On the Nature of Mathematical Education of Engineers: Identifying Hidden Obstacles and Potential for Improvement

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Abstract

Mathematical literacy and competence in mathematics of engineering students are defined and analysed through the structure of conceptual understanding on both levels. The paper provides a view on technology integration in the teaching process, in particular, the ways how technology can contribute to learning needs of engineering students and what difficulties we can expect on that way. Examples that show relationships between different mathematical concepts and using technology are discussed. One of the most important concepts of calculus, limit is considered in detail with applications in MATLAB to identify possible obstacles in learning and students' misconceptions. Potential of conceptual understanding in mathematics for development of students' abilities in engineering applications and computerised calculations is shown. The current situation of the mathematics education of engineers in Australia in the context of the questions raised in the paper is briefly outlined.

Introduction

A high level of prospective engineers' mathematical skills and knowledge is the necessary condition for their future successful work and career. There is a permanently increasing demand for engineering specialists in industry throughout the world. In such circumstances the role of mathematics as provider of any kind of special knowledge for engineering students cannot be underestimated or neglected. It is widely recognised as a specific feature of mathematical education of engineers that to be a good engineer means more than just knowledge and understanding of mathematical theory; it is also the ability to use such knowledge and skills to model different applications with practical outcomes. However, even most of beginning engineering graduate students have not had a good experience with mathematics or have diverse mathematical backgrounds (Bamforth et al. (2007)). Absence of conceptual understanding, which refers to the student's comprehension of mathematical concepts, operations, and relations, leads to a fragmentation of mathematical knowledge in the student's head. In this respect Mitin et al. (2001) emphasised the situation: "As a result, in many specialized engineering courses, such as Control Systems, Solid State Electronics, or Communication Theory, instructors have to begin with a review of the mathematics they to use in the course…Vector analysis stays forever uniquely associated with electromagnetic waves, set operations with computer architecture, and the Laplace transform with circuits and signal processing. The mathematical methods encountered in each course do not evolve into unified patterns which the future engineer would be able to recognize and use universally"(p.vii). The other big concern was raised by Mustoe (1988), who pointed out that "most students lack the ability to apply their mathematical knowledge to non-standard problems. Whereas they may be capable of performing simple exercises in manipulation they fall down badly when required to use the same skills in a different or unfamiliar setting. 'If only I could see how to start' is an all too common remark made by students"(p.ix). Like many other researchers, who deal with engineering students we strongly believe that this situation can be and should be radically improved. There are different views on the methods that should be used to achieve students' better understanding in mathematics. However, we do not have the intention of covering such methods in depth in the paper. Furthermore, this paper does not present specific results of a particular case study or a teaching project aimed on the improvement of students' mathematical skills and comprehension. On the contrary, the paper highlights the key moments in teaching engineering mathematics, which we believe determine the strategic way Mathematics Education of Engineers will follow in 21st century. Moreover, we believe the strong emphasis of the paper on students' conceptual understanding combined with appropriate using and support of technology and strong background in proper mathematical structures, referring to engineering disciplines but not relying on them, will contribute to further discussion to work out the actual tools for radical improvement the current state-of-arts in the near future. As much as possible we tried to demonstrate particular examples in support of the claims and ideas that could be seen too broad or insufficiently described in terms of conference paper. We found this approach workable in many situations and hope readers will appreciate this mixture of global and particular ideas, structures and patterns. At the end of the paper we provide readers with brief outline of the current situation in the mathematical education of engineers in Australia (Mathematics & Statistics (2006)). We have no doubts this last example will serve as one more evidence that the situation in the whole needs to be improved.

Mathematical competence of engineering students

Speaking about mathematical competence of engineering students, we start with definition of mathematical literacy. We understand engineering students' mathematical literacy as the capacity to identify, to understand, and to engage in basic mathematics concepts and make well-founded judgements on mathematical statements. Mathematical competence of engineering students is defined as the ability to work with advanced mathematics concepts by considering the world through a "mathematical frame of mind" (Schoenfeld (1985)). Mathematical competence is related to the process of activating resources (knowledge, skills, strategies) in a variety of mathematical contexts. The way forward from mathematical literacy to competence in mathematics does not seem to be easy for most students. Quite often the competence level remains beyond their highest possible learning achievement that inevitably carries a negative impact on their professional skills and career. On the other hand, students themselves, not all, but quite a significant number of them (Gynnild et al. (2005)), perceive competence in mathematics as the highest level in learning engineering mathematics which not necessary should be achieved. Those students' misconception has, unfortunately, even older roots and origin than anyone could have thought. The core of this challenging transition from literacy to competence lies in development of students' conceptual understanding in mathematics that interacts with technology. Indeed, it is not a difficult task for any student to differentiate or integrate a certain function or evaluate, for example, a volume of any specific 3D-solid using multiple integrals. The difficulties appear in the students' comprehension on any of the mathematical concepts, e.g. what the concept of derivative is, how it can refer to the concept of limit, which physical or geometrical interpretation the differential of a function can have or what uniformly convergent series actually means, etc. In the case of mathematical literacy (Fig.1, left side) student's conceptual understanding is concentrated on separate concepts whose links to each other cannot be identified by students.

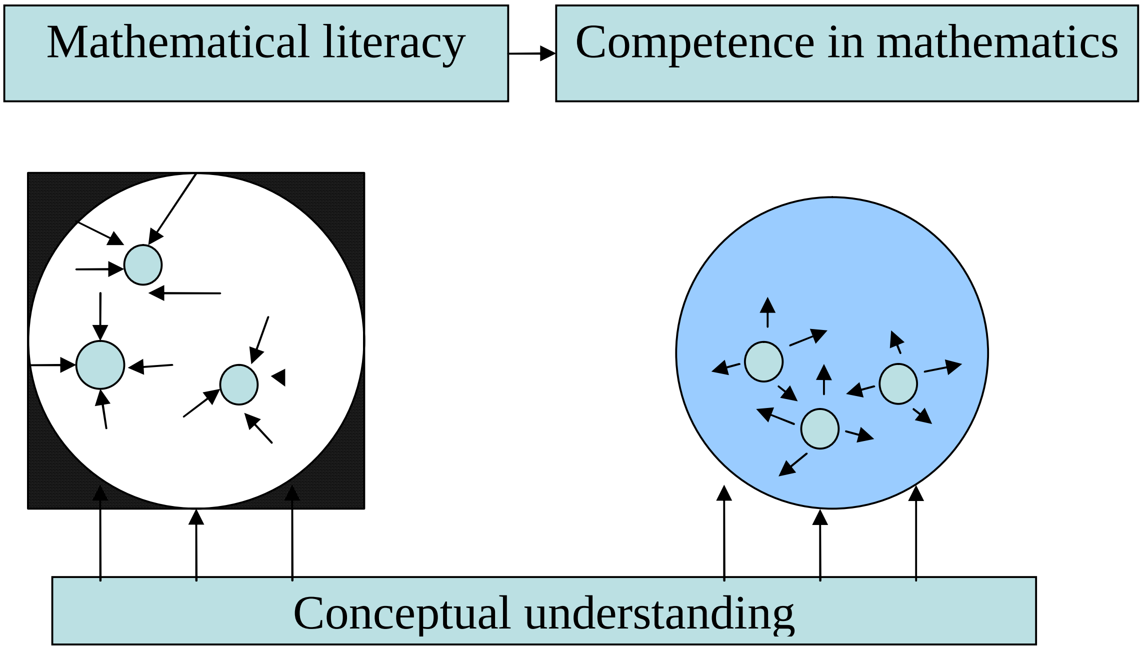

Figure 1. Scheme of the structure of conceptual understanding on two levels

As a consequence, students cannot analyse any significant part of theoretical material as it was initially designed in the course and experience difficulties with most concepts. However, in the competence case (Fig.1, right side) different concepts are considered by students with their relations to each other that leads to further development of conceptual understanding and improvement of the students' competence level. As an example we would like to demonstrate a calculus (functions of one variable) conceptual framework that can be used to motivate students' conceptual understanding in the first year calculus course for engineers. Every student has got the following scheme (Fig.2) on the first week of the semester. During the semester students worked on the scheme regularly to provide their comments and description on every component of the scheme, i.e. on different mathematical concepts and relations between them. At the end of the semester students were involved in the summary session, where the work of each student on the scheme was analysed. Despite the visual simplicity of the scheme, its impact on students' conceptual understanding was even higher than it could be expected, though interaction with technology was observed in both, positive and negative, directions. Now we turn to the analysis of technology influence on literacy and competence level.

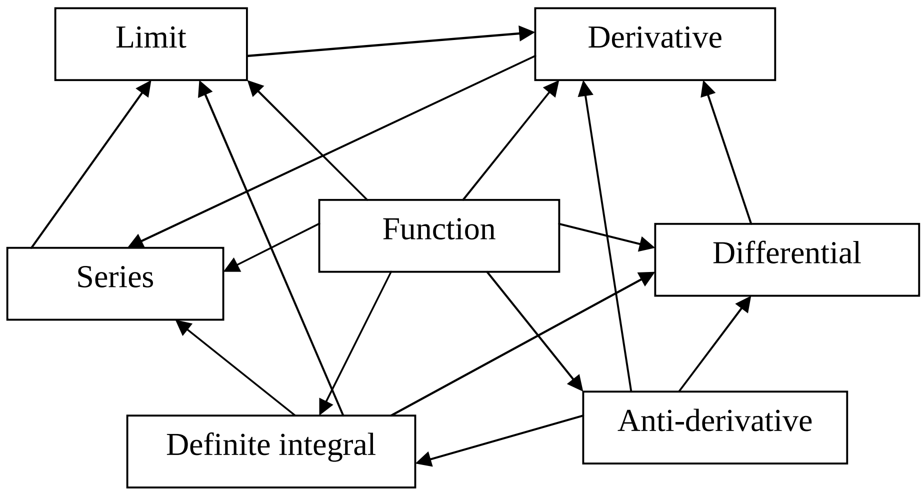

Figure 2. Calculus conceptual framework

Technology: pro and non-contra?

One of the most important issues to deal with in the 21st century will be technology. Despite the role of technology is recognised and looks extremely important nowadays, its influence will be strengthening in the near future. It will require a new look on the integration technology in teaching and learning engineering mathematics. CAS, DGS, statistical software, handwriting using Tablet technology, computer-based learning environments, different mathematical and engineering software will influence the teaching process more and more. Will theoretical frameworks in mathematics education in general and in the mathematical education of engineers in particular be changed dramatically due to the new breakthrough in technology that, apparently, can happen in the next 10-15 years? This question still remains open. Engineering mathematics widely uses the advantage of different mathematical software like MATLAB, Maple, etc. We would like to show two examples of students' obstacles in using technology. The first one refers to the literacy level, the second one deals with the competence level and use MATLAB possibilities for limits evaluation. Both examples emphasise the importance of conceptual understanding in mathematics that can be recalled for use in a wide range of engineering questions.

Example 1

Solve the equation $3^{x+1} = 27$

Student X's write-up

The student used a standard calculator to get the following result:

$$x+1=\log_3 27=$$
$$=\frac{\log_{10} 27}{\log_{10} 3}=\frac{1.4314}{0.47712}=3$$

that provided him with the correct answer x=2. But what a sacrifice that method was where preference was given in the direction of technology even on such a basic level as a standard calculator.

Example 2

Find $\lim_{x\to 0}\frac{6^x-2^x}{x}$.

Student Z's write-up

The best way to estimate this limit is numerically, e.g. in Matlab, or using a calculator. Take x in small steps either side of $x = 0$.

MATLAB Code

h=0.01;
*x=[-3*h:h:-h,h:h:3*h]*;
y=(6.^x-2.^x)./x
y =
1.0585 1.0717 1.0851 1.1124 1.1263 1.1404

We can repeat these calculations for *h=0.001*, *h=0.0001*, etc. to estimate the limit at $x = 0$. (The middle two numbers are at $-h$ and h, closest to $x = 0$.) We should get an answer around *1.0986*.

Again, the answer was correct, though it looked like an approximation. However, if we consider the same limit from pure mathematics point of view, that wouldn't be difficult to obtain exact value *ln 3* as the answer.

$$\lim_{x\to 0}\frac{6^x-2^x}{x}=$$
$$\lim_{x\to 0}\frac{2^x\left(3^x-1\right)}{x}=$$
$$\lim_{x\to 0}\frac{2^x\,x\ln 3}{x}=\ln 3$$

that follows from the following fact:

$$a^x-1\approx x\ln a\left(x\to 0\right),\ a>1.$$

Both examples demonstrate a possibility of conflict between conceptual understanding in mathematics and using technology. Despite the technological aspect being taken into account at least the last 20 years (many significant results were achieved and published), the question of compatibility mathematics and technology, i.e. teaching mathematics using technology in the most effective way requires further research and consideration. The same question with focus on the needs of engineering students looks to be one of the most important questions today and in the near future.

Concluding remarks

Unfortunately the lack of space in the paper for conference proceedings does not give the opportunity to provide the full version of the material. Nevertheless, even the brief description of the questions raised in the paper and approaches to their further investigation and analysis presents for readers those important and significant directions that research on the Mathematics Education of Engineers will address and further develop. All over the world 21st century engineering students differ from their predecessors and that difference will intensively increase, first of all due to the huge impact of technology. The examples mentioned above on obstacles in using technology show the great potential in studying engineering mathematics that students can achieve combining the power of innovative technology with understanding the concepts of fundamental mathematics which slightly changed for the last 100 years. Probably, the supporting balance between engineering students' conceptual understanding in mathematics and the appropriate use of technology will be one of the main challenges in Mathematics Education of Engineers in 21st century. Moreover, we believe that the research on students' successful transition to the competence level over the territory of conceptual understanding and technology, where both components have a huge impact on each other will form a new theoretical framework of the question. We believe it will inevitably happen due to the further and deeper technology penetration in research on the teaching and learning of mathematics as well as in industry development and job requirements for future engineers.

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