modes,

$$\frac{B_{yih}^{(I)\alpha}}{B_{yih}^{(II)\alpha}} = \frac{\cos \left( \beta_{yih}^{(II)\alpha} b_1 \right)}{\varepsilon_{r2} \sin \left( \beta_{yih}^{(II)\alpha} (b_2 - b_1) \right)}$$

and the transcendental equation for the wavenumbers becomes

$$\beta_{yih}^{(II)\alpha} \cot \left( \beta_{yih}^{(II)\alpha} (b_2 - b_1) \right) = \beta_{yih}^{(I)\alpha} \tan \left( \beta_{yih}^{(II)\alpha} b_1 \right).$$

The equations for the other symmetries of the $TM_y$ and $TE_y$ basis functions can be similarly derived.

**B. Mode Matching at the Boundary Between Regions**

The parallel fields $E_T$ and $H_T$ must be continuous at the mode-matching boundary between regions I and II ($x = a_1$, $0 \leq y \leq b_2$). In the case of the electric field, this leads to the equations

$$\sum_{p=1}^{\infty} A_{ph}^{(I)\alpha} E_{Tph}^{(I)\alpha} + \sum_{p=1}^{\infty} A_{ph}^{(I)\alpha} E_{Tpe}^{(I)\alpha} = \sum_{n=1}^{\infty} B_{nh}^{(II)\alpha} \left( K_{nh} E_{Tnh}^{(II)\alpha} + E_{Tnh}^{(II)\alpha} \right)$$

$$+ \sum_{n=1}^{\infty} B_{neh}^{(II)\alpha} \left( K_{neh} E_{Tneh}^{(II)\alpha} + E_{Tneh}^{(II)\alpha} \right).$$

For continuity of the magnetic fields,

$$\sum_{p=1}^{\infty} A_{ph}^{(I)\alpha} H_{Tph}^{(I)\alpha} + \sum_{p=1}^{\infty} A_{ph}^{(I)\alpha} H_{Tpe}^{(I)\alpha} = \sum_{n=1}^{\infty} B_{nh}^{(II)\alpha} \left( K_{nh} H_{Tnh}^{(II)\alpha} + H_{Tnh}^{(II)\alpha} \right)$$

$$+ \sum_{n=1}^{\infty} B_{neh}^{(II)\alpha} \left( K_{neh} H_{Tneh}^{(II)\alpha} + H_{Tneh}^{(II)\alpha} \right).$$
where \( K \) is the right-hand side (RHS) of (10) and (14) for the TM\( ^{\omega} \) and TE\( ^{\omega} \) modes, respectively, and is used to reduce the number of unknown coefficients. The above pair constitute a doubly infinite set of linear equations for the modal coefficients \( A_p \) and \( B_r \). To simplify these equations, and to expand their number to equal the number of unknowns, the electric and magnetic fields in region I were used as testing functions. Only the \( y \)-dependent factors are required, and these have been designated \( e_{q}^{(I)}(y) \) and \( h_{q}^{(I)}(y) \), respectively. The following orthogonality relations are required:

\[
\begin{align*}
\int_{0}^{b_{y}} E_{Tm}^{(I)} E_{q}^{(I)}(y)dy &= 0 \\
\int_{0}^{b_{y}} H_{Tm}^{(I)} e_{q}^{(I)}(y)dy &= 0, \quad \text{for } m \neq q 
\end{align*}
\]

(18)

where \( m \) and \( q \) are the indices used to find wavenumbers \( \beta_{y/m}^{(I)} \) or \( \beta_{y/q}^{(I)} \) [as in (6)] for each mode number \( p \) in region I. That is, (16) and (17) are multiplied by TM\( ^{p} \) or TE\( ^{p} \) testing functions \( h_{q}^{(I)}(y) \) or \( e_{q}^{(I)}(y) \) from region I, respectively, and integrated over the interval \( 0 \leq y \leq b_{y} \) at \( x = a_{y} \), as per Mittra et al. [8]. The infinite set of equations so formed is reduced by truncating the number of basis functions used to a value that can be practically computed and will give a desired accuracy in the solution. The maximum values of the mode indices \( p \) and \( n \) are \( P \) and \( N \), respectively. An equal number of basis functions were used (\( P = N \)) in both regions I and II to alleviate any problems with relative convergence [20], [21]. In matrix form, the equations using the electric field and odd \((y)\) symmetry are for this odd\((y)\) case, the subscripts \( m \) and \( q \) are odd integers only and are equivalent in number to the number of basis functions used \( N \). For the even\((y)\) case, there will be a total of \( N \) even integers (including zero).

In abbreviated form, the matrix equations can be written as

\[
[W][A] = [X][B].
\]

(20)

The magnetic field equations are similar, and in abbreviated form are

\[
[Y][A] = [Z][B].
\]

(21)

C. Propagation Coefficient and Unknown Mode Coefficients of the Structure

A homogeneous system of equations may be formed from (20) and (21) as follows:

\[
\begin{bmatrix}
W & -X \\
Y & -Z
\end{bmatrix}
\begin{bmatrix}
A_T^E \\
A_T^M \\
A_P^E \\
A_P^M
\end{bmatrix} = 0.
\]

(22)

The eigenvalues of (22) are the propagation coefficients of the modes of the structure. These modes can be propagating, evanescent, complex, or backward wave types and are found by substituting a range of propagation coefficient values into the equations and finding those values for which the determinant is zero.

To determine that the propagation coefficients found are physically sensible, and also to find the type of mode each represents, it is essential to calculate the unknown coefficients and plot the field patterns. A selected coefficient is chosen as unity or some appropriate factor. In this paper, the coefficient chosen is that of the first TM mode in region II \( B_{N1}^{TM} \) and, thus, the associated matrix element values are \( b_{N1}^{TM} \) (electric) to \( b_{N1}^{TM} \) (magnetic), as shown. Consequently, the \( B \) coefficients are reduced by one to \( B_r \), and the \( X \) and \( Z \) matrices are reduced by a column to \( X_r \) and \( Z_r \). Hence, (22) can be written as

\[
\begin{bmatrix}
A \\
B_r
\end{bmatrix} =
\begin{bmatrix}
W & -X_r \\
Y & -Z_r
\end{bmatrix}
^{-1}
\begin{bmatrix}
b_{N1}^{TM} \\
\vdots \\
b_{N1}^{TM}
\end{bmatrix}.
\]

(23)

The system of (23) has more equations than unknowns (i.e., overdetermined), but can still be solved for the normalized values of the unknown coefficients by the use of the MATLAB operator "\( \backslash \)." This function gives a least squares solution for these truncated equations and, thus, produces a best fit result [22].

It was found that there is a limit to the number modes that can be used in the system of equations before ill conditioning occurs in the form of rank deficiency (MATLAB gives a warning if the matrix is rank deficient). This is due to the very large numbers that occur when trigonometric functions are evaluated for imaginary arguments. In effect, the computer runs out of
sufficient decimal places to accurately cover the range of the size of numbers in the equation matrix. However, the number of modes required for sufficient accuracy is well below this limit.

Once the coefficients are found, they can then be substituted into the field equations so that the field components can be determined from the sum of the basis functions at a number of spatial grid points, and the resultant field in the structure can be plotted as a superposition of all the components.

III. DISCUSSION AND COMPARISON OF RESULTS
WITH OTHER METHODS

To confirm the validity of this method, the propagation coefficients were calculated for a number of frequency ranges and permittivity values, and compared to the results from other methods. All of the calculations and measurements reported here are for a square cross section, i.e., a square dielectric rod symmetrically located within a square shield. The structure will be characterized by the aspect ratio DR = a2/a1 for the dielectric rod and SDR = a2/a1 for the shield. In the following, the normalization applied by Schwieg and Bridges [6] will be used, where V and B are the normalized frequency and propagation coefficient, respectively:

\[ V = 2a_1/\beta_0 \sqrt{\varepsilon_{r2}} \]
\[ B = \left( \frac{\beta_0}{\varepsilon_{r2}} \right)^2 \]

(24)

The method described in this paper gives the propagation coefficients of the possible modes for each symmetry used. The designations of the modes on the dielectric line in this paper is the same as that used by Marcatili, Goell, and others. Modes will be identified as \( E_{\text{TM}}^m \) or \( E_{\text{TM}}^n \), where x or y denotes the direction of polarization of the main electric field, and \( m \) and \( n \) are the number of maxima in the \( x \)- and \( y \)-directions over the \( xy \)-plane of the dielectric.

A. Comparison of Method Convergence Properties

A comparison of the convergence properties of Goell’s method and the MSW versus the number of basis functions used is shown in Fig. 3. The dielectric rod was square (DR = 1) with \( \varepsilon_{r2} = 37.4 \) and, in free space, with a normalized frequency of \( V = 7 \). The square shield dimension ratio in the MSW method had SDR = 3, which is of sufficient distance from the dielectric (see Fig. 4) so as to be a good approximation of free space. As can be seen, only approximately 7 \( TM^p \) and 7 \( TE^p \) basis functions are required for good convergence. At 11 \( TM^p \) and 11 \( TE^p \) basis functions, the MSW method and Goell’s results are within 1%.

B. Effect on the Propagation Coefficient of the Proximity of the Shield to the Dielectric Rod

The effect of the proximity of the shield on the propagation coefficients of the first few modes to propagate (\( E_{11}^p \) and \( E_{21}^p/\beta_{11}^p \)) is shown in Fig. 4. It can be seen that, for a shield to dielectric dimension ratio value SDR > 2, the shield has only a small effect on the propagation coefficient. These results are verified by considering the situation where the shield size approaches that of the dielectric (SDR = 1). In both cases, the propagation coefficients found using the MSW method approached those calculated for dielectric-filled rectangular waveguide, as shown in this figure.

C. Comparison of Methods Used for Calculation of the Rod Propagation Coefficient in Free Space

With the dielectric in free space, \( \varepsilon_{r2} = 13.1 \) and DR = 1. Fig. 5 shows the propagation coefficients of the first modes to propagate for normalized frequencies from \( V = 4 \) to \( V = 12 \). To simulate a free-space situation, SDR = 3 is used in the MSW program. One can see that there is good agreement with the free-space method of Goell. Some differences at low frequencies are due to the effect of the use of the shield in the MSW program. Modes \( E_{11}^p \) and \( E_{12}^p \) are degenerate, while \( E_{21}^p/\beta_{11}^p \) and \( E_{12}^p/\beta_{21}^p \) are degenerate and coupled (discussed later in Section III-F).
calculation methods of MSW and Goell [4] for a square cross-sectional dielectric-rod waveguide in free space where $B$ and $V$ are the normalized propagation coefficient and frequency, respectively ($\varepsilon_{r1} = 13.1$).

The resultant mode is associated with a degenerate mode is uncoupled and is found to (at 3.5 GHz. The symmetry shows good agreement. The transverse modes have symmetry. Some of the complex, evanescent, and backward wave modes, and $\varepsilon_{r2} = 37.13$. The modes are labeled with their associated symmetry in parentheses.

D. Comparison of Methods for Calculation of the Shielded Dielectric-Rod Propagation Coefficient

The MSW method with eo symmetry shows good agreement with propagation coefficients obtained by Collin [13] and Collin and Ksienki [14] using a boundary-element method and the finite-difference method of Schweig and Bridges. These results are shown in Fig. 6. The mode is $E_{11}^{11}$, DR = 1, SDR = 1.87, and $\varepsilon_{r2} = 2.22$.

E. Propagation Coefficient Verses Frequency Mode Diagram of the Shielded Dielectric-Rod Waveguide

The propagation coefficient versus frequency-mode diagram, of the first few modes to propagate, is shown in Fig. 7. The 12-mm square dielectric has a permittivity $\varepsilon_{r} = 37.13$, the shield is 24-mm square (SDR = 2). This figure shows differences from that of the dielectric image line reported by Strube and Arndt [10]. The MSW method (for a shielded dielectric-rod waveguide with DR = 1 and SDR = 2) reveals a coupled $E_{21}^{0}/E_{21}^{y}$ mode, which is dominant in this structure, and also the $E_{12}^{1}$ mode is associated with a degenerate $E_{11}^{1}$. The $E_{21}^{0}$ and $E_{21}^{y}$ modes and their associated higher order modes do not occur in the dielectric image line and the studies of this structure in [7] and [10] use only a combination of oe and ee symmetry. Some of the complex, evanescent, and backward wave modes for the shielded dielectric rod, mentioned in Section II-C, are also shown in Fig. 7. The symmetry associated with each mode is shown in parentheses.

F. Field Patterns of the First Few Modes to Propagate on the Shielded Dielectric-Rod Waveguide

The transverse electric and magnetic fields of the $E_{11}^{0}$ mode, in one-quarter of the structure, and determined from oe symmetry, are shown in Fig. 8. The calculated propagation coefficient is $\beta_2 = 305.42$ at 3.5 GHz. The $E_{21}^{0}/E_{21}^{y}$ modes have eo symmetry. In a square cross section, using the same parameters, these are degenerate, as they have the same propagation coefficient $\beta_2 = 89.2$. The resultant field plot is a superposition of both modes, and is shown in Fig. 9. These modes are coupled together, as described by Goell [4], such that their propagation coefficients remain locked together for a range of cross-sectional aspect ratios. The coupled modes separate when DR is somewhat greater or less than 1 depending on the frequency. For example if, for this same structure, DR = 4/3 ($a_1 = 8$ mm, $b_1 = 6$ mm) is used, the $E_{21}^{0}$ mode is now uncoupled and is found to have a propagation coefficient of $\beta_2 = 136.4$. The transverse electric-field pattern of the mode is shown in Fig. 10. The $E_{12}^{y}$ mode was found not to propagate. If DR = 3/4 ($a_1 = 6$ mm, $b_1 = 8$ mm) is used instead, the roles of the modes are reversed. The same type of situations occur when the $E_{21}^{0}/E_{21}^{y}$ modes are produced with ee symmetry.
where chosen. The dielectric is the distance between the planes. These propagation modes with dielectric aspect ratio \( \varepsilon_{\text{rod}} \) and \( \varepsilon_{\text{rod}}^\prime \) of a shielded mode with \( \varepsilon_{\text{rod}} \) and \( \varepsilon_{\text{rod}}^\prime \) can then be compared to calculated values from the MSW method at the measured resonant frequency.

The resonant frequencies produced are related to multiple half-wavelengths between the plates and can be measured at a frequency range that covers the first few modes to propagate is shown in Fig. 12. This structure behaves as a resonant cavity where the resonant frequencies produced are related to multiple half-wavelengths between the plates and can be measured at minimum points in the \( S_{11} \) magnitude data. The propagation coefficient at these points can then be calculated from

\[
\beta_c(N) = \frac{\pi N}{L}
\]

where \( N \) is the number of multiple half-wavelengths of the resonant modes that can exist in the shielded dielectric-rod waveguide, and \( L \) is the distance between the plates. These propagation coefficient values \( \beta_c(N) \) can then be compared to calculated values from the MSW method at the measured resonant frequencies.

V. COMPARISON OF CALCULATED AND MEASURED RESULTS

A plot of calculated propagation coefficient over a frequency range that covers the first few modes to propagate is shown in Fig. 12. The coupled \( E_{21}^\text{sp} / E_{21}^\text{sp} \) modes are dominant, and the degenerate modes \( E_{11}^\text{sp} \) and \( E_{11}^\text{sp} \) are cut off at just below 2.9 GHz.
Fig. 12. Calculated propagation coefficient values for the first few modes to propagate, shield dimension ratio SDR = 1.5. The modes are labeled with their associated symmetry in parentheses.

With this frequency range applied to the test setup of Fig. 11, the $S_{11}$ data produced is as shown in Fig. 13. It was found that the frequencies at the resonant dips shown were within 1% of calculated resonant frequencies for the $E_{21}^x/E_{12}^y$ coupled and $E_{11}^x$, $E_{11}^y$ degenerate modes. The $E_{12}^x/E_{21}^y$ mode did not couple to the measurement probe, nor did $N = 5$ for the $E_{21}^x/E_{12}^y$ mode and $N = 11$ for the $E_{11}^x$, $E_{11}^y$ modes.

It can be seen that some of the resonant dips associated with $E_{11}^x$ and $E_{11}^y$ are in pairs and some are not. Some of these are too small to be seen due to the scale of Fig. 13. The pairing indicates that these modes are not quite degenerate in the test unit due to some asymmetry in its dimensions. In these cases, the measurement frequency was averaged. Where only a single resonant dip was measured, it appears that either the $E_{11}^x$ or $E_{11}^y$ mode did not couple sufficiently to the probe to be visible or they overlap.

Fig. 13 also shows that there are no resonances below 2.9 GHz for the $E_{11}^x$ and $E_{11}^y$ modes and, thus, the $E_{21}^x/E_{12}^y$ coupled mode is truly dominant.

The propagation coefficients, calculated from (25) at the measured frequencies for $E_{11}^x$ and $E_{11}^y$, are compared against MSW calculated values in Fig. 14. An estimated permittivity of the dielectric of $\varepsilon = 37.13$ was used. The extremes of the permittivity tolerance for this dielectric are also shown in Fig. 14.

The measured propagation coefficient values for the $E_{11}^x$ and $E_{11}^y$ modes are within 2% of the MSW values above 3 GHz.

Similarly, the measured frequency propagation values for the $E_{21}^x/E_{12}^y$ coupled mode are also within 2%, as shown in Fig. 15.

**VI. CONCLUSION**

A rigorous method for the computation of the propagation coefficients and field patterns of the fundamental modes in a shielded rectangular dielectric-rod waveguide has been presented. The method, based on that of Solbach and Wolff gives closely comparable results to that of Goell in free space and the boundary-element method of Collin, and has been verified by
experiment for the case of a shield close to a dielectric of high permittivity. The method also reveals that when the dielectric is shielded, a dominant $E_{21}'/E_{12}'$ coupled mode exists. The method allows the effect of shield proximity to be assessed and, thus, has application to the design of cavity filters incorporating rectangular parallelepiped or cubic dielectric resonators. It is easily extended to include calculation of both dielectric losses and conductor losses in the shield wall. This will be the subject of a future paper. This study could also be extended to the calculation of the resonant frequencies of fundamental-mode cubic dielectric-loaded cavity resonators.

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REFERENCES


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