Boundary Treatment for Virtual Leaf Surfaces

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Abstract

When working on fitting leaf surfaces for use with virtual plant models [Room et al. 1996] we encountered the unsatisfactory situation of receiving a smooth surface model that is bounded by a piecewise linear curve. To smooth the boundary, we fit a parametric piecewise cubic curve through the boundary data points and extend the surface to the new boundary curve. This method will aid us in the representation of leaf surfaces with arbitrary boundaries in future.

CR Categories: I.3.5 [Computing Methodologies]: Computer Graphics—Computational geometry and object modelling;

Keywords: smooth boundary, surface fitting, triangulation, extended triangles, subdivision, plant modelling

1 Introduction

Tracing the paths of water droplets carrying pathogens across a leaf surface is just one application requiring detailed models of leaf surfaces. Visualizing the situation is another. The central aim of this research is to develop techniques appropriate for creating mathematical models of leaf surfaces from sets of three-dimensional data points.

Our method is based on interpolation using a finite element basis. We first determine a reference plane by a least squares fit and project the data points onto that plane. We apply the Clough Tocher method [Lancaster et al 1986], which returns piecewise cubic polynomials on a triangulation of the projected points. It is a local method that ensures a continuously varying gradient across the whole surface. Evaluation requires the solution of linear systems of order no more than three. However, the boundary of the resulting surface is defined by a piecewise linear contour, passing through the boundary points.

This is sufficient for surface fitting applications where the boundary is not important. For example surface fitting of terrain data can usually be done in a rectangular environment because the domain does not have a natural boundary. For modelling and visualization of leaves, the boundary should capture their smoothness. The problem we are aiming to solve here is the improvement of the piecewise linear boundary given only by the boundary points.

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In our approach, we fit a three dimensional parametric piecewise cubic spline curve through the boundary points, which defines a possible acceptable boundary curve. In this paper, we explore how to extend the surface fit to the new boundary curve.

2 Background

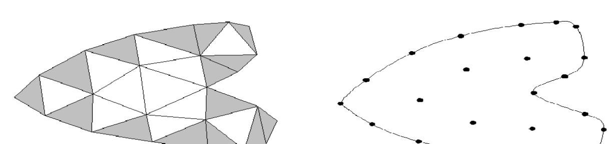
To extend the surface approximation to a curved domain several options are available. A method based on Coons patches [Coons 1964] determines a surface with a curved boundary. Schweitzer's approach [Schweitzer 1996] falls into the category of subdivision curves and surfaces. The boundary curve is created using least squares fitting by smooth subdivision curves. The resulting surface interpolates the boundary curve. Levin's method [Levin1999] is a two-part subdivision scheme. Patches are split into smaller elements everywhere except for the boundary of the mesh. There, additional vertices are placed on a boundary curve, extending the element. This method is iterated until the required resolution is reached. Subdivision takes place everywhere, not just on the boundary. Renka and Cline [Renka et al. 1984] briefly mention extrapolation outside a triangulation. A linear function is passed through the projection of the point to be evaluated and its directional derivative. Although these methods may work well for leaf surface models, we are looking for a simple but effective method that uses the same class of polynomials everywhere.

In order to obtain a surface with a curved boundary using the Clough Tocher method it is necessary to extend the surface into curvilinear triangles situated along the boundary. We need to be able to find a way of treating more complicated boundaries in future and therefore require a method that will deal well with any kind of boundary curve. In this paper we present some approaches for the extension of boundary triangles. Two are based on extrapolation, one of which is a subdivision scheme similar to [Levin1999], and the third is an interpolation method.

Thus far we have obtained satisfactory results with these straightforward methods.

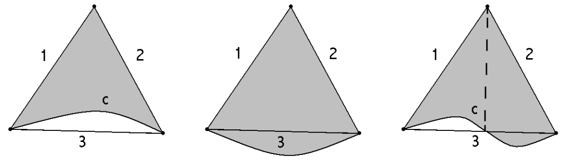
3 Our Approach

In our approach, we fit a three dimensional piecewise cubic, parametric curve through the boundary points whose projection onto the reference plane defines our new boundary (Figure 1b). To extend



a) b)

Figure 1: a) Boundary triangles, b) Parametric boundary curve.



1 2 3

Figure 2: 1. Reduced triangle, 2. Extended triangle, and 3. Subdivision of triangle.

the two-dimensional triangulation (Figure 1a), we use the following algorithm:

The triangles are sorted into boundary and interior triangles (Figure 1a). For each boundary triangle

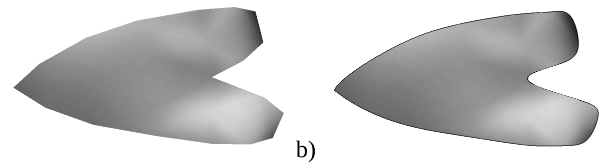
- if *case 1* (Figure 2): reduce triangle.
- if *case 2*: extend triangle applying one of the following:
    1. extrapolate outside original triangle.
    2. linear extension.
    3. subdivide and extrapolate remaining areas.
- if *case 3*: subdivide and apply cases 1 and 2.

Reduce triangle: (*case 1* Figure 2)
If the new boundary curve lies inside the boundary triangle, the triangle is reduced to the area between edges 1 and 2 and the parametric curve segment c.

Extend triangle: (*case 2* Figure 2)
If the boundary curve is outside the triangle, we use one of the following three methods.
The first is an extrapolation of the area outside the triangle. We extend the domain for the piecewise cubic polynomial to the area between original boundary edge 3 and parametric curve segment c. This approach sometimes returns a large deviation if, for example, the surface values of corner points of the boundary triangle differ by a sufficiently large amount. The element is then 'steep' and extrapolation will result in a new boundary that may be far away from the parametric curve.
For the second method we take data from the boundary edge 3 and boundary curve segment c and interpolate by fitting a piecewise linear function which may give a less smooth transition than other methods.
For the third method we apply a subdivision approach. We take the three-dimensional midpoint of the parametric curve segment as an additional corner together with the two boundary points and include the new triangle. Note the possible similarity to Levin's method where the midpoint between the two boundary vertices was chosen. We then extrapolate the two remaining smaller boundary areas. More subdivision steps can be carried out until the extrapolation region is sufficiently small.

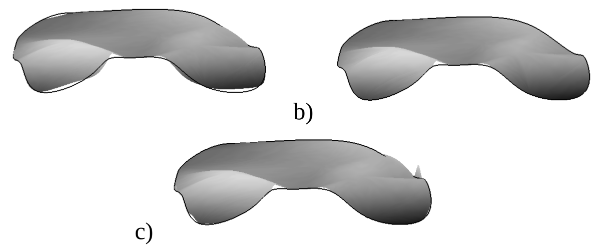
Subdivide triangle: (*case 3* Figure 2)
If the new boundary curve c intersects the boundary edge 3, we subdivide into one element each of case 1 and 2.

Figure 3a) shows the surface fit with the original piecewise linear boundary in the reference plane. Figure 3b) contains the new extended surface. In Figure 4a) we have the surface obtained with the extrapolation approach. The third coordinates of the points on the boundary curve are extrapolated so boundary curve and parametric curve are only identical in two dimensions. The results for the linear extension method can be found in Figure 4b). Note that the parametric curve is the new boundary in three dimensions, apart from where case 1 is applied. Figure 4c) shows the results of the subdivision method where a triangle is added. The remaining two areas are extrapolated, leading to undesirable behaviour on the right.



a) b)

Figure 3: a) Surface with piecewise linear boundary and b) Extended surface



a) b) c)

Figure 4: a) Extrapolation, b) Linear extension, and c) Additional triangle and extrapolation of remaining areas

4 Conclusion

For many surface fitting applications the boundary is not important. Leaves, however, possess a specific boundary. In order to capture this boundary we have implemented a method to extend the fitted surface. We found it to be sufficient to model the smooth boundary of leaf surfaces that are not curled. Of course the quality of the extended boundary relies on the quality of the parametric curve fitted through the boundary points. Some leaves have a more complicated boundary that cannot be described by a smooth curve. Future work will address this aspect of the problem.

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