BIFURCATIONS IN CONVECTION OF INCOMPRESSIBLE FLUID IN A ROTATED SQUARE CYLINDER

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Summary. We consider two-dimensional convection of air in a long horizontal square cylinder two opposite side walls of which are thermally insulated and the other two are maintained at constant but different temperatures. The cylinder is gradually rotated about its horizontal axis so that the direction of the wall-to-wall temperature gradient changes with respect to the gravity vector: the cylinder is consecutively heated from the bottom, the side, the top and vice versa. It is found that a multitude of stationary bifurcating solution exist depending on the inclination angle and the Rayleigh number. Normally and abnormally rotating solutions are defined and distinguished and the bifurcation curve is computed for the first bifurcation resulting in an abnormally rotating convection roll.

STUDY MOTIVATION, PROBLEM FORMULATION AND SOLUTION APPROACH

The interest towards studies of stationary natural convection in simple geometries is associated with the need to achieve better understanding of mechanisms driving complex atmospheric phenomena. It is hoped that studies of the bifurcation nature in laboratory conditions will pave the road towards such understanding. In particular, in laboratory set-ups such as a square cavity the inclination of an enclosure is used to mimic such atmospheric events as the formation of tropical cyclones [1] and paradoxical pollution transfer from densely populated cities to exclusive suburban mountain regions [2]. Steady thermo-convective motions in long closed cavities frequently have the form of rolls with axes aligned with the axis of the enclosure. In such flows fluid particles move along the predominantly planar trajectories perpendicular to the axis of the cylinder. Therefore such motions can be considered as quasi-two-dimensional. In the current work the authors consider a set of two-dimensional solutions in an infinitely long square cylinder with a horizontal axis. Despite such an idealization the two-dimensional study sheds light on the physical nature of realistic steady flows and bifurcations without obscuring them by the influence of complex geometry and three-dimensionality. Somewhat surprisingly, the current two-dimensional computational results for an infinite cylinder are found to be in good agreement with the experimental results of [3] for an inclined cubic air-filled cavity at least for small inclination angles.

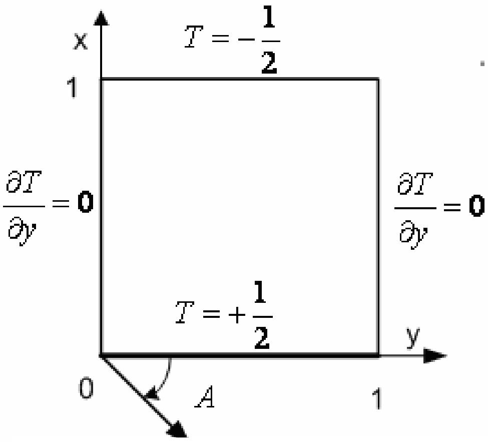

Figure 1. Thermal conditions on the cavity walls. The gravity direction shown by the arrow forms angle *A* with the y axis.

Natural thermal convection in an incompressible fluid contained in a long closed cavity of a square cross-section has been the object of numerous investigations. However most of the known studies deal with two configurations: cavity heated from below or from its side when the gravity is parallel to the vertical cylinder walls. The purpose of the current work is to obtain and classify solutions for an arbitrary inclination angle *A*, see Figure 1. The flows are modelled using spectral Chebyshev-based Petrov-Galerkin numerical method of [4] which is applied to a set of the Boussinesq momentum, energy and continuity equations. The major advantages of this method are that it identically satisfies the divergence-free condition for velocity eliminating costly pressure computations and enables a straightforward even-odd mode decomposition. As a result in the current study we were able to solve a fully nonlinear problem for individual flow modes satisfying the imposed symmetry conditions as well as to study the nonlinear mode interaction leading to the destabilisation of one mode by another. Such features are not available in other conventional numerical methods as finite difference, element, or volume techniques. Steady fully nonlinear solutions (including unstable ones) of a given symmetry were found using Newton iterations and then their linear stability was investigated by solving the corresponding eigenvalue problem for the disturbance amplification rate.

PHYSICAL NATURE OF FLOWS AND BIFURCATIONS

Define the inclination angle *A* so that $A=-90^0$ corresponds to heating from below and $A=90^0$ to heating from above, see Figure 1. In the former case the motionless conduction solution, i.e. mechanical equilibrium, is found to exists up to the critical value of Rayleigh number R_c=2585.02 (the value of 2586 was previously reported in [5]). Beyond this value the conduction solution becomes unstable and is replaced by either clock- or counter-clockwise single roll solutions (which are identical apart from the direction of circulation). Such a transition occurs as a result of a symmetric supercritical pitchfork bifurcation which is confirmed experimentally in [3] for a cubic cavity. Both clock- and counter-clockwise vortices are equally possible. However when the cylinder is slightly rotated this symmetry is broken and there exist three distinct solutions: one emanating from the unstable motionless state at $A=-90^0$, now rotating so that the air rises along the heated inclined surface, and two *stable* solutions emanating from the bifurcation. If the cavity is rotated back to $A=-90^0$ the former vortex will cease to exist regardless of the value of Rayleigh number R. On the other hand the other two would still exist if $R>R_c$. The stable "normal" vortex driving air upwards along the inclined heated surface is more intensive than its "abnormal" counterpart rotating in the opposite direction. In practice, the normal vortex occurs when the inclination angle changes slowly from $A=-90^0$, while an abnormal vortex can result from a hysteresis, which is the feature of an imperfect bifurcation, when the cavity is slowly inclined in one direction and than rapidly rotated in the other. The goal of the current computational effort is to determine the maximum inclination angle for which such an abnormal steady vortex can exist.

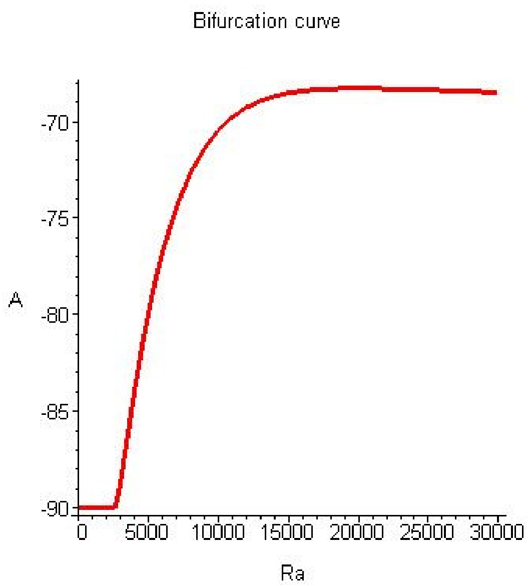

Figure 2. Bifurcation curve: an abnormal vortex (see definition in text) exists under the curve.

To the best of our knowledge the study of convection flow characteristics in a square cavity as a function of the inclination angle from the bifurcation point of view was only attempted in reference [6]. There the cavity with ideally conducting rather than insulated walls was considered. The major attention was devoted to bifurcations and transition between one- and two-vortex solutions existing in a cavity of aspect ratio 2. Stable abnormal vortices were reported for inclination angles of $-89°$ and $-85°$ degrees. In another relevant study [7] the bifurcation diagram defining the parametric region of existence of an abnormal vortex in a circular cylinder heated from a side was computed for Prandtl number $\Pr=1$ using finite differences. For this case the bifurcation curve had a maximum of $-78°$ for $Ra=2Ra_c$. The bifurcation curve for the current configuration is presented in Figure 2. It shows that the abnormal vortex can exist up to a much larger inclination angle of about $-68°$. Recollect that below the bifurcation curve near R_c, in addition to the normal and abnormal stable vortices, another normal but unstable vortex solution exists. Its instability has a saddle type i.e. there exists a direction in the phase space along which this solution is an attractor. Therefore it can be observed experimentally or numerically for some (not necessarily short) time. For larger values of R additional 2-, 3- and so on vortex solutions appear as a result secondary bifurcations of the motionless state which however we do not have space to discuss here.

In conclusion note that the curve in Figure 2 and the bifurcation curve obtained in [7] both have extrema. However the extremum reported in [7] is much sharper. Therefore we conclude that the parameter range in which an abnormal vortex solution can exist for a square cylinder is much larger than that for a circular one. This suggests that the rate with which the existence region of an abnormal vortex reduces with Rayleigh number is negatively affected by the boundary corners and by the ratio of the surface area of the cavity and its volume.

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