Transverse Injection from a Hypersonic Cone

A dissertation submitted by

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in fulfilment of the requirements of

ENG4112 Research Project

towards the degree of

Bachelor of Engineering (Mechanical)

Submitted: October, 2004
Abstract

The purpose of this project was to investigate sonic gas injection into a hypersonic free stream from a conical geometry base. The project emphasis was image based data acquisition and analysis and consequently the work performed included considerable development of computational image analysis tools.

The major achievement in the image analysis section was the prototyping of an accurate image rotation method using a grid-based colour interpolation approach in MATLAB. This allowed the rotation of flow visualisation footage obtained from the USQ Gun Tunnel so that the frame-to-frame orientation was consistent.

Further achievements included the edge detection and analysis work performed on footage of a hot air jet issuing into quiescent surrounds. The results from this work included a method of jet edge detection that worked in spite of marked variations in brightness between frames.

A numerical solution was obtained, using MATLAB, for the Taylor-Maccoll equations that govern hypersonic flow around conical geometry. The solution to this pair of non-linear differential equations benefited significantly from the crossing point detection capabilities of the MATLAB ODE solvers.

Finally the transverse injection footage obtained from model testing in the USQ Gun Tunnel allowed the quantification of injection based parameters such as jet penetration into the free stream and fore-body conical shock angle.
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Certification of Dissertation

I certify that the ideas, designs and experimental work, results, analyses and conclusions set out in this dissertation are entirely my own effort, except where otherwise indicated and acknowledged.

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Acknowledgments

Sincere thanks is extended to my supervisor Dr David Buttsworth for his professional assistance, for his wise council and encouragement throughout the course of this project. Appreciation also to Dr Ahmad Sharifian for his supervisory contribution. Many thanks also to my parents, Gary and Catherine for their support and proof-reading. Last but not least gratitude is expressed to Ainslie for her contribution to the preservation of my equilibrium throughout.

DAVID B. T. SERCOMBE

University of Southern Queensland

October 2004
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Chapter 1

Introduction

At present the Supersonic Combustion Ramjet engine or SCRamjet stands as one of the most promising candidates as an air-breathing propulsion device for a hypersonic (greater than Mach 5) flight vehicle. The engine differs from a conventional jet engine by having supersonic flow through the combustion chamber. The high-speed flow results in a short residence time in which a good fuel-air mix has to be achieved (in the order of a millisecond). The short residence time coupled with slow mixing which is characteristic of compressible flow regimes results in the retardation of effective fuel-air mixing relative to subsonic mixing regimes. A variety of techniques have been developed to enhance or augment fuel-air mixing in the combustor and these include:

- Fuel injection parallel to the free stream with enhanced nozzle geometry (Hyper-mixers).
- Injection perpendicular to the free stream (Transverse Injection).

Parallel injection requires injection nozzle geometry geared to the creation of stream-wise vorticity (which leads to enhanced turbulent mixing), otherwise ineffective mixing occurs resulting in inefficient combustion. On the other hand, total pressure losses tend to be lower with this mode of injection resulting in a higher total cycle efficiency if effective fuel-air mixing takes place. Transverse injection generally results in a good mix over a short distance but the presence of a normal jet impinging on the supersonic
free stream flow results in additional shock structures and hence, larger total pressure losses.

This project concerns the study of transverse injection from a conical model into a hypersonic test flow. Studies conducted on transverse injection to date have generally concentrated on sonic or supersonic injection of gas into a free-stream test flow and so injection from a conical model could be viewed as a progression in geometric complexity of the test environment. So the broad aim of the project is:

“... to analytically and experimentally investigate the fuel-air mixing process associated with injection from a cone at hypersonic conditions (as a model for SCRAMJet mixing studies).”

Transverse gas injection has further application in the field of Thrust Vector Control (TVC) where the pressure forces created on the surrounding geometry by the injected gas stream are used for directional control. Application can also be demonstrated in the field of Film Cooling, where object surfaces are thermally protected from surrounding high temperature gas flow by a cooler gas film that hugs the surface. The reasons for testing a conically shaped model include:

- The shape of the model can be primarily viewed as an academic abstraction of physical geometry such as the fore-body of a scramjet or the nose of a high speed flight vehicle which are usually conical or near-conical.

- The axisymmetric nature of the cone means any analytical or numerical models pursued are simpler (quasi 2-dimensional instead of 3-dimensional) which is of real importance when studying complex flow behaviour.

- There is some military interest in studies on conical/near-conical geometry because of possible impact on ‘nose-cone’ design.

Having established a basis of credibility for the project the broad project aims would be to:
1. Research current designs, techniques and principles used to enhance fuel-air mixing in scramjet engines and how these relate to the conical model developed.

2. Develop a computational model for Conical Flow to aid in flow behaviour prediction.

3. Research fluid flow visualisation techniques, with an emphasis on those presently in use at the USQ Gun Tunnel Facility.

4. Develop computational tools to enhance the flow visualisation facilities within the USQ Gun Tunnel Facility.

5. Conduct experiments with a hot air jet in quiescent surrounds as an evaluation of point (4).

6. Conduct experiments using the conical/injection model and the USQ Gun Tunnel Facility and associated flow visualisation/data acquisition facilities.

7. Critically evaluate the analytical/computational models developed against the experimental data collected.

8. Identify eddy formation and behaviour in the hot air jet described in point (5) from the flow visualisation data.

9. Identify and quantify/qualify the mixing behaviour identified in the conical/injection model experiments.

The USQ Gun Tunnel facility is where all of the testing of the model will take place. The gun tunnel (seen in Figure 1.1) is capable of producing a Mach 7 test flow for approximately 25 ms. The facility also incorporates flow visualisation equipment which will be used along with the other measurement and data logging equipment to study the mixing structures that occur during testing. As mentioned above some ancillary software tools are required for the visualisation equipment to function at its full potential.

By taking footage of a hot air jet issuing into the quiescent laboratory surrounds some evaluation of these tools can be done. The footage can also be used to evaluate the visualisation equipment as a whole through the identification and tracking of the high speed convection eddies that form in the turbulent hot jet. The structure identification and tracking methods used for the hypersonic cone experiments will be developed
1.1 Overview of the Dissertation

This dissertation is organized as follows:

**Chapter 2** contains a review of available literature related to this project.

**Chapter 3** considers a numerical solution to super/hypersonic conical flow.

**Chapter 4** details the experimental apparatus used.

**Chapter 5** describes work done in the area of image analysis and the computational tools developed.

**Chapter 6** examines image based description and analysis of a hot air jet issuing into a quiescent environment.

Figure 1.1: The USQ Gun Tunnel which is capable of a Mach 7 hypersonic test flow. The driver tank and the barrel are illustrated leading into the nozzle. The optics channel, which runs perpendicular to the barrel, contains the flow visualisation equipment (Buttsworth 2003).

with the hot air jet footage. The flow visualisation facilities are further described in Chapter 4.

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**Chapter 6** examines image based description and analysis of a hot air jet issuing into a quiescent environment.
1.1 Overview of the Dissertation

Chapter 7 analyses the images and data gathered from the gun tunnel experimentation.

Chapter 8 concludes the dissertation and suggests areas of further investigation.
Chapter 2

Literature Review

In an attempt to gain an historical perspective on the development of the scramjet, Curran (2001) also expands upon development contributions from individual countries such as Australia and selected European countries, though the article shows a heavy slant towards progress within the USA. Further literature pertaining to this topic falls fairly neatly into literature which deals with Transverse Injection and literature relating to Flow Visualisation and Imaging, though obviously some measure of overlap is apt to occur.

2.1 Transverse Injection

Seiner, Dash & Kenzakowski (2001) considers the various techniques and methods developed to enhance the fuel-air mix in a scramjet combustor from an historical perspective and is useful for placing transverse injection in the context of application to scramjet mixing augmentation. The authors of this article raise the issue of total pressure losses and their subsequent impact on the cycle efficiency of a scramjet engine that primarily utilises transverse injection. These pressure losses result from the bow shock that occurs upstream of the injection plume. The article does highlight however, that the separation zones that occur around the nozzle have desirable flame-holding abilities, and interest in transverse injection, its variations and its broader applications currently
2.1 Transverse Injection

remains (Jacobsen, Gallimore, Schetz, O’Brien & Goss 2002).

The quantity of literature available on transverse injection of gas into a supersonic free stream is mountainous. Schetz (1980) presents one of the earlier collations of work done in this field and recounts the areas where greater understanding of the behaviour in transversely injected jets could be applied; namely, fuel injection in scramjet combustors and thermal protection (so called film cooling). While the work being discussed here could be considered fairly rudimentary, an important point is arrived at; that penetration of the injected gas stream is not practically increased by increasing injection pressure, varying the shape of the injector or attempting to employ supersonic injection. In most experimental cases discussed, testing regimes are defined through a ratio of injection to free stream momentum flux:

$$\bar{q} = \frac{\rho_{inj} U_{inj}^2}{\rho_\infty U_{\infty}^2} = \frac{(\gamma P M^2)_{inj}}{(\gamma P M^2)_{\infty}}$$  \hspace{1cm} (2.1)

This ratio $\bar{q}$ is used to designate testing regimes in some more recent work (Jacobsen et al. 2002) which J. A. Schetz has been a party to, confirming what was established earlier is still present in more current work. While this article mainly relates to a variation on transverse injectors (multi-holed transverse injectors known as ‘aeroramps’) it demonstrates interest in free stream transverse injection and variations on the theme (such as transverse injection from a hypersonic cone).

VanLerberghe, Santiago, Dutton & Lucht (2000) looks into sonic, normal gas injection into a supersonic free stream with an emphasis on quantifying the actual mixing taking place, as opposed to the level of injected stream penetration into the cross flow. Most of the data gained in the testing done was from a Planar Induced Laser Fluorescence (PILF) imaging set up which is capable of taking a planar ‘snapshot’ of the different gas species and their interaction. Such a setup is not in place at the USQ facility so, for the most part, the article is of academic interest only. The article does make mention of the agreement of numerical studies into transverse jet mixing behaviour with experimental data in areas such as wall pressure and jet penetration but highlights the general disparity between results that occur when considering the velocity field, jet separation and the amount of mixing that takes place.
2.1 Transverse Injection

The paper by Li & Ni (2003) deals with supersonic, transverse injection into a supersonic free stream (so-called ‘interacted flow’), with an emphasis on flow visualisation using a Schlieren optical system similar to that presently in use at the USQ Gun Tunnel. Figure 2.1 from the article is one of the better qualifying descriptors of interacted flow encountered to date, describing the gross flow field characteristics expected in experiments of this kind. The paper compares the level of injected jet penetration in the two experimental cases presented through a dimensionless ratio $h/D_j$ where $D_j$ is the reference jet dimension (in this case the nozzle throat diameter) and $h$ is the height of the bow shock at the centreline of the nozzle (illustrated in Figure 2.2). Interestingly the experimental regimes presented are designated by a ratio of injection to free stream pressure $p_j/p_\infty$ in contrast to the momentum flux ratio $\bar{q}$ employed by Schetz (1980). This difference is reflected in the final result where a significant increase in the injection pressure $p_j$ results in a comparatively small increase in dimensionless penetration $h/D_j$.

![Figure 2.1: Flow field interactions and structures commonly present when a gas is injected perpendicular to a compressible free stream (Li & Ni 2003).](image)

Li, Ni & Sun (2003) presents both free jet and interacted jet flow studies, with the former being of little consequence to the work pursued in this project. The transverse injection (interacted jet) experimental apparatus includes a large circular array of pressure transducers surrounding the injection nozzle so as to study pressure behaviour near the plate. The pressure readings near the plate would tend to reflect flow behaviour in the boundary layer/separation zone, meaning that the results would be more applicable to TVC and film cooling than the study of mixing behaviour. The pressure distribution
2.2 Conical Flow

With a conical test piece as the focal point of this project some investigation into super/hypersonic flow behaviour around conical objects was deemed necessary. A classical conical flow solution was proposed by Taylor & Maccoll (1933) and is presented in Anderson (1990). Anderson (1990) shows the derivation of the solution, the final
2.3 Flow Visualisation

result being a pair of non-linear ordinary differential equations that yield the velocity field behind the conical shock (see Figure 2.3). The solution is non-analytical and so a step-by-step guide is given to find a numerical solution. This guide would prove quite useful when formulating the conical flow solution into a computer script in order to get a more numerically precise solution. As well as presenting the solution some discussion is made on how the numerical results compare with observed experimental data and the limitations of the solution (specifically, when the shock becomes detached).

Other sections of the book include data tables for Isentropic, Oblique and Normal Shock relations as well as information on the generalities and peculiarities of hypersonic flow. These include the very thin oblique and conical shock layers that occur at higher Mach numbers as well as the propensity for greater boundary layer growth and thickness in this flow regime.

Figure 2.3: Behaviour of the flow field around a cone behind the conical shock as given by the Taylor–MacColl Solution (Anderson 1990).

2.3 Flow Visualisation

The work of Smits & Lim (2000) includes an entire chapter on flow visualisation techniques for compressible flow regimes. Schlieren methods and techniques are given fairly comprehensive treatment and further discussion of the Schlieren method and its use in
2.3 Flow Visualisation

the gun tunnel is included in Chapter 4. A relationship between the change in relative intensity $\Delta I/I$ at the image plane and the density gradient is presented, for gases (Merzkirch 1987):

$$\frac{\Delta I}{I} = \frac{K f_2}{s} \int_{\gamma_1}^{\gamma_2} \frac{\partial \rho}{\partial z} dy$$

where $K$ is the Gladstone-Dale constant and $\gamma$ is the ratio of specific heats for the gases in question, $f_2$ is the system focal length and $s$ is a focal point dimension. The directions $y$ and $z$ are perpendicular and parallel to the optical path respectively. What this equation highlights is that the relative change in intensity seen at the viewing plane is proportional to the integral sum of the density gradients along the optical path. The directionality of $y$ is determined by the orientation of the knife edge places at the focal point of a Schlieren system.

There have been techniques developed to determining flow field density from Schlieren images based on the deflection of a series of lines or a speckle pattern, from a background frame, due to the presence of density gradients. Details of such a ‘Background Oriented Schlieren’ (BOS) technique is given by Meier (2002). While the prospect of such a system is tantalising, its usefulness is fairly heavily restricted to highly planar flows and thus of limited value in the study of axisymmetric flow domains such as conical injection.

The flow visualisation facilities in place at the USQ Gun Tunnel include a high-speed, Cranz-Schardin Camera capable of producing five Schlieren images during a typical gun tunnel run. The system in place is based extensively on one developed by Brouwer (1999) for the University of Queensland. Brouwer (1999) includes optical ray-traces of the Cranz-Schardin system developed, as well as footage from the testing where a measure of spherical abberation becomes evident when the system is extended to capture multiple frames. This distortion is acknowledged in the main body of the thesis and is attributed to a large angle of incidence between the beams produced by the LEDs and the optical axis. The thesis also justifies the use of LEDs as a light source over the more traditional use of spark gaps.
2.3 Flow Visualisation

An earlier incarnation of the present visualisation system in place is described in a technical report by Buttsworth & Alfock (2003). The report mainly concerns the design and construction of a system designed to produce a short pulse of light of known duration from an LED source, as a source of illumination for the Schlieren imaging system but includes a description of the optical componentry in situ.

Another Schlieren image system illuminated by two spark gaps is presented in Papamoschou (1989), where the spark gaps are fired in succession over a known time interval $\delta t$ of the order of a few micro-seconds. Test footage is presented of two supersonic shear layers formed by the mixing of two select gas species. In each case a structure in the turbulent shear layer is tracked in successive frames in a manner similar to that employed in the experiments conducted in this project. It could be said that some imagination is required to see the structure progression over the two frames.

Tsai & Bakos (1998) present an account of a scramjet combustor study undertaken after an upgrade of the flow visualisation facilities at the NASA HYPULSE test facility. The upgrade included a multiple frame Cranz-Schardin style camera very similar to the one employed at the USQ gun tunnel, for the purposes of acquiring Schlieren images. Interestingly their camera also used high intensity LED sources for test field illumination. The Schlieren images acquired were used to determine the angle of the oblique shock that occurred on the combustor model and from these measurements, in conjunction with pressure measurements, interferometry data and numerical simulation, inferences are made about transient combustor characteristics.

Verma (2002) looks into Mach 9 flow over a conical model using a laser based Schlieren system which projects onto a vertical series of photo-diodes as a means of tracking unsteady conical shock behaviour. The Schlieren system intensifies the shadow area produced by the density gradient in the flow causing a drop in photodiode voltage in the area where the shock occurs. A couple of behaviours were observed such as ‘flapping’ and ‘rippling’ of the shock, though the article does mention that the accuracy of any findings is curtailed because of the large distance between photodiodes. While such a system is not in place at the USQ facility, the ability to take sequential images allows quantitative observation of transient or unsteady behaviour in shock structures, such as those found in this article.
2.4 Chapter Summary

Studying the behaviour of transverse injection from a hypersonic cone primarily through flow visualisation, represents an amalgamation of a few distinct fields - those of conical flow, transverse injection and flow visualisation. While articles of flow visualisation techniques relevant to this type of experimentation are quite often tested using conical flow or transverse injection experiments, no work that deals specifically with normal gas injection into a hypersonic conical flow field has been identified. This could be seen in both a positive and a negative light, as no-one appears to have studied this particular experimental setup to date leaving no direct foundation upon which to build. So it was necessary to seek out information from the related fields (as shown) and determine their application to the area being studied.
Chapter 3

Conical Flow

3.1 Introduction

The proposed model to be tested in this project is essentially a right cone. As outlined in Chapter 1 the axisymmetric shape of a cone lends itself to analysis as a quasi-2D case. There exists a model of supersonic flow over a right cone (presented by Anderson (1990)) known as the Taylor-Maccoll Solution (Taylor & Maccoll 1933). The solution quantitatively describes flow behaviour behind the conical shock structure that forms due to the presence of a conical impediment in the free stream.

3.2 Taylor-Maccoll Equations

The main solution proposed is presented in the form of a nonlinear ordinary differential equation (ODE) in planar spherical coordinates. A fundamental assumption of the solution is that flow properties such as the velocity or Mach number are constant along rays that extend out from the tip of the cone at a planar angle $\theta$. This assumption has been experimentally validated (Anderson 1990). The velocity field is described by:
\[
\frac{\gamma - 1}{2} \left[ 1 - V_r'^2 - \left( \frac{dV_r'}{d\theta} \right)^2 \right] \left[ 2V_r' + \frac{dV_r'}{d\theta} \cot(\theta) + \frac{d^2V_r'}{d\theta^2} \right] - \frac{dV_r'}{d\theta} \left[ V_r' \frac{dV_r'}{d\theta} + \frac{dV_r'}{d\theta} \frac{d^2V_r'}{d\theta^2} \right] = 0
\] 

(3.1)

where \( \gamma \) is the ratio of specific heats and \( V_r' \) is the radial component of the dimensionless velocity \( V' \) which is defined as:

\[
V' = \frac{V}{V_{max}}
\]

(3.2)

\( V_{max} \) is a theoretical maximum velocity (not to be confused with the speed of sound \( a \)) and is constant for a given flow. The angular component of the velocity is given as:

\[
V_\theta' = \frac{dV_r'}{d\theta}
\]

(3.3)

A relationship between dimensionless velocity \( V' \) and the Mach number \( M \) is also given such that:

\[
V' = \left[ \frac{2}{(\gamma - 1) M^2} + 1 \right]^{-1/2}
\]

(3.4)

Using the above relationship the Mach number can be found for any given velocity along a property ray. Fluid properties such as density, temperature and pressure can be found along a given ray using the Mach number and standard isentropic flow property relations. In this way a complete picture of the flow field can be developed from the dimensionless velocity values. A more rigorous derivation of these equations is presented in Anderson (1990).
3.3 Conical Flow Solution

Equation 3.4 is nonlinear and as such, an analytical or closed form solution doesn’t exist, so it was necessary to devise a numerical solution. Anderson (1990) details the procedure for finding numerical data by hand and it was desirous to formulate this procedure into a MATLAB script as a computational solution. MATLAB contains a number of numerical ODE solvers, the most ubiquitous of these being ode23 which is an adaptive, Runge-Kutta style solver. The idea was to use this solver to step through small angles $\Delta \theta$ and find the respective dimensionless velocity components $V_r'$ and $V_\theta'$ at that angle $\theta$.

3.3.1 Boundary Values

A crucial part of any numerical solution incorporates the determination of starting points or boundary values. Unfortunately no flow velocity data is readily available at the cone angle $\theta_c$ which would be the logical starting point. It is possible however, using the standard relations derived for oblique shocks to determine the velocity components behind the conical shock. Using these values the solver could advance in steps of $\Delta \theta$ until $V_\theta' = 0$ which occurs at the face of the cone. While these boundary values are readily available it means that the solver starts at the shock angle $\theta_s$ and works through to the cone angle $\theta_c$.

Figure 3.1 shows the geometry of the boundary values as well as illustrating the direction the solver takes. The Mach number behind the shock ($M_2$) and the flow deflection angle $\delta$ can be determined using standard relations for oblique shocks (the planar equivalent of conical shocks) and $V'$ can be found using Eq. 3.4. The relations were formulated into a MATLAB script such that boundary values could be found for a given $\theta_s$ and free-stream Mach number $M_\infty$. 
3.3 Conical Flow Solution

3.3.2 Solver Formulation

In order to obtain results from the MATLAB solver Eq. 3.1 had to be rewritten as a pair of equations in state-variable form. This meant that the solver returned values for both \( \frac{dV_r'}{d\theta} \) and \( V_r' \) and hence solved both components of \( V' \) at once.

The \textit{ode23} solver is generally set up to solve over a certain range of the dependant variable \( \theta \) and as such the solver \textit{options} parameter was set to flag when \( V_{\theta}' \) approached zero. As \( V_{\theta}' \) came close to zero the solver was stopped and an estimation was made of the angle \( \theta \) where the zero point occurred. In practice the final \( V_{\theta}' \) value was never exactly zero but consistently of the order of \( 10^{-18} \). By flagging values and causing the solver to cease it was possible to get a much closer estimation of the final cone angle \( \theta_c \) as well as avoiding the waste of computational time by solving for redundant values of \( \theta \).
3.3 Conical Flow Solution

3.3.3 Solution Iteration

As mentioned in Subsection 3.3.1 the solver script essentially operates back-to-front, as it is more usual to have a cone angle $\theta_c$ for which it is more desirable to know the shock angle $\theta_s$ and corresponding flow properties behind the resulting conical shock than the reverse. This was certainly the case with the test piece in question. It was obviously necessary to run the solver script repeatedly until the known $\theta_c$ and the found $\theta_c$ converged on a single value.

A bisection iteration method was employed as the results obtained from this sort of iteration usually converge relatively quickly. Simply, $\theta_c$ values are calculated for initial upper and lower $\theta_s$ bounds and at the midpoint between the upper and lower bounds, a logic function is employed to ascertain whether the midpoint $\theta_c$ is higher or lower than the known $\theta_c$ and correspondingly this value then becomes the new upper or lower bound. The iteration is continued ad nauseum until the found $\theta_c$ converges to within a predetermined tolerance.

The final MATLAB script suite ran quite quickly and for a cone angle $\theta_c = 10$ degrees a solution was obtained to within $10^{-5}\%$ of the known $\theta_c$ after 33 iterations. Conical and near-conical geometry is often encountered in super/hypersonic flow experiments and so the scripts developed will most certainly have broader application than just in this project.
This chapter dealt with the development of a concise computational solution to the Taylor-Maccoll conical flow solution. The derivation of the Taylor-Maccoll equations is readily available and therefore not included in the chapter’s content. The resulting suite of MATLAB scripts that form the solution solve quickly and to a quite acceptable level of numerical accuracy $10^{-5}\%$. 
Chapter 4

Apparatus

4.1 Introduction

The physical experimentation conducted in this project required a sizable amount of equipment. The vast majority of this equipment was already in situ at the commencement of this project, while some was developed or has undergone some development since commencement.

In short, the apparatus used falls fairly neatly into a number of categories:

1. The USQ Gun Tunnel.
2. The Flow Visualisation Facilities in place in the USQ Gun Tunnel.
3. The Conical Model used.
4. Apparatus associated with the hot air jet testing.

This chapter aims to expand upon the above categories in relation to the physical testing conducted.
4.2 The USQ Gun Tunnel

The purpose of the USQ Gun Tunnel would succinctly be described as: to provide a hypersonic (nominally Mach 7) flow of a test gas of sufficient duration to facilitate the collection of usable data. As would be expected, the mechanism through which this is accomplished is somewhat complicated and is itemised with reference to Figure 1.1 thus:

1. The yellow driver tank is pressurised to the nominal free stream stagnation pressure required for the run.

2. The aluminium diaphragm that separates the barrel from the driver tank bursts, either from the pressure in the driver tank or manually using a plunger.

3. The sudden diaphragm burst drives a plastic piston down the barrel at speed. The piston’s motion sets up shock waves in the test gas in front of the piston.

4. As the piston nears the end of the barrel, the pressurised test gas ruptures a cellophane secondary diaphragm positioned between the barrel and the nozzle. This secondary diaphragm separates the test gas from the test section which is normally evacuated to a near vacuum.

5. The test gas is expanded, isentropically, to Mach 7 via the nozzle which is of converging-diverging profile.

6. The expanded gas enters the test section which has circular ports on each side allowing observation of the flow.

7. The test gas then terminates the dump tank.

The entire test section and dump tank is on rails and can be rolled backwards and separated from the nozzle during setup. The optics channel is discussed with the flow visualisation apparatus covered in Section 4.3. The instrumentation bench contains the data-logging equipment in the form of Tektronix TDS 210 and TDS 2104 digital oscilloscopes used to record data from pressure transducers and the firing of ancillary equipment such as cameras. This bench also contains the HC11 microprocessor unit.
4.2 The USQ Gun Tunnel

Figure 4.1: Plot of the Nozzle Stagnation Pressure reading taken from a typical gun tunnel run. The black bars give an indication of the usable steady-state flow time (around 18 ms).

that is used to control the entire process from activating the plunger to puncture the diaphragm to activating the gas injection system and firing the cameras. This is no mean feat given that a typical gun tunnel provides a usable test flow of around 18 ms.

Aside from the flow visualisation equipment, ancillary gun tunnel equipment includes a gas injection system consisting of a pressure tank and a fast-acting valve that allows the injection of gas under pressure into the test section during a gun tunnel run. This particular system allowed the injection of gas during the testing that was undertaken.

Figure 4.1 shows a stagnation pressure reading taken of the test gas immediately in front of the nozzle. The usable flow takes place just after the initial pressure spike. This pressure reading is monitored by the HC11 microprocessor and the spike shown is used as a trigger for the injection equipment and the firing of the cameras.

The setup procedure for a run typical of the type conducted is as follows:

1. The test section and dump tank are rolled back separating them from the rest of
the tunnel.

2. The nozzle is unbolted from the barrel and slid out of the line of sight of the barrel facilitating removal of the piston.

3. The barrel is unbolted from the driver tank and the ruptured aluminium diaphragm is removed.

4. With the barrel bolted loosely to the driver, a cleaning piston is driven down the barrel from the nozzle end using a plate and pressurised line bolted to the nozzle end.

5. The aluminium diaphragm is replaced, the piston is reinserted into the top end of the barrel and the barrel is bolted onto the driver tank.

6. The ruptured cellophane diaphragm is discarded and a new diaphragm is attached to the nozzle. The nozzle is now bolted back onto the barrel.

7. Assuming the model is set up, the test section and dump tank can now be rolled up to the nozzle and the whole section evacuated ready for firing.

8. The gas injection valves are checked and once in position the injection tank can be pressurised.

Once all of the above is performed all that remains is to set the cameras and data-logging equipment running and to pressurise the driver tank. The run is initiated by diaphragm rupture of operator command.

4.3 Flow Visualisation Apparatus

A brief introduction to the flow visualisation facilities in place at the USQ Gun Tunnel was given in Section 2.3. The visualisation system in place is based around a Schlieren optical system. This type of optical system is based on the bending of light as it passes through a density gradient in the fluid under observation. This bending of the light results in areas of varying intensity when the light is projected onto a viewing plane (such as a camera or a ground-glass screen).
4.3 Flow Visualisation Apparatus

Figure 4.2: A schematic of the multiple frame Cranz-Schardin camera system employed in the USQ Gun Tunnel (Buttsworth 2003).

The basic schematic of the optical system used is illustrated in Figure 4.2. The raytrace shows the point sources, which in this case are LEDs and the collimation of the light through the test section. The light is then focused onto a knife edge, which ideally is positioned so as to cut off a portion of the light, and finally the resulting image is projected onto a viewing plane. The focusing of the image onto a knife edge is considered the hallmark of the Schlieren system and results in a more visible change in intensity than would be expected from a shadowgraph-style setup. Further dissection of the compressible flow imaging techniques including the Schlieren method can be found in Smits & Lim (2000).

As stated in Section 2.3 the camera system shown is known as a multiple frame Cranz-Schardin camera. The multiple frames are achieved through the use of multiple light sources and a corresponding array of cameras. The use of LEDs as light sources permits pulses of very short (1.5 – 38\(\mu\)s) duration and using multiple sources allows for time lapses in the order of a few milliseconds between frames. During a typical gun tunnel run the system is able to take five successive frames over an 18 ms effective run duration.

Figure 4.3 shows the aluminium cross that supports the four Samsung 1/3" CCD cameras, with the fifth Nikon E2n digital camera in the background. The digital camera has its own image storage setup whereas the four CCDs are each fed into a separate Samsung SV660B VCR. The digital acquisition and analysis of the Video footage is covered in Chapter 5. While ideally the output from the CCDs should have been captured directly, there was some question over whether sufficient computing power would be available to perform the digital capture of four video sources, running at 25 fps (frames
Figure 4.3: Shown is the aluminum array containing four CCD video cameras and the Nikon digital camera used to record images from gun tunnel runs (Buttsworth 2003).

A separate laser diode is also mounted on the source end of the optics channel and is used in conjunction with a height jig to ensure the alignment of all lenses and mirrors.

Attempts were made during the course of this project to improve upon the current system and to overcome some of the limitations that it posed. Of principal concern was the amount of light available to illuminate the test field; any increase in the amount of light would allow for the use of shorter illumination times in an effort to freeze fine structure more effectively. With this goal in mind minor modifications were made to the pulsing system that allowed the LEDs to be replaced with laser diodes. Initial results were far from encouraging as passing the laser light down the fiber optic cables that had previously been used for the LEDs resulted in randomised variations in intensity. These variations are known as ‘laser speckle’ and are the result of marked phase cancellations exacerbated by the narrow bandwidth of the laser light.

Positioning the laser diodes directly at the source removed the speckle and as an added benefit, allowed for very precise calibration of the optical system. The problem was that any particulate matter in the optical path caused the appearance of very obvious
diffraction fringes. Having minimised the occurrence of such fringes, a non-injection gun tunnel run was performed, the result of which can be seen in Figure 7.1. In terms of illumination the laser diodes were a marked improvement over the LEDs and the illumination time used for Figure 7.1 was around 1.5 $\mu$s. The quality of the Schlieren image produced by the laser diodes is quite poor in comparison to the results obtained using the LEDs and while in this case the results were still usable it was decided to remove the laser diodes and continue with LED illumination for the time being.

4.4 The Hypersonic Cone

A justification of the type of model employed was included in Chapter 1. Essentially the model is a cone with a centrally machined hole which ends in a converging annulus, the thickness of which can be adjusted by screwing the top section of the cone in or out. An impression of the model is given in Figure 4.4 and full technical drawings of the model are included in Appendix B.

The purpose of the model is to allow sonic gas injection via the converging annulus into hypersonic flow around the cone proper. By ensuring that the annulus area is small in comparison to the central hole and sufficient tank pressure is available it was assumed
4.4 The Hypersonic Cone

4.4.1 Pressure Transducer Holder

The conical model contained machined sections which allowed pressure transducers to be fitted in order to yield injection pressure readings that could be compared with readings taken further upstream and tank pressure readings. A holder was developed and made such that a piezo-resistive pressure transducer could be fitted. The holder is shown in place in Figure 4.5 and full drawings are included in Appendix B.

While the holder itself was constructed prior to experimentation some difficulty was experienced in obtaining seals, so the holder was not employed during testing. This was not viewed as a great loss given that readings were still available from the piezo-electric transducer that was already in place on the gas injection system. On top of this, the results obtained from separate piezo-resistive pressure transducer experimentation in the gun tunnel were subject to a great deal of electro-magnetic interference, casting
doubt over the usefulness of such readings.

4.5 Hot Air Jet Apparatus

It was decided that some test footage was required as a means of evaluating the flow visualisation facilities and for use in the development of some ancillary software tools, the formulation of which is detailed in Chapter 5. An Arlec EHG998 1600W heat gun of the type used for stripping paint was used to create a jet of hot air issuing into the quiescent laboratory surrounds.

In order to film the hot air jet the test section of the tunnel was rolled back and a wooden location jig was used to position the jet. Slight variations were made to the control program on the HC11 microprocessor to allow a continuous sequence to be filmed. This did not limit the exposure time used but meant that successive frames on a single camera could only be taken on a 50Hz cycle.
4.6 Chapter Summary

This chapter has summarised and then detailed the experimental apparatus used during testing for this project. It has included a description of the functions and procedures of the USQ Gun Tunnel, detailed the flow visualisation facilities in place, discussed the conical model used during testing and shown the apparatus used to gain evaluative footage from the flow visualisation facilities.
Chapter 5

Image Analysis

5.1 Introduction

This chapter discusses the image analysis software and techniques that were developed and implemented in the USQ Gun Tunnel Facility. The tools are an addition to the high-speed video capture facilities, previously set in place at the facility, that form the core of the facility’s fluid flow visualisation componentry. This componentry includes the Schlieren optical system, a digital still camera and four CCD cameras feeding into four VCRs. Further details of the componentry and their setup are given in Chapter 4.

The specific objective was to develop the capacity to sequentially order the digitally captured video and digital still picture frames producing a continuous digital video sequence. This was further complicated as the optical system used to split the images displayed on each of the CCDs produced some measure of rotation. This meant that a method of digital counter-rotation would have to be developed in order for the final sequence to be useable.

The ability to view a coherent sequence of images is very important in the high speed flow visualisation aspect of experimental data collection. It better allows qualitative, and eventually quantitative, description of high speed (hypersonic) fluid flow, especially when observing transient flow or structure formation in mixing - although the applica-
tion is by no means limited to these scenarios.

5.2 Image Acquisition

For the purposes of development, a series of video records were made, using the Schlieren optical system, of a hot air jet issuing into quiescent surroundings. The apparatus used is further described in Chapter 4. This experiment provided a large amount of sequential video footage with which to refine and develop the image capture process and the associated software tools.

The video frames produced by the four CCD cameras were recorded by four Samsung VCRs onto four blank video cassettes. To produce an effective 25 frames per second (fps) the recorders operate at an interlaced 50 Hz cycle. This means a video frame comprises of two fields displayed sequentially at 50Hz. In the apparatus used, LED illumination (hence footage capture) of the test section happens only during the duration of one field or one half of the frame. By making use of this half-frame recording and having the four VCRs recording in sequence, four half-frames can be recorded during one cycle with a period of approximately 0.02s. A typical firing and recording sequence, during one 50Hz cycle, would be as follows:

1. Recording on Field 1 on Video 1
2. Recording on Field 1 on Video 2
3. Recording on Field 1 on Video 3
4. Recording on Field 1 on Video 4
5. Recording on Field 2 on Video 1 etc.

As it is impossible to synchronise which field is being recorded on each of the tapes, it is important to realise that recording might start on Field 2 on any particular tape, with the next half-frame being captured on Field 1 of the following frame. In a practical sense, when the system is used as primarily intended, capturing footage from Gun
5.3 The Image Manipulation Software

Tunnel experiments, the duration of the experiment is such that only four half-frames are recorded, with the time interval between LED firing adjusted to suit the period of interest.

The analogue video footage recorded during the experiment was digitally captured and converted into AVI (Audio-Video Interleaved) digital movie files using a Pinnacle Video Capture Card and the associated Pinnacle Studio 5 video editing software. The footage from the four video cassette tapes was captured at a resolution of 576 \times 720 pixels. The conversion to AVI format was done using a ‘Cinepak’ codec (compressor-decompressor) as this was found to be compatible with the MATLAB 6 software that was used to create the tools required to manipulate the digital version of the footage. The test footage used comprised of approximately 30 digitally captured frames of actual Hot Air Jet footage and approximately a second’s worth of cross-hair footage (used to obtain common points on each of the four videos) from each of the four video cassettes. An LED illumination time of 13\,\mu s was used on all of the test footage.

5.3 The Image Manipulation Software

The captured footage was manipulated using MATLAB 6. Principally this software was chosen because of its in-built image manipulation tools and ease of programming, allowing scope for the development of tools more specifically applicable to the frame (or image) manipulation required in this instance. The tasks to be accomplished included the rotation of the individual frames in each of the four recordings to a common horizontal axis and the re-ordering of the four rotated digital video’s into a single playable sequence and in the case of the Hot Jet experiments the resultant ‘high frame-rate’ sequence being used to visually track hot-air convection eddies.

MATLAB possesses the ability to read AVI movies into a MATLAB movie format. This format sets the movie data up in a 1 \times n structural array, where n is the number of frames in the sequence. Each of the frames contains the fields colormap and cdata. The colormap field usually contains the MATLAB colour map, which may be the default RGB (Red-Green-Blue) or some other colour map defined by MATLAB or other
image processing or creation software. In the case of ‘truecolor’ images and movies the *colormap* field is empty. It follows then that in ‘non-truecolor’ frames the *cdata* field contains the colour values, that correspond to the colour map used, in an $m \times n$ array, where $m$ is the number of rows of pixels and $n$ is the number of columns of pixels. When the frames are truecolor, the *cdata* field is an $m \times n$ 3-dimensional array containing the pixel colour values (Red, Green and Blue) at each of the points $(m, n)$. The captured footage was outputted by the Pinnacle software in the default, truecolor format.

The MATLAB movie format, by virtue of its structural array format, is well suited to footage manipulation, principally due to the ease with which frames can be extracted as images, manipulated and reformatted into a movie. It also allows for straightforward subsequent extraction of rows or columns of pixels from an individual frame or image. The way in which MATLAB plays digital video can be essentially described as a sequential display of still digital images on a set of graphical axes identical to those used for displaying digital images. For any of the above mentioned formats, the colour value/s at any point $(m, n)$ in the *cdata* field can be displayed on a pixel based set of axes that ascribes a unit distance between pixels. This graphical display can be used with MATLAB’s function discovery facilities to fit lines or curves to identifiable features within the image.

### 5.4 Frame Extraction and Re-ordering

The Digital Video (DV) format used by the Pinnacle software essentially sequentially displays 25 image frames per second. The capture process, in order to produce these 25 frames, has to use some method of combining the two fields used in the interlaced analogue video system into one non-interlaced digital frame. Typical methods used by capture software include, in the simplest case, combining two fields into one frame having alternating lines from each of the fields, to more complex cases where some form of colour interpolation is done on the combined frame in order to ‘smooth out’ large image variations between fields, with the purpose of creating a more representative vision of captured footage (Watkinson 1994).
As the footage was recorded with the intent of using only the image captured by one field (not an image combined with one captured 0.02s later) it was imperative that no interpolation of the captured footage take place, thus preserving the integrity of the data captured and allowing for the separation of the fields into distinct frames. Fortunately the Pinnacle Studio 5 editing/capturing software contains a field overlay option, such that fields could be combined without any software interpolation taking place. The captured footage itself proved a testament to the method of field combination. When a single captured frame was enlarged and viewed in MATLAB, a variation in light and dark areas was exhibited between rows of pixels. These variations were not in keeping with expected row-to-row variations, and produced an effect similar to that of ‘ghosting’. This effect was most pronounced on selected videos, in the initial and final frames, where the first captured field corresponded to the second field recorded on the VCR (illustrated in Figure 5.1). When the footage was captured it was combined with a blank first field resulting in a digital frame where every other line is blank. A similar situation was apt to occur on selected final frames as well.

The above situations were further tested by a test script (see Appendix C) developed
in MATLAB that extracted alternating lines from a captured frame and stored them as separate images. Where the frame was taken from the middle of the hot jet footage, the result was two distinct and separate frames (see Figure 5.2) that varied in content sufficiently to indicate the above suppositions were correct. When the same test script was trialed on an initial frame that exhibited alternating blank lines, the resulting pair of images (see Figure 5.3) showed one containing a Schlieren image of the hot air jet and the other completely blank. The blank lines in the initial and final frames also proved useful in ascertaining whether the first field was displayed on the odd or even lines. This made it possible to tell which of the images, extracted by the test file, came first without having to resort to visual observation of the images to ascertain the order. In all of the cases tested the image extracted from the even lines proved to be that captured first.

Having established that the captured frames could be separated, and the correct manner in which to re-ordered the separated frames, it was now possible to separate and re-order each of the captured hot air jet videos. As evidenced in Figures 5.3 and 5.2 the separated frames would be half height (288 × 720) unless the absent alternating lines were filled. It was considered necessary to fill the blank lines in order to preserve the dimensional representation of the frame, especially if the frame were to be eventually utilised for some form of quantitative data extraction. Possibilities included (1) using a blank or neutral filler line colour and (2) filling the blank line with the data from the previous line. The latter was chosen, in spite of the fact that it might be viewed as data creation. The basis for this was that it would be more suited to the method of frame rotation that was then under consideration, as it would be less likely to significantly alter the data. The method developed to rotate the frames is expanded upon later in Section 5.5. The footage taken of the cross-hairs was not separated and re-ordered as the footage was completely static. The original composite frames were seen as more useful for getting reference points in order to accurately and consistantly rotate and realign the four videos.

It is interesting that the separated digital videos from each of the recorders contained exactly 60 frames of hot air jet images, indicating that no frames were lost or ‘dropped’ during analogue recording or digital capture.
5.4 Frame Extraction and Re-ordering

Figure 5.2: A pair of half-height, separated frames from the test footage. Note that the images display distinct and different eddy formations confirming a progression of time between the recording of the two.

Figure 5.3: A pair of half-height, separated initial frames from the test footage. During capture a field containing recorded data has been combined with the preceding, blank field. When the fields were separated one of the images contained recorded data; the other is blank.
5.5 Frame Rotation

The positioning of the CCDs within the optical system, meant that there was a relative rotation of the image between videos as is seen in the non-rotated montage Figure 5.4. There is also a difference in intensity between the videos but this was seen as only a minor impediment to high speed sequential playback. In order for the four sets of footage to be collated and played as a single sequence, each of the videos would have to be counter-rotated back to an arbitrary datum (eg. the horizontal or vertical axis). Initially it was thought this might be achieved through a transformation of the data field, essentially shifting the colour values from one position \((m, n)\) in the array, through an angle of rotation, to another. While this could be considered an established image rotation technique its limitations primarily revolve around accuracy of rotation. The smallest angular division available is limited by the resolution of the image or frame and when the colour values are shifted, the pixels won’t be displayed as rotated pixels but rather as non-rotated pixels, creating some measure of image distortion. This method also assumes that there is little or no skew in the frames such that the only requirement is rotation of the whole frame through some arbitrary angle. With these limitations in mind it was highly desirable to find a more accurate, less distorting method of image/frame rotation.

5.5.1 Cross-Hair and Reference Points

In order to make the best use of a more accurate rotation method, it was very important to ensure points of common reference could be accurately located on the captured footage. Footage of a cross-hair set in place during the capturing of the experiment footage, was able to provide us with the necessary points of reference. This cross-hair footage was recorded immediately before the hot air jet footage was recorded (ie. under the same conditions).

The cross-hair used (shown in Figure 5.5) consisted of a clear transparency sheet with a pair of perpendicular lines and a set of concentric circles centred at the intersection of the perpendicular lines. Once the footage had been digitally captured, it was possible,
in MATLAB, to find equations that described the perpendicular lines in the image’s own coordinate system. More specifically, this was accomplished by the selection of a fairly large number of pixels that represented a line and performing a least-squares fit of its corresponding coordinates. This mathematical description provided two angles of relative rotation (ideally orthogonal to one another) and a common centre point for each of the four sets of footage. The representational accuracy of each of the fits was evaluated visually and demonstrated in Figure 5.6. It must be noted that the centre reference point, being formed at the intersection of the two lines, is not an approximate ‘centre pixel’ but a decimal coordinate in the image’s own coordinate system. Having established a method of getting points of reference with a precision greater than one pixel, it was necessary to develop a frame rotation method that was in keeping with this new-found level of precision.
Figure 5.5: An extracted frame from the footage of the cross-hairs. Note that the white and black lines, which would normally be perpendicular are not and that the concentric circles are somewhat elliptical, indicating a level of aberration within the frame. Axes shown are the default MATLAB image axes.

Figure 5.6: The same extracted frame as seen in Figure 5.5 from the cross-hair footage, showing the lines fitted to the cross-hairs for the purpose of providing reference points for frame rotation.
5.5 Frame Rotation

5.5.2 Frame Rotation Techniques

The more traditional technique of image rotation described earlier in Section 5.5 was abandoned in favour of one developed in the course of this project. The technique revolves around the creation of a grid of points, with a unit distance between points, in the image coordinate system and performing a 2-dimensional linear interpolation of the colour values, in the \textit{cdata} field, that surround each of the points in the grid. The fundamental assumption behind this technique is, that the colour values at a point inbetween the centre of two pixels (or four pixels in a 2-dimensional case) can be approximated by a value inbetween the values adjacent. This assumption would be considered valid where the physical distance between points represented or captured at a given pixel location, is small. MATLAB was used to create the grid and perform the linear interpolation at each of the grid points, on each of the 3 colour layers (Red, Green and Blue). By creating the grid about the centre reference point and with reference to the relative angular rotation in each of the footage sets the resulting array of colour values displayed the image or frame parallel to the reference datum (the horizontal or vertical axis), thus achieving rotation of the captured image or frame at a level of precision theoretically similar to that achieved when fitting lines to the cross-hairs.

The method used to create the interpolation grid was influenced greatly by the discovery of some measure of skew or optical aberration in the captured footage. While there exists a few possible causes of this distortion, the most likely explanation results from the optical system in place. The optical system in place at the USQ Gun Tunnel Facility is an LED illuminated Cranz-Schardin Camera similar to one developed for use by the University of Queensland’s compressible flow visualisation facilities (Brouwer 1999). In the multiple camera configuration used by Brouwer (1999), a measure of spherical aberration was identified in their test images. This was attributed to the large angle of incidence between the light beams from the LEDs and the so-called ‘optical axis’. Evidence of distortion in the hot air jet footage could be best viewed in the cross-hair footage, where the ‘perpendicular’ lines were markedly non-perpendicular. The lines however, were still straight and by performing the least squares regression on the cross-hair footage it was possible to quantify the level of angular skew present in each of the four sets of test footage. In consequence, it was now feasible to introduce an
appropriate counter-skew into the interpolation grid that would display the rotated image free of the angular-skew component of the distortion.

5.5.3 Grid Creation

The interpolation grid used in each of the four cases was created such that the interpolated (rotated) image displayed the line parallel to the hot air jet’s flow direction (see Figure 5.5), parallel to the horizontal axis. The grid consisted of coordinates \((x, y)\) centred around the reference centre in the image’s coordinate system. The location \((m, n)\) in the grid coordinate array corresponded to the final interpolated image’s pixel location \((m, n)\), where \(m\) is the row and \(n\) is the column. Thus the set of coordinates at \((1, 1)\) in the grid coordinate array corresponded to the pixel colour values at \((1, 1)\) in the final rotated image. In order to counter the angular skew present, each of the rows in the interpolation grid were created parallel to the line that is approximately parallel to the direction of flow (the ‘white’ line in Figure 5.5), while the columns were created parallel to the line that is normally perpendicular to the white line (the ‘black’ line in Figure 5.5).

The first stage of grid creation involved determining the coordinates of the initial point where \((m, n) = (1, 1)\), illustrated in Figure 5.7). Using the two line angles \(\theta_1\) and \(\theta_2\) measured from the horizontal axis, we can use the rule of cosines to determine the distance \((D)\) of the initial point from the centre reference \((x_c, y_c)\) viz:

\[
D^2 = m_b^2 + n_b^2 - 2m_bn_b\cos(\pi + \theta_1 - \theta_2) \tag{5.1}
\]

where \(m_b\) and \(n_b\) are the number of rows and columns in front of the centre reference point \((x_c, y_c)\) respectively. The length \((D)\) can be resolved into a horizontal component \(x_{comp}\) and a vertical component \(y_{comp}\).

\[
x_{comp} = D\cos(\theta_1 + \theta_3)
\]

\[
y_{comp} = D\sin(\theta_1 + \theta_3) \tag{5.2}
\]
where $\theta_3$ is the angle made by the white line and the distance line $(D)$:

$$\theta_3 = \sin^{-1} \left( \frac{m_b \sin (\pi + \theta_1 - \theta_2)}{D} \right) \quad (5.3)$$

Thus the coordinates $(x_1, y_1)$ of the initial point, in the image’s local coordinates can be found as:

$$x_1 = x_c - x_{comp}$$

$$y_1 = y_c - y_{comp}. \quad (5.4)$$

Having established a starting point from which to build the interpolation grid, the physical position $(x, y)$ of any grid position $(m, n)$ could be determined by the following pair of equations:

$$x = x_1 + (n - 1) \cos(\theta_1) - (m - 1) \cos(\pi - \theta_2)$$

$$y = y_1 + (n - 1) \sin(\theta_1) + (m - 1) \sin(\pi - \theta_2) \quad (5.5)$$

As such an array of grid-points with which to perform the colour interpolated rotation described in Subsection 5.5.2 could be constructed.

### 5.5.4 Cross-hair Rotation

The frame or image to be rotated was ‘padded’ with a black border in order to compensate for grid points that fell outside the frame’s domain. The resulting padded frame is shown with an interpolation grid overlayed in Figure 5.8. The frame is shown in the default image coordinate system used by MATLAB. It demonstrates how the grid was constructed with the rows parallel to the horizontal white line and, correspondingly, the columns parallel to the vertical black line. The angular skew apparent in the test footage is highlighted by the visible skew of the grid.
5.5 Frame Rotation

Figure 5.7: This figure shows the geometry used to construct the interpolation grid. The angular skew has been accentuated for illustrative purposes.

Figure 5.8: A ‘padded frame’ from the cross-hair footage shown with the interpolation grid overlayed. (Note: Only every 20th point is shown for clarity.)
After performing the linear interpolation of the colour values at each of the grid points and storing those values in an image array the resulting rotated frame (Figure 5.9) shows not only a change in orientation but a much improved level of perpendicularity between the horizontal and vertical lines. This improvement is further witnessed by the nozzle outline which now appears square.

5.6 Test Footage Rotation

The theory and code developed was formulated into a succinct script suite designed to rotate single frames and images or AVI sequences such as the test footage being used. The test footage initially consisted of four AVI videos of about thirty frames duration, representing a total recording period of approximately 1.2s. Due to the half-frame nature of the Schleiren system used, the total number of frames extracted stood at 60 frames per video. It could quite well be considered a testament to the optical and recording systems that no discrepancy was observed in the number of frames in each video due to missed or ‘dropped’ frames.

Each of the videos were run through the rotation scripts and were reordered into a sin-
Figure 5.10: Four sequential frames from the final reordered sequence. The top left is video 1 with the top right being the footage from video 2. Videos 3 and 4 make up the bottom left and right respectively.

gle sequence comprised of 240 frames, meaning that the resulting continuous sequence of the hot air jet footage represented a theoretical frame rate of 200 frames/second. A selection of sequential frames from the final sequence is shown in Figure 5.10. The frames appear in the same orientation and the nozzle outline seems free of angular distortion. Some variation in brightness is noticeable as well as small variations in magnification between successive frames. These variations result from minor discrepancies in the optical system and while they can be minimised by careful setup, it is unrealistic to expect that they could be removed entirely.

The playback of the final sequence is not entirely without variation but the result is quite good given it is comprised of footage that is essentially from four separate sources. While transient observation and analysis of the recorded data would have been quite possible without any rotation of the footage, the alignment of the videos with common axes is of great benefit to this process.

### 5.7 Grid Development

The cross-hair employed for the hot air jet testing provided sufficient reference geometry with which to rotate the video frames. It was decided for the gun tunnel experi-
mentation to vary the cross-hair geometry to better suit the anticipated experimental requirements. The cross-hair used for the gun tunnel work consisted of a grid of lines of 10 mm spacing. A rotated frame is shown in Figure 5.11 where the grid used can be viewed in place along with the silhouette of the cone.

This benefit of the new grid geometry was two-fold. Firstly it facilitated a better understanding of the level and type of distortion that had been indentified from previous cross-hair footage and secondly, it allowed the conversion of digitised distances (in pixels) into physical dimensions (in millimetres).

The angular component of the skew has been removed during frame rotation and the resulting image (Figure 5.11) is markedly free from aberration with the only observable distortion being a slight curvature of the straight lines as they tend towards the edge of the frame. The effect of this aberration was minimal as most of the observable structure in the gun tunnel footage was towards the centre of the frame.
By finding the equivalent spacing of the grid elements, in pixels, it was possible to determine the conversion factor between measured pixel and physical distances. The grid spacing was measured on the rotated image by fitting two lines to a pair of parallel cross-hair grid lines and a third to a line perpendicular to the first two. The intersections of these fitted lines were easily attainable and the distance between these points formed the grid spacing of the image. The line fitting and intersection calculations were handled by a MATLAB script.

Fitting lines to a pair of vertical lines (ie. the horizontal grid spacing) returned a distance of 75.3 pixels which for a 10 mm physical grid space results in a conversion factor of 0.133 mm/pixel. The rotated frame appeared to show a measure of elongation in the vertical direction and so for completeness the process was repeated for the vertical grid spacing. The lines fitted to the image are shown in Figure 5.12. The returned grid spacing was 75.1 pixels indicating that elongation in either the horizontal or vertical direction was virtually non-existant and the appearance of such elongation most probably resulted from perceived optical illusion.
In summary this chapter covered the computational analysis applied to image data collected by the flow visualisation facilities in place at the USQ Gun Tunnel Facility. Specific mention was made of the image acquisition and extraction process in preparation for the footage to be rotated and reordered. The development of the colour interpolation rotation technique used to rotate the images was covered in the context of its application to test-case footage of a hot air jet issuing into the quiescent room surrounds.
Chapter 6

Hot Air Jet Analysis

6.1 Introduction

This chapter deals with analysis and description of the flow behaviour shown in the footage of the hot air jet. While the primary purpose of this footage was to aid the optimisation of the flow visualisation and image acquisition systems (detailed in Chapter 5), it was also deemed appropriate to make further use of the footage and, if nothing else, establish a methodology or approach that could be applied to the footage gained during gun tunnel experimentation. In an effort to establish this methodology the information presented in this chapter initially considers qualitative description of the flow domain and then proceeds to the quantitative analysis aspect.

6.2 Qualitative Flow Domain Descriptions

6.2.1 Gross Flow Characteristics

Figure 6.1 shows a typical frame from the final sequence created from the hot air jet footage. To the left of the frame, the nozzle of the heat gun is shown in silhouette with the swirling mass of hot air shown flowing across the frame. While it is of little consequence, there is some disparity between the horizontal orientation of the image
and the physical orientation of the jet (which was vertical during experimentation).

On the actual heat gun there is a flat plate just inside the nozzle which acts as a turbulisor, presumably to ensure consistency of air temperature across the jet in an effort to avoid ‘hot spots’. The flow shown is somewhat directional, with small but consistent change in mean jet diameter throughout the length of the frame. These properties would be viewed as desirable in a heat gun, as the purpose of such a device would be the consistent heating of a localised area.

The frame shows a marked darkening towards the bottom of the frame and this identifies the image as a Schlieren image with the knife edge parallel to the flow direction. The domain shown is, by its very nature, a complex 3-dimensional flow domain. With this in mind, the bulk of the jet shown must be viewed as an integral sum of the density gradients encountered by the light along its path through the hot air jet.

### 6.2.2 Transient Observations

The primary advantage in having a multiple frame imaging system in place is the ability to observe changes in flow structure with time; put simply, the observation of transience. This transient observation capability is extremely useful in tracking the growth and movement of turbulent structure. In the case of the hot air jet, the bulk of the image
was an integral density sum or a 2-dimensional expression of a markedly 3-dimensional nature, this meant that valid observation and tracking of turbulent structure was only possible on the edges of the jet.

Figures 6.2-6.5 show frames 21-24 from the hot air jet sequence. Shown is the growth and movement of a convection eddy of the jet edge in successive frames. Figure 6.2 shows the structure as little more than a bulge in the side of the main jet (shown by the arrow), whereas in the next frame, Figure 6.3 the ‘bulge’ has grown in size and structure and shifted further downstream. In Figure 6.4 we see less downstream shift but the structure appears to have nearly broken away from the jet proper. In the final frame, Figure 6.5, the fully-detached structure is now faintly observable as the air in the structure cools and its density becomes indistinguishable from the surrounding air. This progressive property assimilation is witnessed in the series depicted by a lessening in the intensity change where the structure occurs.

6.3 Quantitative Analysis

In the broader sense, the visualisation of a flow domain is of primary benefit to areas such as experimental confirmation (ie. proving something did actually happen), general descriptions of flow domains (such as that done above in Section 6.2) and to facilitate
Figure 6.3: Frame 22 from the hot air jet sequence.

Figure 6.4: Frame 23 from the hot air jet sequence.

Figure 6.5: Frame 24 from the hot air jet sequence.
6.3 Quantitative Analysis

comparison with and validation of CFD (Computational Fluid Dynamics) models. To move beyond this scope is to enter the world which seeks to gain hard data from images, in essence to shift from the world of qualitative general description to that of quantitative data collection.

For the same reasons outlined in the qualitative analysis, the domain of interest is limited to the edge of the jet. With this in mind it was proposed to consider a means of jet edge detection and investigate the changes that occur in the jet edge in successive frames.

6.3.1 Jet Edge Detection

A proposed method of jet edge detection was initially to convert the frame from a RGB (Red-Green-Blue) format to an HSV (Hue-Saturation-Value) format using MATLAB where the ‘V’ (value) represents a measure of the pixel’s intensity. As the Schlieren method primarily results in variations in intensity, the so-called ‘jet edge’ would be indicated by a marked change in the intensity value. Physically the jet edge present in the top half of the frame would be detected by working from the top, row-by-row down the column of pixels until the intensity reached a predetermined cut off value. This yielded a series of points representing the jet edge.

A general variation in frame-to-frame brightness was mentioned in Chapter 5, resulting from subtle variations in the setup of each of the optical paths in the Cranz-Schardin camera. While this was of minor consequence to the creation of a single sequence, it represented a problem in that without taking into account these variations it would be impossible to use a single cut off intensity value on successive frames. These variations were overcome by considering the change in actual intensity \((I_{m,n})\) from the average intensity \((I_{ave})\) of the top half of the frame so that:

\[
\Delta I = I_{m,n} - (I_{ave})
\]  

(6.1)

where \(\Delta I\) is the change in intensity.
6.3 Quantitative Analysis

Figure 6.6: Plot of the change in intensity $\Delta I$ looking down a typical column of a typical frame.

Figure 6.6 shows a plot of the change in intensity looking down a typical column of pixels from a frame of the hot air jet footage. The plot shows a spike in the $\Delta I$ value which marks the edge of the jet. Through careful choice of the intensity cutoff and the starting row of the detection script this method proved quite successful at returning position values for the edge of the jet.

Figure 6.7 shows a frame from the jet footage (in the default MATLAB image coordinate system) that has been converted to HSV using the `rgb2hsv` command (which for some reason resulted in a predominately blue hue). The red crosses designate the detected edge of the jet using the methodology described above. The fine line fitted through the middle of the points was used to normalise the location of the detected points.

Figure 6.8 shows the following frame from that shown in Figure 6.7. Again the converted image is visible, along with the detected edge and the line about which the points were normalised. What is important to note however, is that though there is significant variation in mean brightness (or intensity) between the successive frames it was possible to use a single $\Delta I$ cutoff value. This meant that it was quite possible to consistently detect the jet edge position on successive frames using a single cut off value.
Figure 6.7: Frame 30 from the hot air jet sequence shown with a plot of the detected edge points and the line used to normalise the position of the points.

Figure 6.8: Frame 31 from the hot air jet sequence shown with a plot of the detected edge points and the line used to normalise the position of the points. The same intensity cut off value has been used for the successive frames.
6.3.2 Jet Edge Cross-Correlation

The final step in the analysis of the hot air jet was to compare the detected jet edges. This was accomplished using a statistical cross-correlation of the two positional data sets from a pair of successive frames. The cross-correlation was performed using the MATLAB Signal Processing Toolbox and more specifically the `xcorr` command which outputs the measure of correlation between the two data sets at various lags. By finding the lag at which the data sets seemed to best correlate, it was hoped, at the time, using the found lag (in pixels) and the known time gap between successive frames to determine what might be termed a ‘convective velocity’ of the edge of the jet.

In practice, the results from the cross-correlation were not particularly encouraging with the majority of cases tested (ie. pairs of successive frames analysed) returning a correlation vs lag plot similar to the one shown in Figure 6.9 where the best correlation was found at a lag of zero pixels. This would tend to indicate very little correlation exists between the majority of successive frames. There are a number of reasons why this would be the case, principally:

- The flow domain being studied is a highly turbulent, 3-dimensional field and as such moving and changing a great deal in a very short space of time (ie. too much change is occurring in the 5ms between frames).

- Turbulent structures would be more sharply identifiable if the exposure time was reduced.

- The 3-dimensional effects still operate to some extent on the jet edges which casts some shadow over the validity of a 2-dimensional treatment of the edge.

- The small variations in the optical paths that form the multiple frame camera and the placement of the knife edges would quite likely result in some camera-to-camera variation in sensitivity to the density gradients.

The time gap between hot air jet frames was determined from the physical limit of the multiple frame camera if the sequence was to be infinitely continuous. If the experiment was to be re-run with the specific purpose of edge detection and cross-correlation, a
6.3 Quantitative Analysis

Figure 6.9: Plot of the cross-correlation performed on Frames 21-22. The above plot indicates that the best correlation occurs at a lag of zero.

A shorter time gap could be readily employed at the cost of only being able to capture four frames in succession. A reduction in exposure time is also quite feasible, the trade-off however, is a loss in total image intensity. Finally, with very careful setup, variations between cameras can be minimised resulting in less variation in sensitivity and brightness.

Despite all of this there was one slightly more reasonable result from an analysis conducted on Frames 22 and 23. These frames were previously shown in Section 6.2. The images are shown again, along with the detected edges in Figures 6.10 and 6.11. The image plots show the edge of the structure (discussed in Section 6.2) and its subsequent detection.

The cross-correlation of these detected edges showed the best correlation occurring at a lag of $-7$ pixels and this is shown in Figure 6.12. This would tend to indicate that the edge of Frame 23 leads the edge of Frame 22 by 7 pixels. Assuming a conversion factor equal to that found in Chapter 5 of 0.133 mm/pixel and a time gap of 0.005 s this results in a convective edge velocity of around 186.00 mm/s. While the shape of
Figure 6.10: Frame 22 is shown in the default MATLAB image coordinate system. The crosses indicate the detected edge points.

Figure 6.11: Frame 23 is shown in the default MATLAB image coordinate system. Again the crosses indicate the detected edge points.
6.3 Quantitative Analysis

the correlation curve indicates a comparatively strong correlation at and around this lag value, it must be said that these results must be seen in terms of order of magnitude only and that their validity is overshadowed by the marked inconsistency with which comparable results can be obtained from other frame edge comparisons.

Figure 6.12: Plot of the cross-correlation performed on Frames 22-23. The above plot indicates that the best correlation occurs at a lag of $-7$. 
This chapter dealt primarily with the analysis on the hot air jet footage that was conducted as a follow up to the creation of a single sequence (discussed in Chapter 5). Initially the focus was on the qualitative analysis of a series of frames that showed the growth and progression of a markedly identifiable turbulent eddy structure and then progression was made from descriptive qualitative study to a data-gathering quantitative analysis. On the quantitative front, a demonstration was given of a fairly simple edge detection technique and subsequent cross-correlation of detected jet edges.
Chapter 7

Gun Tunnel Experimentation

7.1 Introduction

This chapter expounds upon the culmination of the work completed and discussed in all previous chapters, namely the regime of experimentation conducted in the hypersonic test flow. The approach taken bears similarity to the gross approach developed and expanded upon in Chapter 6, where the footage obtained from testing was first described in qualitative manner and then the flow structures subjected to quantitative analysis.

All gun tunnel runs analysed employed a single diaphragm with a gauge driver tank pressure reading of 3.2 MPa. The diaphragm was manually punctured each run.

7.2 Experimental Conical Flow

Some experimentation took place where the conical test piece was subjected to the gun tunnel flow without gas injection taking place. This was done as a means of generating a purely conical flow regime in order to facilitate direct comparison between experimental flow structures and results obtained from the numerical model of conical flow detailed in Chapter 3. While flow properties such as pressure and velocity details are beyond the
present scope of possible analysis (given the current facilities), the conical shock angles obtained from the numerical model and the flow visualisation footage obtained during the gun tunnel run offer an opportunity for direct comparison and hence a measure of model/experimental validation.

7.2.1 Numerical Shock Angle

The results obtained in Chapter 3 indicate that for a free-stream stagnation pressure $P_o$ of around 2.65 MPa and a Mach No. of 7, the angle of the conical shock ($θ_s$) is found to be 12.90 degrees. This is the shock angle that was compared with the experimental value of shock angle obtained. The stagnation pressure used was found from pressure transducer data available from single diaphragm runs that had been previously conducted.

7.2.2 Experimental Shock Angle

Figure 7.1 shows Frame 1 from the gun tunnel run GT34. This figure formed the primary basis for determination of the experimental conical shock angle. The odd diffraction pattern observed in this figure can be attributed to the laser diode light source used. As well as the conical flow aspect, this particular run was also used to test laser diodes as a possible replacement for the LEDs that had been used previously. This illumination experimentation is detailed in Section 4.3.

The frame shows the presence of the conical shock at an acute angle to the cone (in silhouette). The acuteness of this angle could be considered a distinguishing trait of hypersonic flow (Anderson 1990). As the physical geometry of the cone is known (ie. the cone angle ($θ_c$) is known) all that remains is to determine the angle between the outline of the cone and that of the shock. This was accomplished by fitting two lines to the image (shown in Figure 7.2) and calculating the angle between the line $Δθ$ where:

$$Δθ = \tan^{-1}(m_1) - \tan^{-1}(m_2)$$  \hspace{1cm} (7.1)
Figure 7.1: A frame obtained from the gun tunnel run GT34. Shown is the conical shock structure that has developed around the hypersonic cone.
7.2 Experimental Conical Flow

Figure 7.2: Shown above are the lines fitted to determine the conical shock angle that occurred during experimentation.

and $m_1$ and $m_2$ are the respective gradients of the fitted lines. Using this technique of the frame from GT34 an experimental cone angle of 13.9 degrees was found.

7.2.3 Shock Angle Comparison

The disparity of nearly a degree between the numerical shock angle and the experimental shock angle was significant given the cone angle itself is 10 degrees. There are however a few reasons as to why this difference may exist:

1. Boundary layers tend to be thicker in hypersonic flow regimes (Anderson 1990) making the object in the flow appear thicker to the inviscid flow outside the boundary layer (Anderson 1990). The numerical conical flow model deals with purely inviscid flow behaviour and the fact that supersonic shear layers are comparatively thin makes an assumption of inviscid flow more reasonable. In the case of hypersonic flow, the combination of more acute shock angles and thicker boundary layers makes inviscid flow behaviour assumptions less reasonable as the numerical model would tend to underestimate the conical shock angle.
2. The surface of the cone would be described as a general machined finish. Very small surface irregularities would tend to ‘trip’ the flow, contributing to boundary layer growth and encroachment upon the inviscid layer.

3. Confines of space within the test section meant that the apex of the cone actually sat a distance inside the nozzle. This was unavoidable and less than ideal as the test flow would not have been fully expanded to Mach 7 as it encountered the cone.

With the last point in mind and as a comparison the numerical model was re-solved with a free stream Mach no. of 5.5 and this yielded a shock angle of 13.84 degrees. This does not necessarily mean that the Mach number is lower than expected but is designed to give an indication of the types of errors that were of significance (both in the numerical modelling and experimentation). Given that the same physical experimental set up was employed for the gas injection runs, if nothing else this analysis indicates the type of errors that were likely to be of significance.

7.3 Pre-experimental Analysis

Some pre-experimental analysis and modelling was performed in order to better understand some expected flow field characteristics and aid in experimental case selection.

7.3.1 Analytical Modelling

In order for the injected gas to exit the annulus at the speed of sound, it was necessary to ensure two parameters were satisfied, namely that the total annulus area was much less than the upstream cavity area and that the static pressure immediately surrounding the annulus (the so-called ‘back pressure’ \( P_b \)) was less than the static pressure of the injection gas at the annulus exit \( P_{inj} \). If the gas exit pressure was less than the back pressure \( P_b \) then the injection flow would be ‘choked’ and would not reach sonic speeds.

A simple model was developed to estimate \( P_b \) based on the results from the numerical
Conical Shock

Normal Shock

Annulus

Figure 7.3: This figure illustrates the model developed to estimate the annulus back pressure $P_b$.

Conical flow solution as it was anticipated that the injection flow behaviour would principally take place behind the conical shock. A diagram of the model is shown in Figure 7.3. Essentially the flow properties of the property ray that occurred nearest to the cone were considered with the bow shock structures normally seen in standard transverse injection models (see Figure 2.1) approximated by a normal shock placed just in front of the annulus. This bow shock approximation is based on the idea that property changes across a bow shock, as it approaches a perpendicular orientation to the flow, approach those of a normal shock (Anderson 1990). So for a free-stream stagnation pressure $P_{o1} = 2.65 \text{ MPa}$ and a Mach number of 7, the solver returns the following properties:

$$
M_2 = 5.49 \\
\frac{P_{o2}}{P_2} = 2.408 \text{ MPa} \\
\frac{P_{o2}}{P_2} = 918.4
$$

where $P_{o2}$ is the stagnation pressure behind the conical shock and the Mach number $M_2$ is for the ray nearest to the face of the cone. The pressure ratio $P_{o2}/P_2$ is obtained using standard isentropic flow relations from the ray Mach number. The static pressure along this ray would be:
The static pressure ratio across the normal shock is given by Anderson (1990) as:

\[
\frac{P_3}{P_2} = 1 + \frac{2\gamma}{\gamma + 1} (M_s^2 - 1)
\]  

(7.2)

where \( P_3 \) is the estimated back pressure \( P_b \) and \( \gamma \) is a ratio of specific heats. For this purpose a value of \( \gamma = 1.4 \) was assumed yielding:

\[
\frac{P_b}{P_2} = 1 + \frac{2 \times 1.4}{1.4 + 1} (5.49^2 - 1)
\]

\[
\frac{P_b}{P_2} = 34.97
\]

therefore:

\[
P_b = 34.97P_2
\]

\[
P_b = 91.7 \times 10^3 \text{ Pa}(\text{abs}).
\]

This process yields an estimate of the static back pressure of around 91.7 kPa. It is important to realise that while the fundamental assumption of normal shock behaviour would be considered valid, this assumption would tend to produce results that overestimate the back pressure as property change is more violent across a normal shock and while assumed normal shock behaviour is useful for simplified analysis, actual normal shock behaviour would represent a significant total pressure loss which in turn would have a negative effect on the cycle efficiency of a theoretical scramjet.
7.3.2 Experimental Case Selection

Having established a guide back pressure with which to work, the next step was to look at experimental case parameters. In short the aim was to study and observe the flow structures evident for a couple of $\bar{q}$ values (see Section 2.1 for definition) and, using the back pressure calculated, examine the effects of fully expanded vs choked injection flow.

The physical parameters available for variation included adjustment of the annulus gap on the cone itself, variation in the driver tank pressure, which in turn affected the free-stream stagnation pressure ($P_{o \infty}$) and the injection tank pressure which governed the injection stagnation pressure ($P_{o inj}$). In order to keep the variation in set-up to a minimum, it was decided to standardise the annulus gap and the driver tank pressure as the range of injection tank pressures available would allow sufficient variation in $\bar{q}$ to be achieved.

Ultimately two runs were conducted (labelled GT35 and GT36) from which usable images and data could be obtained. The first run (GT35) had a tank pressure of 150 kPa (gauge), with estimates made at the time that this figure would result in a static injection pressure ($P_{inj}$) greater that the back pressure determined in Subsection 7.3.1 and hence fully expanded flow. For the second run (GT36) the injection tank pressure was 50 kPa (gauge) with the intention that this would result in an underexpanded injection jet. The differing $P_{o inj}$ values obtained by varying the tank pressure caused variation in the static injection pressure $P_{inj}$ which in turn resulted in direct variation in the value obtained for $\bar{q}$. Final values for $\bar{q}$ were not obtained until injection pressure measurements were obtained after the runs were conducted.

7.4 Footage Presentation

The following Figures 7.4 - 7.7 and Figures 7.8 - 7.11 are from GT35 and GT36 respectively. As with all other footage obtained the images presented here were recorded onto VHS tape, digitally captured and re-oriented and de-skewed using the scripts de-
Figure 7.4: The first frame from the GT35 run. Note the angle of the forebody conical shock and the bow shock that forms the outer boundary of the injection plume.

veloped and described in Chapter 5. Both cases were illuminated using LEDs. In the footage from GT35 an exposure time of around $13\mu s$ was used and the resulting images appeared overexposed in places. With this in mind the exposure time was reduced to $10\mu s$. In both cases the gap between frames is 3 ms.

7.5 Transverse Injection - Qualitative Analysis

This section deals primarily with the analysis conducted on the flow visualisation footage obtained from the gun tunnel runs GT35 and GT36. All of the analysis conducted is with reference to the previous Section 7.4.
Figure 7.5: The second frame from the GT35 run. The structure that is forming on the end of the nozzle indicates that the back pressure of the nozzle is larger than the static nozzle pressure.
7.5 Transverse Injection - Qualitative Analysis

Figure 7.6: The third frame from the GT35 run.
Figure 7.7: The fourth and final frame from the GT35 run.
Figure 7.8: The first frame from the GT36 run. Note the effect of a reduction in exposure time in terms of frame saturation and intensity.
Figure 7.9: The second frame from the GT36 run. There is a marked similarity between the gross structures that occur in this frame and those seen during GT35.
Figure 7.10: The third frame from the GT36 run.
Figure 7.11: The fourth and final frame from the GT36 run.
7.5 Transverse Injection - Qualitative Analysis

7.5.1 Gross Structure

Figure 7.4 shows the first frame from the GT35. In this frame we can clearly see the presence of an impinging jet and its apparent effect on the rest of the flow domain. The variation in intensity between the top and bottom halves of the frame shows that the knife edge in the Schlieren system was oriented parallel to the free stream (horizontal Schlieren).

The first important feature to note is the marked bright patch (or dark patch depending on which half of the frame is under consideration) that appears on the face of the cone, where the injected gas enters the flow around the cone. This bright patch indicates a marked change in fluid density at this point which would be consistent with pressure matching behaviour exhibited by an exiting jet. This coupled with the shape and expansive nature of subsequent jet behaviour is a very good indication that the jet has in fact been sonically expanded.

On the forebody of the cone the outline of the conical shock can be seen at a much less acute angle to the surface of the cone. It would also appear that the presence of the jet has resulted in a large recirculation zone along the surface of the cone which has been responsible for the detachment of the conical shock.

Finally, further along the cone the outer boundary or the total penetration of the jet is bounded by another marked density change indicating the presence of a bow shock structure. The jet itself, after the initial pressure matching, exhibits some initial mixing structure (shown by the dark patches near the jet plume). Eventually the jet plume properties assimilate with those of the surrounding flow.

Considering Figure 7.8, which is the first frame from GT36, the gross structure bears marked similarity to that of the frame just discussed, especially the injection jet pressure matching previously seen. The fact that the shape and nature of this section bears marked qualitative similarity to that of Figure 7.4 is a good indication that the transverse jet in this case has also been sonically expanded. This is not an unreasonable occurrence given that, if anything the model developed would tend to overestimate the back pressure.
7.5 Transverse Injection - Qualitative Analysis

While similar structures appear to be present in both frames, differences can be seen in terms of levels of shock detachment and jet penetration. The fact that in both cases the jet would appear to have been expanded to Mach 1 merely makes numerical comparison of shock angle and jet penetration seen in the two cases more reasonable.

7.5.2 Transient Observations

The most noticeable change shown in both the GT35 and GT36 sequences is the development of an axial oblique shock structure off the nozzle after the first frame. The presence of this structure results in some discontinuity of the bow shock that forms the outermost border of the injected jet. The apparent lack of this type of structure in the first frames from both gun tunnel runs strongly influenced the decision to restrict initial gross structure description to these frames.

The presence of this type of structure indicates that the free stream flow is being turned inward due to the pressure in the test section being greater than the static pressure of the free stream flow. Giving initial consideration to the the static pressure of the free stream jet $P_\infty$ which for isentropic gas expansion is given in Anderson (1990) as:

$$\frac{P_o}{P_\infty} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\gamma/(\gamma-1)}$$

which for a Mach number of 7 and $\gamma$ of 1.4 gives:

$$\frac{P_o}{P_\infty} = 0.414 \times 10^4$$

given that $P_o = 2.65$ MPa (as used previously):

$$P_\infty = \frac{2.65 \times 10^6}{0.414 \times 10^4} = 640.09 \text{ Pa.}$$
Before firing, the test section is evacuated and the pressure of the test section is monitored via a digital pressure sensor, which could be used to establish atmospheric pressure in the gun tunnel and a mercury manometer, which gave an indication of absolute test section pressure. While a near perfect vacuum would have been ideal this is always difficult to achieve and generally the absolute pressure in the test section prior to firing was around the 400 - 500 Pa mark. This gives a good indication as to why we progressively see the formation of nozzle shock structures as initially the static nozzle pressure and the back pressure of the test section would be well matched. With time however, more of the test gas enters the test section and dump tank resulting in a rise in the back pressure such that the free stream is now ‘choked’.

This case highlights the benefits of having a flow visualisation setup in terms of the identification of free stream anomalies and subsequent error in measured flow properties resulting from these anomalies. That is not to say the occurrence of such structures could not have been predicted prior to testing using a pressure comparison similar to that shown but rather to show how the equipment can be used to identify these shock structures and qualify their impact on the flow domain.

Another notable transient effect observed relates to Figures 7.8 - 7.11. There appears to be a gradual increase in the intensity of the bow shock that borders the jet plume. This is best attested to if the top half of the images are considered with the gradual darkening of the shock in relation to its surroundings indicating a more pronounced change in the local density gradient and hence an increase in the severity of the shock.

In general the discussion to date has been restricted to gross structure. This primarily relates to the fact that the effective shutter speeds employed do not appear to readily capture fine structure (such as shear or mixing layers). Always in image acquisition it is a trade off between field illumination time and total image intensity; if the illumination time is too large, structural detail is blurred; if too short then the lack of intensity begins to impede effective analysis. While some steps were taken to improve the overall level of light intensity (see Section 4.3) an improved mechanism of field illumination is still desirable.
7.6 Transverse Injection - Quantitative Analysis

In any experimentation it is always useful to obtain numerical data to complement experimental observations. This section deals with the measurements obtained from the experimentation.

7.6.1 Experimental Case Designation

As stated in Section 7.3.2 the two experimental cases were to be considered based on $\bar{q}$ values (ratios of injection to free stream momentum flux). Given the definition of this ratio (see Section 2.1):

$$\bar{q} = \frac{(\gamma PM^2)_{\text{inj}}}{(\gamma PM^2)_\infty}$$

(7.5)

where $P$ is the static pressure, $M$ is the Mach number and $\gamma$ is the ratio of specific heats. For both the experimental cases air was used for both the free stream and injectant gas, thus $\gamma_{\text{inj}}/\gamma_\infty \approx 1$. Hence Equation 7.5 simplifies to:

$$\bar{q} = \frac{(PM^2)_{\text{inj}}}{P M^2}_\infty$$

(7.6)

For both cases $P_\infty$ was known as were the respective Mach numbers. The decision was made to assume that all injectant streams were fully isentropically expanded, as full expansion appeared to be occurring in both Figures 7.4 - 7.7 and 7.8 - 7.11. While there was some evidence that indicated the presence of the cone in the nozzle affected the full expansion of the free stream and the effects of the progressive ‘choking’ of the flow were also of consequence, these effects were difficult to accurately quantify and therefore assumed to exert minimal effect on the full expansion of the test flow to Mach 7. It must be stated that these effects are of consequence and, having been identified, steps must be taken to ensure the minimisation of such effects in future experimentation.

A fault in the data-logging equipment resulted in no pressure data being available from
GT35 so initial consideration is given to the GT36 run. Given a previously calculated \( P_\infty \) value of 640.09 Pa and free stream/injection Mach numbers of 7 and 1 respectively all that was needed was the static injection pressure. The pressure transducer on the injection stream gave a mean injection stagnation pressure of \( P_{o \, inj} \) of 124.15 kPa (a plot of the pressure readings obtained is included in Figure 7.12), so using the isentropic pressure ratio equation (see Equation 7.3) we find:

\[
\frac{P_{o \, inj}}{P_{inj}} = 1.893 \\
P_{inj} = \frac{124.15 \times 10^3}{1.893} \\
P_{inj} = 65.6 \times 10^3 \text{ Pa}
\]

So for GT36 the momentum flux ratio was:

\[
\bar{q} = \frac{65.6 \times 10^3}{640.09 \times 7^2} \\
\bar{q} = 2.091
\]
With the exception of $P_{o \text{ inj}}$ all the parameters used to determine $\bar{q}$ for GT35 remained the same. It was possible to estimate $P_{o \text{ inj}}$ for GT35 by considering the losses in total pressure that occurred in GT36. From the evacuation of the test chamber it was possible to determine that the local atmospheric pressure was 94.9 kPa thus yielding an absolute tank pressure of 144.9 kPa for GT36. Compared with the transducer reading this shows a 14.23% loss in total pressure. Assuming that the same percentile pressure losses would occur for GT35, which had a gauge tank pressure of 150 kPa, $P_{o \text{ inj}}$ can be estimated:

\[
P_{o \text{ inj}} = (1 - 0.1423) \times (150 + 94.9)
\]

\[
P_{o \text{ inj}} = 209.8 \text{ kPa}
\]

For GT35 this yields a $\bar{q}$ of 3.54.

### 7.6.2 Image Based Data Acquisition

As discussed in Chapter 2, the quantification of jet penetration was defined by the parameter $h/D_j$. For the experimental work conducted there was also interest in observing changes in the forebody conical shock angle relative to the different experimental cases and the no-injection case. Furthermore, there was keen interest to exploit the multi-frame nature of the flow visualisation set-up and look into transient changes in the parameters investigated.

The jet dimension $D_j$ in this case was the gap in the annulus and was determined using a number of means. Measurements taken using a profile projector indicated a gap dimension of 0.33 mm. Some doubt was cast over the validity of this value when a value of 0.43 mm was obtained using a set of filler gauges. A final gap dimension was determined from the background images obtained, that show the cone in silhouette, using the script developed to determine the pixel conversion factor in Chapter 5. The three fitted lines and their intersection is shown in Figure 7.13. This returned a gap
The angle of the forebody conical shock was calculated using the scripts that had previously been used to determine the conical shock angle of the no-injection case GT34 (see Section 7.2). Again the procedure was repeated for all frames in the interest of transient observation. It was also pertinent to compare the injection cases to that of the run that was without gas injection. Qualitative comparison had indicated that the presence of the injection plume had resulted in a larger shock angle than observed in the conical flow run. Figure 7.14 shows an example of the lines fitted to the outline of the cone and the edge of the shock.
Figure 7.14: A plot showing the two lines that were fitted to a rotated frame in order to determine the conical shock angle for the injection cases.

Table 7.1: The conical shock angle and jet penetration measurements taken from the footage of GT35 for a $\bar{q}$ of 3.54.

<table>
<thead>
<tr>
<th>Time $\Delta t$ (ms)</th>
<th>Shock Angle $\theta_s$ (degrees)</th>
<th>Jet Penetration ($h/D_j$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>19.2</td>
<td>20.4</td>
</tr>
<tr>
<td>3</td>
<td>18.5</td>
<td>25.6</td>
</tr>
<tr>
<td>6</td>
<td>19.5</td>
<td>24.7</td>
</tr>
<tr>
<td>9</td>
<td>18.2</td>
<td>24.1</td>
</tr>
</tbody>
</table>

The results obtained for GT35 are tabulated in Table 7.1 and similarly for GT36 in Table 7.2. Considering first the jet penetration, it can be seen that the level of penetration is consistently higher for GT35 and this was to be expected given a larger value for $\bar{q}$. Indeed, the dimensionless jet penetration level of GT35 is around 26.7% greater than that seen in GT36 and this is consistent with the observation made by Schetz (1980) that $h/D_j$ is principally a function of $\bar{q}$. In both cases the level of penetration when $\Delta t = 0$ (Frame 1) is somewhat smaller than subsequent values. With time $h/D_j$ appears to vary around a steady-state value (if, given the complexity of the flow field under consideration, such a state exists).

In contrast, the shock angles obtained showed little change with $\bar{q}$ for the two cases in
Table 7.2: The conical shock angle and jet penetration measurements taken from the footage of GT36 for a $q$ of 2.091.

<table>
<thead>
<tr>
<th>Time $\Delta t$(ms)</th>
<th>Shock Angle $\theta_s$ (degrees)</th>
<th>Jet Penetration ($h/D_j$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>19.02</td>
<td>15.7</td>
</tr>
<tr>
<td>3</td>
<td>19.1</td>
<td>17.7</td>
</tr>
<tr>
<td>6</td>
<td>18.4</td>
<td>17.7</td>
</tr>
<tr>
<td>9</td>
<td>18.2</td>
<td>18.3</td>
</tr>
</tbody>
</table>

Figure 7.15: This image plot also shows the two lines that were fitted to the fourth frame from GT36. Of note is the visible bulge in the shock structure that is highlighted by the fitted line.

spite of the differences in jet penetration that occurred between cases. The presence of the impinging jet has certainly made the shock angle comparably larger than that observed during pure conical flow. In both examples the shock angle does not exhibit a great level of temporal stability and indeed the subtle oscillations in shock angle, shown in Table 7.1, would tend to indicate the occurrence of shock ‘flapping’ of the type observed by Verma (2002). As an aside, a shock ripple (also identified by Verma (2002)) can be seen to occur in Figure 7.15 in which a distinct bulge appears as a deviation from the line fitted to the shock.

In future experimentation it would be good to consider image acquisition at various
time values, in particular time values taken earlier than those seen here. The results presented indicate that, for the most part, the flow field under consideration has reached a nominally steady state and while observable transient changes have occurred in the testing conducted, the images and data give little indication of transient structure formation. Taking the photos earlier during testing would facilitate comparison between experimental results and those obtained from other sources such as transient CFD simulation.

It is important to note that the sources of error identified in Section 7.2, namely the presence of a section of the cone in the nozzle and the eventual choking of the free-stream, were still present during the gas injection runs. In future experimentation the effect of these errors will either have to be quantified or steps taken to ensure that they are minimised. A new, larger nozzle had been designed prior to the commencement of this project that would have allowed better positioning of the cone’s physical geometry, and while the design of the nozzle had been finalised it had not been constructed at the time of testing. Choked nozzle flow and the subsequent structures observed could be easily minimised or eliminated by using a double-plate diaphragm in conjunction with a higher driver tank pressure.

In spite of these issues, the results and observations obtained from the gun tunnel experimentation were very encouraging given the fact that this is the first time this geometrical set-up has been tested. If nothing else, the procedures and tools developed to obtain the results shown will facilitate continued experimentation using this model with the ultimate aim of such experimentation being the development of a correlation linking jet penetration with the $\bar{q}$ values.
7.7 Chapter Summary

This chapter detailed the results and observations obtained from the gun tunnel experimentation conducted. The first section dealt with pure conical flow experimentation and the comparison of results obtained through experimentation and numerical simulation. The issue of experimental case selection and designation for the transverse injection runs was covered with reference to an analytical model developed to estimate the back pressure around the gas injection exit. The final sections dealt with qualitative and quantitative description and results obtained using the images captured by the flow visualisation equipment.
Chapter 8

Conclusions and Further Work

The analysis and experimentation performed for this project has covered a broad range of both subject matter (such as hypersonic conical flow, optics and flow visualisation in addition to the title subject), and emphasis (design and development, analysis or numerical simulation and physical experimentation). Despite the wide ranging scope this project commenced with specific objectives and thus it is fitting to conclude by considering the achievements that have been made in relation to those objectives.

8.1 Achievement of Project Objectives

The achievements of this project are listed in relation to the project objectives detailed in the project specification in Appendix A.

1. Current designs and design principles used to enhance mixing in scramjet models to date were addressed in Chapters 1 and 2. Notably the point was made that the model studied during this project had a wider range of applications than simply that of scramjet forebody geometry.

2. The numerical solution developed to the Conical Flow model was presented in Chapter 3. This solution was successfully developed into a succinct MATLAB script and aided in the prediction of the back pressure around the conical injection
3. Investigation took place into fluid flow visualisation techniques with a specific emphasis on the Schlieren optical system employed in the multiple frame imaging system in situ. The generalities of flow visualisation setups and techniques were discussed in Chapter 2, while the actual system presently used in the USQ Gun Tunnel was covered in Chapter 4.

4. Chapter 5 showed the development of software-based tools that enhance the flow visualisation and imaging facilities as well as highlighting the attempts made to overcome some of the limitations presently imposed by the current setup. The most notable achievement of this section was the development and implementation of an interpolation frame rotation technique that enabled rotation of frames to a high degree of accuracy and removed the angular skew that had occurred during image acquisition.

5. The experimental apparatus used to conduct the hot air jet experiments is described in Chapter 4. The footage obtained from this experiment aided in the development of the frame rotation tools, the culmination of which was the creation of a continuous 240 frame sequence of the hot air jet.

6. The early sections of Chapter 7 include discussions of the gun tunnel runs that proved suitable for analysis. In addition to these runs mentioned a number of other runs were conducted for set-up and diagnostic purposes.

7. Chapter 7 contains the comparison between the experimental and numerical results obtained for the instance of pure Conical Flow. There was some measure of disparity between the numerical and experimental results and possible reasons for this difference were included as part of this critical comparison.

8. Having rotated and reordered the footage of the hot air jet, the edge of the jet was analysed with an emphasis on transient structure change and this analysis is described in Chapter 6. The most notable accomplishment would be the identification of convective structure in a series of four frames and the attempts at quantifying the change in this structure.

9. Finally the conical/injection footage obtained from gun tunnel runs GT35 and
GT36 was subjected to both qualitative and quantitative analysis which was covered in Chapter 7. It was established that the structure observed during transverse injection from a hypersonic cone bears marked qualitative similarity to that of pure transverse injection. The quantitative analysis showed comparative jet penetration levels between injection cases as well, indicating that the ratio of momentum flux (and subsequent jet penetration distance) had minimal effect on the forebody conical shock angle. The images acquired from this experimentation also allowed the observation of changes in these identifiable structures with time.

8.2 Further Work

Possibly the first item that would fall into the category of ‘further work’ would be the comparison of results obtained from the numerical conical flow solution with results obtained from so-called ‘linearised’ models. This is not entirely necessary and is mainly for mathematical completeness.

There were several areas of improvement in the flow visualisation area that were identified during the course of this project. Principally an increase in the intensity of the light source used to illuminate the test field would be of significant benefit. This increase could possibly be achieved by using a different type of light source (although the results obtained using laser diodes were not particularly encouraging) or by increasing the number of illumination sources for a single optical path. A greater amount of available light would help to freeze fine mixing structure with a shorter exposure time.

While on the topic of flow visualisation, any increase in the number of frames that can be captured during a gun tunnel run would also be of great value. It is most probably highly impractical to increase the number of optical paths (cameras and VCRs) due to space and cost constraints but the possibility of using multiple light sources of differing colours that are recorded onto a single frame was raised, with some investigation required as to digital colour filtration and subsequent frame separation.

At present the rotational scripts developed require a very large amount of computing time to perform their function. The time required is acceptable given that the scripts
are essentially a prototype of an idea and MATLAB is a very good language for concept development and prototyping. Never-the-less, it would still be perhaps desirable to have the scripts reformulated into a lower level language (such as C++) which would allow for faster, more succinct programs that did not require ancillary software to run.

If further edge analysis of the hot air jet is desired it would be useful to reshoot a single sequence (four frames) of the hot air jet using a smaller exposure time and a shorter time gap between frames. This would allow more consistent and reliable results to be obtained from the jet edge analysis scripts that were developed as part of this project.

A CFD program called mb-cns has been developed by the University of Queensland specifically to simulate 2-dimensional transient hypersonic flow. It would be useful to compare the results obtained from a CFD model of the cone (with and without gas injection) with experimental results obtained.

Finally further testing of the conical model using the procedures and tools developed during this project is needed in order to more fully understand the flow behaviour. For the pure Conical Flow case it would be useful to perform further comparison between experimental and numerical results, having addressed the sources of error highlighted in Chapter 7. Of specific interest to the study of transverse injection would be the development of a correlation between the ratio of momentum flux $\bar{q}$ and the jet penetration parameter $h/D_j$ based on a larger number and range of experimental results. It would also be advantageous to determine experimentally or numerically the point where the injected flow becomes choked. A more accurate determination of what the back pressure around the nozzle exit is would help to estimate the total pressure losses associated with transverse injection from conical geometry and compare the impact of such losses, in terms of theoretical scramjet cycle efficiency, with that of pure transverse injection. This further experimentation would also help to highlight issues of the highly transient nature of this flow domain.
References


REFERENCES


Appendix A

Project Specification
ENZ 4111/4112 Research Project PROJECT SPECIFICATION

FOR:  David Bruce Sercombe

TOPIC: Transverse Gas Injection from a Hypersonic Cone (previous title: Scramjet Mixing Augmentation)

SUPERVISORS: Dr. David Buttsworth, Dr Ahmad Sharifian

PROJECT AIM: This project aims to analytically and experimentally investigate the fuel-air mixing process associated with injection from a cone at hypersonic conditions (as a model for SCRAMJet mixing studies).

PROGRAMME: Issue A.2, 26 October 2004

1. Research current designs, techniques and principles used to enhance fuel-air mixing in scramjet engines and how these relate to the conical model developed.

2. Develop a computational model for Conical Flow.

3. Research fluid flow visualisation techniques, with an emphasis on those presently in use at the USQ Gun Tunnel Facility.

4. Develop computational tools to enhance the flow visualisation facilities within the USQ Gun Tunnel Facility.

5. Conduct experiments with a hot air jet in quiescent surrounds in as an evaluation of point (4).

6. Conduct experiments using the conical/injection model and the USQ Gun Tunnel Facility and associated flow visualisation/data acquisition facilities.

7. Critically evaluate the analytical/computational models developed against the experimental data collected.

As time permits:

8. Identify eddy formation and behaviour in the hot air jet described in point (5) from the flow visualisation data.

9. Identify and quantify/qualify the mixing behaviour identified in the conical/injection model experiments.

AGREED: ______________ (Student)  ______________ (Supervisor)

(dated) __/__/
Appendix B

Technical Drawings

B.1 Introduction

The following pages contain all the technical drawings of the hypersonic cone model and the piezo-resistive pressure transducer holder.
B.2 Hypersonic Cone
B.2 Hypersonic Cone

Figure B.1: The body of the hypersonic cone.
Figure B.2: The tip of the hypersonic cone.
Figure B.3: Internal routing and thread holders for the hypersonic cone.
B.3 Pressure Transducer Holder
Figure B.4: The body of the pressure transducer holder.
Figure B.5: The cap of the pressure transducer holder.
Figure B.6: The pressure transducer holder shown assembled with the body of the hypersonic cone.
Appendix C

MATLAB Code
This appendix contains all the MATLAB scripts and functions written during the course of this project.

C.1 Conical Flow

clear;
c1c;
% flow_propertycalc.m;
% Taylor-Maccoll conical flow solution using equations given by
% Anderson, 1990, "Modern Compressible Flow (with Historical
% Perspective)"
% Script co-ordinates the overall solution. Outputs are the
% isentropic property ratios and some static/stagnation values for the
% flow field at the cone angle
% David BT Sercombe, 3 January 2004
% David R Buttsworth,
% Preset values for free stream Mach no. (M) Cone Angle (thetac) and
% Ratio of Specific Heats (g) are specific to the experimental cone
% in use.
M=7;
% g=1.4;
% calculates the conical shock angle
thetas=find_cone_shock_angle(M,thetac,g);
% returns the flow field solution for the given cone angle
[v,mn1]=taymacsol2(M,thetas,g);
% output and answer handling script
% resolves the radial and angular velocity components into a
% single ray velocity value
vdash=sqrt((v(:,1).^2)+(v(:,2).^2));
% converts the velocity values into Mach numbers
mach=sqrt(2./(((vdash.^(-2))-1).*(g-1)));
% find the Mach number of the ray nearest the cone
% mfin=mach(length(mach));
% calculates the Temperature ratio for the ray nearest the cone
To_T=1+(((g-1)/2).*mfin.^2);
% calculates the Pressure ratio for the ray nearest the cone
Po_P=(1+(((g-1)/2).*mfin.^2)).^(g/(g-1));
% calculates the Density ratio for the ray nearest the cone
rho_rh=(1+(((g-1)/2).*mfin.^2)).^(1/(g-1));
% Calculated the Stagnation pressure ratio (after/before conical shock)
Po2_Po1=(((g+1)/2.*mn1.^2)./(1+(((g-1)/2).*mn1.^2))).^(g/(g-1))./((((2*g/(g+1)).*mn1.^2)-((g-1)/(g+1))).^(1/(g-1)));
% asks user for the free stream stagnation pressure
Po1=input('What is the Free Stream Stagnation Pressure (P_o1) in MPa?:');
% calculates the post shock stagnation pressure
Po2=Po2_Po1*Po1;
% answer printing scripts
fprintf('For a Cone Angle of %.0f degrees and a Free Stream Mach no. ...
of %.0f
The Shock Wave Angle is %.2f degrees
The Temperature Ratio ...
(To/T_2) is %.4f at the Cone Angle
The Pressure Ratio (Po_2/P) is %.4f...
The Density Ratio (po_2/p) is %.4f',thetac,M,thetas,To_T,Po_P,rho_rh)
fprintf('The Stagnation Pressure Ratio (After/Before Shock) (Po_2/Po_1)...
is %.4f
Hence the Stagnation Pressure behind the shockwave (P_o2) is %.4f...
MPa',Po2_Po1,Po2)

function thetas=find_cone_shock_angle(M,thetac,g);
% find_cone_shock_angle(M,thetac,g);
% Taylor-Maccoll conical flow solution using equations given by
% Anderson, 1990, "Modern Compressible Flow (with Historical
% Perspective)"
% M - Mach number upstream of shock
% thetac - cone angle (in degrees)
% g - ratio of specific heats
% David BT Sercombe and David R Buttsworth, 22 December 2003
% Function iterates through a series of shock angles using a bisection
% iteration technique until the found cone angle = the known cone angle
% sets the maximum number of iterations
nitermax=50;
% sets the iteration tolerance
tol=1e-9;
% returns the accuracy (initial value for the ’while’ loop)
accuracy=tol*2;
% initial value for the ’while command’
niter=0;
% sets the maximum shock angle
thetasmax=60;
% finds the corresponding cone angle
coneangmax=find_cone_angle(M,thetasmax,g);
% sets the minimum shock angle
thetasmin=asin(1/M)*180/pi+0.2;
% finds the corresponding cone angle
coneangmin=find_cone_angle(M,thetasmin,g);
% ’while’ loop continues the iteration until the solution
% converges to within the tolerance or exceeds the maximum
% number of iterations allowable
while (niter<nitermax)&&(accuracy>tol)
% finds the mean of the shock angle
thetasmean=(thetasmax+thetasmin)/2;
% finds the corresponding cone angle
coneangmean=find_cone_angle(M,thetasmean,g);
% ’if’ command determines whether the found value
% forms the new upper or lower bound
if coneangmean>thetac
    thetasmax=thetasmean;
else
    thetasmin=thetasmean;
function coneang=find_cone_angle(M,thetas,g);
% find_cone_angle.m;
% Taylor-Maccoll conical flow solution using equations given by
% Anderson, 1990, "Modern Compressible Flow (with Historical
% Perspective)"
% M - Mach number upstream of shock
% thetas - shock wave angle (in degrees)
% g - ratio of specific heats
% Limiting cone angle is set to 0.1 degrees
% David BT Sercombe, 18 December 2003
% David R Buttsworth, 22 December 2003

global gamma
m1=M; % mach no.
gamma=g; % ratio of specific heats
% starting shock angle
thetas=thetas.*(pi)/180;
% stream deflection angle just behind the wave
delta=atan(2.*cot(thetas).*((m1.^2).*((m1.^2).*(sin(thetas).^2)-1))./(m1.^2).*(gamma+cos(2*thetas))+2));
% normal component of the free stream Mach No.
mn1=m1.*sin(thetas);
% normal component of the post shock Mach no.
mn2=sqrt(((mn1.^2)+(2/(gamma-1)))./(2*gamma./*(gamma-1))).*(mn1.^2)-1));
% calculation of post shock Mach no.
m2=mn2./sin(thetas-delta);
% initial dimensionless component of the post shock velocity
v_in=(2/((gamma-1).*((m2.^2)))+1).^(1.5);
% radial and normal components of the velocity calculated
% as boundary conditions
v_rin=v_in.*cos(thetas-delta);
v_thin=v_in.*sin(thetas-delta);
% lower bound for cone angle (limiting case)
endtheta=0.1.*(pi)/180;
% 'options' command switches on event detection (crossing point
detection) and refines the final solution by a factor of 4
options=odeset('Events',on,'Refine',4);
% 'ode23' solver returns the numerical solution for the current shock angle
[theta,v]=ode23('taymaceqn',[thetas, endtheta],[v_rin, v_thin], options);
% converts the angle values to degrees
theta2=theta.*(180/(pi));
% returns the cone angle from the numerical values found
coneang=theta2(length(theta2));
% EOF

function [v,mn1]=taymacsol2(m1,thetas,g);
% taymacsol2.m
% Taylor-Maccoll conical flow solution using equations given by Anderson, 1990, "Modern Compressible Flow (with Historical Perspective)"
% M - Mach number upstream of shock
% thetas - shock wave angle (in degrees)
% g - ratio of specific heats
% Limiting cone angle is set to 0.1 degrees
% David BT Sercombe, 18 December 2003
% David R Buttsworth, 22 December 2003
% Slight variation on find_cone_angle.m used to find the conical flow properties of the final solution (found through iteration)
% declaration of global variables
global gamma
g=gamma;
% shock wave angle in radians
thetas=thetas.*(pi)/180;
% stream deflection angle just behind the wave
delta=atan(2.*cot(thetas).*(((m1.^2).*(sin(thetas).^2)-1)./...((m1.^2).*(gamma+cos(2*thetas))+2)));
% normal component of the free stream Mach No.
mn1=m1.*sin(thetas);
% normal component of the post shock Mach no.
mn2=sqrt(((mn1.^2)+(2/(gamma-1)))/...((2*gamma/(gamma-1)).*(mn1.^2)-1));
% calculation of post shock Mach no.
m2=mn2./sin(thetas-delta);
% initial dimensionless component of the post shock velocity
v_in=(2/((gamma-1).*m2.^2)+1).^(-.5);
% radial and normal components of the velocity calculated as boundary conditions
v_rin=v_in.*cos(thetas-delta);
v_thin=v_in.*sin(thetas-delta);
% lower bound for cone angle (limiting case)
endtheta=0.1.*(pi)/180;
% 'options' command switches on event detection (crossing point
% detection) and refines the final solution by a factor of 4
options=odeset('Events','on','Refine',4);
% 'ode23' solver returns the numerical solution for the current
% shock angle
[theta,v]=ode23('taymaceqn',[thetas, endtheta],[v_rin, v_thin], options);
% converts the angle values to degrees
theta2=theta.*(180/(pi));
% returns the cone angle from the numerical values found
coneang=theta2(length(theta2));
% EOF

function [value, isterminal, direction] = taymaceqn(t,g,flag);
% taymaceqn.m
% Taylor-Maccoll conical flow solution using equations given by
% Anderson, 1990, "Modern Compressible Flow (with Historical
% Perspective)"
% Function contains the differential equations in state-variable
% format for use with the 'ode23' numerical differential equation
% solver.
% David BT Sercombe and David R Buttsworth, 20 December 2003
% specification of global variables
global gamma
% state variable form of the differential equation governing
% conical flow
gdot(1)=g(2);
a=(gamma-1)/2;
c=a.*(2*((g(1)).^3-g(1))+(g(2)-g(2).*g(1).^2-g(2).^3).*cot(t)-2*g(1).*g(2).^2)-(g(1).^2.*g(2).^2);
d=a.*((g(2)).^2+(g(1)).^2 - 1) + (g(2)).^2;
gdot(2)=c./d;
% events detection flagging routine set up to detect when g(2)
% (angular velocity) is zero, stop the iteration from proceeding
% and provide an estimation of where the crossing point occurs
if (nargin<3)|isempty(flag);
    value=[gdot(1);gdot(2)];
extelseif flag=='events';
    value=g; % values to check
    isterminal=[0;1]; % terminal on g(2)
    direction=[0;-1]; % detects falling slope
else
    error(['unknown flag''flag']); % error flag
end
% EOF
C.2 Image Analysis

clear;
clc;
% bmp_split_test.m
% A small test program to determine which line in the image corresponds
% to the first field played, by splitting the captured image
% (images are all bitmaps)

% Pinnacle frame grab at the beginning of Video 4
% Reads the grabbed frame into the MATLAB image structure
data1=imread('hjv4GrabbedFrame1.bmp');
% determines the size of the image array
dasize=size(data1);
% field splitting
im1=data1(1:2:(dasize(1)-1),:,:);
im2=data1(2:2:(dasize(1)),:,:);

% Pinnacle frame grab at the end of vid 4
% Reads the grabbed frame into the MATLAB image structure
data2=imread('hjv4GrabbedFrame 2.bmp');
% field splitting
im3=data2(1:2:(dasize(1)-1),:,:);
im4=data2(2:2:(dasize(1)),:,:);

% Pinnacle created avi (image from frame extraction for Video 2
% comparison to Video 4)
% Reads Video 2 from the hot jet data for splitting purposes
B=aviread('hotjetv2');
% extracts the image data from the 7th frame
data3=B(7).cdata(1:dasize(1),:,:);
% field splitting
im5=data3(1:2:(dasize(1)-1),:,:);
im6=data3(2:2:(dasize(1)),:,:);
% exports one of the extracted fields as a bitmap image
imwrite(im5,'hjcrhairsv3split.bmp','bmp')

% EOF

C.2 Image Analysis

clear;
clc;
% image_rotate.m
% Driver script for the image rotation software suite. Inputs are
% an image to be rotated in 'jpeg' format and an image of the reference
% cross hair also in 'jpeg' format. The rotated image is outputted
% as a 'jpeg'.
% Currently set up for '600' x 750 frame size (optimal for gun tunnel
% images) - frame size can be varied by adjusting the no_rows and
% no_cols parameters.
% Remember to rename output file each time an image is rotated
% Created 17/02/04 - Last Modified 26/08/04 - David B.T. Sercombe
C.2 Image Analysis

% importing image data into MATLAB image format from jpegs
vidframe=imread('rotate_filename.jpg','jpg');
vidcross=imread('crosshair_filename.jpg','jpg');

% calls the 'framesplitter' function to separate the odd and even
% fields from both the image and cross-hair frames
frameimage7=framesplitter(vidframe);
frameimage=framesplitter(vidcross);

% calls the 'fit_point_grabber' function to grab the line fit points
% from the cross-hair image (user picks fit points)
[x_pixels1,y_pixels1,x_pixels2,y_pixels2]=fit_point_grabber(frameimage);

% calls the 'crosshair_fit' function that fits straight lines to the
% points grabbed above (returns two angles and a midpoint)
[theta1,thetab,midx,midy]=crosshair_fit(frameimage7,x_pixels1,...
   y_pixels1,x_pixels2,y_pixels2);
theta1=thetaa;
theta2=thetab;

% specifies the amount that the image is 'padded' for rotation
high=250;
wide=150;

% calls function to add a black border to the frame being interpolated
% ('frame padding') and returns the padded image
frame=border_addition(frameimage7,high,wide);

% adjusts the midpoint for the now padded frame
pointinx=midx+wide;
pointiny=midy+high;

% parameters that specify the distance the interpolation grid starts
% in front of the midpoint found above (variable m_b and n_b in the
% accompanying derivation)
row=200;
col=300;

% 'while' command handles user inputted decision. This section of the
% code allows the user to add 180 deg to the parallel and perpendicular
% lines to overcome any quadrant anomalies. The first line and diagonal
% point are plotted on the image and each angular combination can be
% tested until the grid consistency is achieved.
dec=8;
while dec~=1;
   % (With angular skew correction)
   % calculates the length 'D'
leng=sqrt(row^2+col^2-2*row*col*cos(pi-(theta2-theta1)));
   % calculation of theta_3
   theta3=asin(row*sin(pi-(theta2-theta1))/leng);
   % calculation of the starting points of the interpolation grid
   % (x_1,x_2)
   grid_startx=pointinx-leng*cos(theta1+theta3);
   grid_starty=pointiny-leng*sin(theta1+theta3);
   % parameters that determine grid size
   no_rows=650;
   no_cols=700;
   % array based calculation of the x and y positions of the
   % first line in the interpolation grid
   k=[1:no_cols];
   km1=k-1;
xline=grid_startx+km1*cos(theta1);
yline=grid_starty+km1*sin(theta1);
% calculation of the end points (the diametric opposite of the
% initial points) to give an indication of grid size.
xend=grid_startx+(no_cols-1).*cos(theta1)-(no_rows-1).*cos(pi-theta2);
yend=grid_starty+(no_cols-1).*sin(theta1)+(no_rows-1).*sin(pi-theta2);
% plots the padded frame 'frame'
image(frame)
% sets the plot axis to that of the image
axis image
hold on
% plots the initial point as a red cross
plot(xline(1),yline(1),'rx')
% plots the first line as a series of blue crosses
plot(xline(20:20:no_cols),yline(20:20:no_cols),'bx')
% plots the end point as a blue cross
plot(xend,yend,'x')
% proffers a graphical menu presenting the options for grid
% correction available and handles the decision making process.
dec=menu('Angular Correction','Happy With Grid',...
   'Perpendicular + Pi','Perpendicular + 0','Parallel + Pi',...
   'Parallel + 0');
if dec==2
    theta1=thetaa+pi;
elseif dec==3
    theta1=thetaa;
elseif dec==4
    theta2=thetab+pi;
elseif dec==5
    theta2=thetab;
else
    end
clf; % clears the plot window after user decision
end
% closes current figure
close;
% having determined the angular correction the rest of the interpolation
% grid points are calculated. This section handles full grid creation
% using as much of an array based approach as possible
for h=[1:no_rows];
    gridx(h,:)=xline;
    gridy(h,:)=yline;
end
o=[1:no_rows];
bex=((o-1)').*cos(pi-theta2);
bey=((o-1)').*sin(pi-theta2);
for p=[1:no_cols];
    xtra(:,p)=bex;
    ytra(:,p)=bey;
end
% final interpolation grid points array
x_int=gridx-xtra;
y_int=gridy+ytra;
e=size(frameimage7);
m=size(frame);
% creation of border values for interp2 function
y4=[1:m(1)];
x4=[1:m(2)];
tic; % start computing timer
% interpolation loop
counter=1 % 'for' loop counter
% first 'for' loop cycles through each line of
% the interpolation grid determining line colour values
for i=1:h(length(h));
    % conversion of image from 'unit8' format to a
    % double precision floating point array
    frame_doub=double(frame)/255;
    % second 'for' loop cycle through each colour (RGB)
    % for one line and find the interpolated values
    for f=1:3;
        % extracts colour base (RGB)
        colour_base=frame_doub(:,:,f);
        % interpolates colour values of the grid points
        linecolour1=interp2(x4,y4,colour_base,x_int(i,:),y_int(i,:));
        % converts back to 'unit8' format from 'double' format
        line_colour(:,:,f)=uint8(round(linecolour1*255));
    end
    % counter advancement
    counter=counter+1
    % storing of interpolated colour values (the rotated image)
    imagefin(i,:,:)=line_colour;
end
% exports the rotated image as a jpeg
imwrite(imagefin,'rotated_image_filename.jpg','jpg')
t=toc;% end computing timer
% loads and plays excerpt from Handel’s Messiah when finished
load handel
wavplay(y)
% EOF

function im_split=framesplitter(M);
% framesplitter.m
% % Function file splits each of the images into the separate fields and
% % allows the user to select which field is kept and which is discarded.
% % Created 20/02/04 - Last Modified 25/08/04 - David B.T. Sercombe
% determines input pixel array size
sis=size(M);
% extraction of 2nd field from the inputted image
field2=M(1:2:(sis(1)-1),:,:);
% creation of composite frame that is the same size as the input
% picture by alternately doubling the single extracted field
l=[1:2:(sis(1)-1)];
frame2(l,:,:)=field2(1:size(field2),:,:);
% extraction of 1st field from the inputted image
field1=M(2:2:(sis(1)),:,:);
% creation of composite frame that is the same size as the input
% picture by alternately doubling the single extracted field
n=[1:2:(sis(1)-1)];
frame(n,:,:)=field1(1:size(field1),:,:);
o=[2:2:(sis(1))];
frame(o,:,:)=field1(1:size(field1),:,:);
% 1st subplot of the 1st extracted field
C.2 Image Analysis

```matlab
subplot(2,1,1)
image(frame);
% 2nd subplot of the 2nd extracted field
subplot(2,1,2)
image(frame2);
% constructs a decision menu that allows the user to select which
% field to keep
k=menu('Image to Keep','Top','Bottom');
if k==1
    im_split=frame;
else
    im_split=frame2;
end
% closes current figure
close
% EOF
```

```matlab
function [x_pixels1, y_pixels1, x_pixels2, y_pixels2] = fit_point_grabber...
(frameimage);
% fit_point_grabber.m
% The following script allows the user to pick points for use in fitting
% lines to the captured cross-hair footage. The ginput command continues
% to record the points until 'Enter' is pressed. The points are rounded
% (to get their pixel values) and outputted to the driver script.
% Created 26/02/04 DBTS modified 26/08/2004 - David B.T. Sercombe
% initial array designation
xtot1=[];
ytot1=[];
% plot title
title('Cross-Hairs - Parallel Line');
% 'for' loop allows three sets of fit points to be picked. This allows
% the user to zoom to various parts of the cross-hair footage.
for m=[1:3];
    % displays cross-hair image
    image(frameimage)
    % plot title
    title('Cross-Hairs - Parallel Line');
    % wait until 'Enter' is pressed;
    pause;
    % 'ginput allows the user to pick points from the image plot'
    [x1,y1] = ginput;
    % concatenates currently picked points with those previously picked
    xtot1=cat(1,xtot1,x1);
ytot1=cat(1,ytot1,y1);
end
% rounding of the picked points to represent pixel values
x_pixels2=round(xtot1);
y_pixels2=round(ytot1);
% close current figure
close;
% below is a repetition of the above for the selection of the other set
% of points for the perpendicular line
```
function [angleperp, angleparr, midpointx, midpointy] = crosshair_fit... (frameimage7, x_pixels1, y_pixels1, x_pixels2, y_pixels2);
% crosshair_fit.m
% Function file takes the points grabbed using fit_point_grabber.m
% and fits straight lines to them using a least squares fitting routine.
% The outputs are the angles of the two fitted lines and the point
% where they intersect.
% Created 19/02/04 - Last Modified 26/02/04 DBTS 27/07/2004 DBTS
% uses 'polyfit' command to fit a least squares line fit to the
% two sets of grabbed points
p1 = polyfit(x_pixels1, y_pixels1, 1);
p2 = polyfit(x_pixels2, y_pixels2, 1);

% gradient and offset extraction
m1 = p1(1);
m2 = p2(1);
c1 = p1(2);
c2 = p2(2);

% calculation of the point of intersection between the two fitted lines
midpointx = (c2 - c1) / (m1 - m2);
midpointy = polyval(p2, midpointx);

% gradient assignation
grad1 = m2;
grad2 = m1;

% calculation of the angles of the two lines from their found gradients
angleperp = atan(grad1); %
angleparr = atan(grad2); %
% EOF

function [image_border] = border_addition(data, a, b); % border_addition.m
C.2 Image Analysis

% Function file that takes inputted image data 'data' and pads that image with a black border. Input parameter 'a' and 'b' determine the thickness of the border (in pixels)
% Created 18/02/04 - Last Modified 26/08/04 - David B.T. Sercombe
% determines the size of the input pixel array
sis=size(data);
% sets the colour of the pixel border (black at present)
black=[0 0 0];
% creates the horizontal part (top and bottom) of the border
% from the frame size details
black_add1(1:a,1:sis(2),1)=black(1);
black_add1(1:a,1:sis(2),2)=black(2);
black_add1(1:a,1:sis(2),3)=black(3);
% creates the vertical part (sides) of the border
% from the frame size details
black_add2(1:sis(1)+2.*a,1:b,1)=black(1);
black_add2(1:sis(1)+2.*a,1:b,2)=black(2);
black_add2(1:sis(1)+2.*a,1:b,3)=black(3);
% progessive concatenation of the image array
% with the created border arrays until the final 'padded' image is created
c=cat(1,black_add1,data);
d=cat(1,c,black_add1);
e=cat(2,black_add2,d);
image_border=cat(2,e,black_add2);

clear;
c1c;
% jpg_pic_creation.m
% Script reads four images from a gun tunnel run and creates a collage of the four images. Frame 1 is displayed top left, Frame 2 is displayed top right, Frame 3 bottom left and Frame 4 bottom right.
% Created 20/04/2004 - Last Modified 24/08/2004 - David B.T. Sercombe
% Frame 1
a=imread('Frame1_filename.jpg','jpg');
% Frame 2
b=imread('Frame2_filename.jpg','jpg');
% Frame 3
c=imread('Frame3_filename.jpg','jpg');
% Frame 4
d=imread('Frame4_filename.jpg','jpg');
dataa=a(12).cdata;
datab=b(12).cdata;
datac=c(12).cdata;
datad=d(12).cdata;
e=[dataa datab];
f=[datac datad];
imo=[e ; f];
clear;
clc;
% previewer.m
%
% Script reads a movie in 'avi' format into the MATLAB movie format and
% allows the user the display each frame in sequence. Navigation between
% frames is achieved via a menu of function.
%
% Created 10/07/04 - Last Modified 25/08/04 - David B.T. Sercombe
% reads the avi movie into the MATLAB workspace
mo=aviread('vikingsplit');
% frame counter
frame=1;
% extracts the pixel data from the movie structure
image(mo(frame).cdata);
% title string and plot title function
titl=sprintf('Frame %.0f of %.0f',frame,length(2))
title(titl);
k=1;
% determines the length of the movie (in frames)
length=size(mo)
% 'while' command allows the user to move forward or backward through
% the sequence of frames and displays the frame and its number.
while k<3;
% creates the menu and the option system
k=menu('Frame Preview','Forward','Back','Ok')
% 'if' command handles the frame advancement or regression
if k==1;
% frame advance
frame=frame+1;
% 2nd 'if' command ascertains if the end of the movie has been
% reached and if so, returns to the beginning of the sequence
if frame>length(2)
    frame=frame-length(2);
end
% image plotting and title handling
image(mo(frame).cdata);
titl=sprintf('Frame %.0f of %.0f',frame,length(2))
title(titl);
elseif k==2;
% same as before except that the frame progression is reversed
frame=frame-1;
if frame<=0
    frame=length(2)-frame;
end
else
% image plotting and title handling
image(mo(frame).cdata);
titl=sprintf('Frame %.0f of %.0f',frame,length(2))
title(titl);
end
% EOF
C.3 Hot Jet Analysis

clear;
clc;

% hotcross.m

% Driver script for the edge detection of the hot air jet.
% The script finds the edge points of two successive frames
% and returns the cross correlation of the two frames using
% the 'xcorr' command

% Created 16/07/2004 - Last Modified 24/08/04 - David B.T. Sercombe
% reads the avi sequence into the MATLAB movie structure.
mov=aviread('hotjet_total.avi');
% extracts the image data from the movie frame structure
frame1=mov(22).cdata;
% returns the detected and normalised edge points
[x1_vals y1_vals]=detect(frame1);
% a repeat of the above for the next frame in the sequence
frame2=mov(23).cdata;
[x2_vals y2_vals]=detect(frame2);
% 'xcorr' command returns the cross correlation values
% and the lags used for ease of plotting
[correl, lags]=xcorr(y1_vals, y2_vals);
% opens a new figure window and plots the correlation values
% vs the lags
figure;
plot(lags, correl)
% EOF

function [x_pix, dis] = detect(frame);
% detect.m

% Function file is designed to detect the top edge of the hot
% air jet input image 'frame' from 'hotcross.m' by looking at
% the change in relative intensity along each column

% Created 15/07/04 - Last Modified 30/08/04 - David B.T. Sercombe
% converting the image to from RGB to HSV
im_hsv=rgb2hsv(frame);
% size determination of the image
s=size(im_hsv);
rows=s(1);
cols=s(2);
% parameters set the starting and ending columns under consideration
% for edge detection
startcol=110;
endcol=440;
% intensity cut-off - when the relative intensity along a column reaches
% this cut-off the edge of the jet is registered
cutoff=.075;
‘for’ loop cycles through all the columns between 'startcol' and 'endcol'.

for i=[1:endcol-startcol];
    row starting point
    r=45;
    initial value for 'while' loop
    val=0;
    calculates average intensity of the top half of the frame
    ave=mean(mean(im_hsv(1:225,startcol:endcol,3)));
    'while' loop cycles through each row of the current column
    until the intensity value is greater than the cut-off
    while val<cutoff & r<=rows;
        val=im_hsv(r,i+startcol,3)-ave;
        r=r+1;
    end
    records the row and column where the jet edge has been detected
    y_det(i)=r;
    x_det(i)=i+startcol;
end

% opens a figure window
figure
% plots the image
image(im_hsv);
hold on
% plots the detected edge in red crosses
plot(x_det,y_det,'rx')
y_dis=round(rows./2)-y_det;
% 'polyfit' command performs a least squares fit on the detected edge
coefs=polyfit(x_det,y_det,1);
% assigns the gradient from the fit coefficients
grad=coefs(1);
% calculates an array of positional values from the fitted line
xline=[startcol+1:endcol];
yline=polyval(coefs,xline);
% plots the fitted line in yellow
plot(xline,yline,'y')
% calculation of the position of the edge points,
% normalised about the fitted line
h=yline-y_det;
theta=atan(grad);
dis=h.*cos(theta)+50;%
x_pix=xline+h.*sin(theta);
% opens a new figure window and plots the
% normalised edge points
figure;
plot(x_pix,dis,'x')
EOF
C.4 Gun Tunnel Analysis

clear;
clc;
"dist_find.m"

% Script allows the user to fit two near parallel lines and
% a third line to an cross hair image and determines the
% distance between the intersection. Used to find the
% grid spacing and the annulus gap dimension
% Created 19/07/2004 - Last Modified 29/08/04 - David B.T. Sercombe

% reads image into MATLAB image structure
vidframe=imread('gt36v1cross_rot.jpg','jpg');
% runs the 'point_grabber' function to retrieve the picked points of
% the first line
[x1,y1]=point_grabber(vidframe,1);
% runs the 'point_grabber' function to retrieve the picked points of
% the second line
[x2,y2]=point_grabber(vidframe,2);
% runs the 'point_grabber' function to retrieve the picked points of
% the third line
[x3,y3]=point_grabber(vidframe,3);
% 'polyfit' command fits a line to the points of the first line
% (parallel)
p1=polyfit(x1,y1,1);
m1=p1(1);
b1=p1(2);
% 'polyfit' command fits a line to the points of the second line
% (parallel)
p2=polyfit(x2,y2,1);
m2=p2(1);
b2=p2(2);
% 'polyfit' command fits a line to the points of the third line
p3=polyfit(x3,y3,1);
m3=p3(1);
b3=p3(2);
% calculates the intersection of the first and third line
x_int1=(b3-b1)/(m1-m3);
y_int1=polyval(p1,x_int1);
% calculates the intersection of the second and third line
x_int2=(b3-b2)/(m2-m3);
y_int2=polyval(p2,x_int2);
% determines the distance vector between the two intersections
dist=[x_int2-x_int1 y_int2-y_int1];
% finds the magnitude of the distance vector
length=norm(dist);
% creates arrays of the fitted line points
x_points=[100:500];
y1_points=polyval(p1,x_points);
y2_points=polyval(p2,x_points);
y3_points=polyval(p3,x_points);
% plots the image
image(vidframe)
hold on
C.4 Gun Tunnel Analysis

% plots the three fitted lines
plot(x_points,y1_points,'r',x_points,y2_points,'g',x_points,y3_points)
% EOF

-clear;
clear;
% and_find.m
% Script fits two lines to an image in order to determine conical shock
% angles from gun tunnel imagery. The user picks the points for two
% lines and the angle between them is calculated.
% Created 18/07/2004 - Last Modified 29/08/04 - David B.T. Sercombe
% reads image into MATLAB image structure
vidframe=imread('Gun_Tunnel_filename.jpg','jpg');
% runs the 'point_grabber' function to retrieve the picked points of
% the first line
[x1,y1]=point_grabber(vidframe,1);
% runs the 'point_grabber' function to retrieve the picked points of
% the second line
[x2,y2]=point_grabber(vidframe,2);
% 'polyfit' command fits a line to the points of the first line
p1=polyfit(x1,y1,1);
m1=p1(1);
b1=p1(2);
% calculates the angle of the first line from the gradient
angle1=abs(atan(m1));
% 'polyfit' command fits a line to the points of the second line
p2=polyfit(x2,y2,1);
m2=p2(1);
b2=p2(2);
% calculates the angle of the second line from the gradient
angle2=abs(atan(m2));
% returns the shock angle for a cone angle of 10 degrees
shock_angle=(angle1-angle2).*180/pi + 10;
% calculates an array of points for both fitted lines
x_points=[100:500];
y1_points=polyval(p1,x_points);
y2_points=polyval(p2,x_points);
% plots the image
image(vidframe)
hold on
% plots the fitted lines on the image
plot(x_points,y1_points,'r',x_points,y2_points,'g')
% EOF

function [x_pixels,y_pixels]=point_grabber(frameimage,lineno);
% point_grabber.m


% Allows the user to pick points from an image and rounds the points
% to reflect the pixel values. Used to fit lines to structures and
% cross-hairs.
%
% Created 19/07/2004 - Last Modified 29/09/04 - David B.T. Sercombe

xtot1=[];
ytot1=[];

% title string and title presentation
str=sprintf('Points for line no. %.0f',lineno);
title(str);

% 'for' loop allows the user to pick points from an image
% three times consecutively - so different parts of the line
% can be zoomed to.
for m=[1:3];
    % plots the image
    image(frameimage)
    % adds the title
    title(str);
    % waits until enter is pressed
    pause;
    % records the points picked by the user
    [x1,y1] = ginput;
    % adds the current selection of points to those picked previously
    xtot1=cat(1,xtot1,x1);
ytot1=cat(1,ytot1,y1);
end

% rounding of the points (pixels)
x_pixels=round(xtot1);
y_pixels=round(ytot1);
% closes current figure
close;
% EOF

clear;
cclc;
% height.m
%
% Script used to determine the jet penetration height (in pixels).
% The user selects the centre line of the annulus gap and where the
% cone surface and jet edge fall on that line.
% Created 18/08/2004 - Last Modified 29/09/04 - David B.T. Sercombe

% reads image into MATLAB image structure
vidframe=imread('gt35v4_rot.jpg','jpg');
% plots the image
image(vidframe)
% adds a title to the plot
title('Please pick the approximate gap centre')
% waits until enter is pressed allowing user to zoom
pause;
% allows the selection of one point as the centre of the gap
[x,y]=ginput(1);
% rounds the picked points
x=round(x);
% returns an array of centre line values
rows=[50:10:600];
cols=x*ones(size(rows));
hold on
% plots the centre line on the image
plot(cols,rows)

% picking the edge of the cone (as above)
title('Please pick the edge of the cone on the line')
pause;
[x1,y1]=ginput(1);
y1=round(y1);

% pick the edge of the shock (as above)
title('Please pick the edge of the bow shock on the line')
pause;
[x2,y2]=ginput(1);
y2=round(y2);

% closes the current figure
close;
% returns the distance of jet penetration in pixels
dist=y2-y1;

% EOF
Appendix D

Gun Tunnel Run Notes

D.1 Introduction

This appendix contains generalised notes on each of the gun tunnel runs conducted. It is included for the purpose of completeness and to add perspective to the work that was done.

D.2 GT29

Model Tested: Hypersonic Cone with gas injection
Date: 12/07/2004
Driver Tank Pressure: 3.2 MPa (gauge)
Injection Tank Pressure: 200 kPa (gauge)
Run Specific Notes: No images or run data recorded due to catastrophic failure of piezoresistive pressure transducer. No injection appeared to have taken place.

D.3 GT30

Model Tested: Hypersonic Cone with gas injection
D.4 GT31

Model Tested: Hypersonic Cone with gas injection
Date: 14/07/2004
Driver Tank Pressure: 3.2 MPa (gauge)
Injection Tank Pressure: 150 kPa (gauge)
Optical Set-up: Shadowgraph (knife edge removed)
Run Specific Notes: Gun tunnel firing occurred with oscilloscope pressure data recorded. LEDs failed to fire - no images recorded. Piezoelectric pressure transducer signal amplification changed from 1 MPa/volt to 10 MPa/volt to get larger LED trigger signal.

D.5 GT32

Model Tested: Hypersonic Cone with gas injection
Date: 21/07/2004
Driver Tank Pressure: 3.2 MPa (gauge)
Injection Tank Pressure: 200 kPa (gauge)
Optical Set-up: Vertical Schlieren (LED illumination)
Run Specific Notes: Gun tunnel firing occurred with oscilloscope pressure data recorded. Injection pressure data indicated minimal injection took place. LEDs fired with images recorded. Images were much too dark for useful analysis. LED exposure time increased.

D.6 GT33

Model Tested: Hypersonic Cone with gas injection
Date: 22/07/2004
Driver Tank Pressure: 3.2 MPa (gauge)
Injection Tank Pressure: 200 kPa (gauge)
Optical Set-up: Vertical Schlieren (LED illumination)
Run Specific Notes: Gun tunnel firing occurred with oscilloscope pressure data recorded. Injection pressure data again indicated minimal injection took place. LEDs fired with images recorded. Images were much too dark for useful analysis with the possible exception of frame 1.

D.7 GT34

Model Tested: Hypersonic Cone without gas injection
Date: 17/08/2004
Driver Tank Pressure: 3.2 MPa (gauge)
Injection Tank Pressure: 150 kPa (gauge)
Optical Set-up: Vertical Schlieren (laser diode illumination)
Run Specific Notes: Gun tunnel firing occurred with oscilloscope pressure data recorded. Laser diodes fired with images recorded. Exposure time around 1.5 μs. Best image occurs at frame 1. Illumination level quite acceptable - captured flow detail
very ordinary compared with LEDs. LEDs reinstalled.

D.8 GT35

Model Tested: Hypersonic Cone with gas injection
Date: 18/08/2004
Driver Tank Pressure: 3.2 MPa (gauge)
Injection Tank Pressure: 150 kPa (gauge)
Optical Set-up: Horizontal Schlieren (LED illumination)
Run Specific Notes: Gun tunnel firing occurred with no oscilloscope pressure data recorded due to premature triggering. LEDs fired with four good images recorded. Illumination level quite acceptable (full optical set-up was conducted prior to commencing the run). Exposure time around 13 µs.

D.9 GT36

Model Tested: Hypersonic Cone with gas injection
Date: 18/08/2004
Driver Tank Pressure: 3.2 MPa (gauge)
Injection Tank Pressure: 50 kPa (gauge)
Optical Set-up: Horizontal Schlieren (LED illumination)
Run Specific Notes: Gun tunnel firing occurred with oscilloscope pressure data recorded. LEDs fired with four good images recorded. Illumination level quite acceptable on all frames. Exposure time around 10 µs.
D.10  GT37

Model Tested:  Beagle - 2  
Date:  26/08/2004  
Driver Tank Pressure:  3.2 MPa (gauge)  
Optical Set-up:  Horizontal Schlieren (LED illumination)  
Run Specific Notes:  Performed the image acquisition and rotation.  
LEDs fired with four good images recorded. Illumination level quite acceptable on all frames. Exposure time around 10 $\mu$s.

D.11  GT38

Model Tested:  Muses - C  
Date:  n.d.  
Driver Tank Pressure:  3.2 MPa (gauge)  
Optical Set-up:  Vertical Schlieren (LED illumination)  
Run Specific Notes:  Performed the image acquisition and rotation.  
LEDs fired with four good images recorded. Illumination level quite acceptable on all frames (frame 3 a little dark). Exposure time around 10 $\mu$s.
Appendix E

Image Gallery

E.1 Introduction

The following pages contain all the images from gun tunnel runs that were rotated using the software expanded upon in Chapter 5 that either proved unsuitable for analysis or were captured as confirmation for other models undergoing testing in the USQ Gun Tunnel.
E.2 GT31

GT31 was conducted without the knife edge in place, resulting in the following two shadowgraph images. The shadowgraph technique is generally considered less sensitive than the Schlieren method and tends only to highlight areas where a large density change is occurring such as the bow shock bordering the injection plume.

The injection tank pressure used for this run was the same as that of GT35 (150 kPa). Shadowgraphs require a slight variation in optical setup and so only the first and second cameras returned clear images.
Figure E.1: The first frame from the GT31 run. The strong bow shock highlighted in the shadowgraph can be seen bordering the injection plume.

Figure E.2: The second frame from the GT31 run.
E.3 Viking

These images are of a model of the Viking probe and were captured prior to commencement of this project. The model allows for gas injection on the front surface to simulate the gas dissociation or oblation that occurs during atmospheric re-entry.

What was particularly noteworthy was the fact that no cross-hair footage was taken at the time of the run and subsequent rotation was performed by fitting two lines to the silhouette of the geometry. The knife edge in the camera was perpendicular to the free stream.
Figure E.3: The first frame from the testing of the Viking model. The prominent bow shock is shown along with the highly turbulent post-shock zone.

Figure E.4: The second frame from the testing of the Viking model.
Figure E.5: The third frame from the testing of the Viking model.

Figure E.6: The fourth frame from the testing of the Viking model.
E.4 GT37

The GT37 gun tunnel run was used to test some quick response thermocouples and the images presented here were used for experimental confirmation purposes. The model shown in silhouette is a model of the Beagle-2 re-entry vehicle. The knife edge was set parallel to the Mach 7 free stream.
Figure E.7: The first frame from the testing of the Beagle-2 model. The strong bow shock caused by the model’s geometry can be seen directly in front of the model outline.

Figure E.8: The second frame from the testing of the Beagle-2 model.
Figure E.9: The third frame from the testing of the Beagle-2 model.

Figure E.10: The third frame from the testing of the Beagle-2 model.
E.5 GT38

The GT38 gun tunnel run was used to test an aerodynamic drag force measurement system and the images presented here were used for experimental confirmation purposes. The model shown is a model of the Muses-C re-entry capsule. The knife edge was set perpendicular to the free stream.
Figure E.11: The first frame from the testing of the Muses-C model. The strong bow shock can be seen directly in front of the model outline.

Figure E.12: The second frame from the testing of the Muses-C model.
Figure E.13: The third frame from the testing of the Muses-C model.

Figure E.14: The fourth and final frame from the testing of the Muses-C model.