A generalized analytical model for Joule heating of segmented wires

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Abstract

This paper presents an analytical solution for the Joule heating problem of a segmented wire made of two materials with different properties and suspended as a bridge across two fixed ends. The paper first establishes the one-dimensional governing equations of the steady-state temperature distribution along the wire with the consideration of heat conduction and free-heat convection phenomena. The temperature coefficient of resistance of the constructing materials, and the dimension of the each segmented wires

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were also taken into account to obtain analytical solution of the temperature. COMSOL Numerical solutions were also obtained for initial validation. Experimental studies were carried out using copper and nichrome wires, where the temperature distribution was monitored using an IR thermal camera. The data showed a good agreement between experimental data and the analytical data, validating our model for the design and development of thermal sensors based on multi-segmented structures.

**Keywords:** Joule-heating, segmented wire, steady-state temperature, heat conduction, free convection, infra-red thermal camera

1. Introduction

Micro electro mechanical system (MEMS) based micro heating devices attract a great deal of interest from the research community, as they provide a wide range of applications and advantages[1, 2]. The major advantages include low power consumption, fast thermal response time, long operating time, high reliability and relatively low fabrication cost using micromachining technology. These devices are the centre pieces in various applications such as fluid flow sensing [3], gas sensing, and bio-thermal therapy. With their capability to reduce the overall power consumption, microheaters have been investigated widely for gas sensors [4-12], flow sensors [13-16], humidity sensors[17, 18] and bio-thermal therapy[19, 20]. In particular, MEMS microheaters are becoming increasingly important in portable electronics applications, where low voltage and low power are required [21, 22]. To minimize the power consumption in microheaters, the knowledge of heat-loss mechanism is paramount as the heat loss to the supporting substrate dominates usually. Recent works have focused
on suspended beam design where, the heat conduction through the beam to the anchors have been considered as primary heat loss mechanism [23-27].

In recent years, many researchers have investigated the temperature distribution on the heater surface using analytical and finite-element based approaches. Briand et al[28] and Dumitrescu et al[29] described the thermal behaviour of microheaters on square shaped membranes using commercial tools such as COSMOS and MEMCAD. Similarly, Faglia et al. proposed a numerical technique using SOLIDIS-FEM for detecting environmental pollutants using a platinum based double spiral shaped heater[30]. Iwaki et al. developed a 2D numerical model using FEMLAB to study the thermal characteristics of calorimetric gas flow sensors[31]. Swart et al. reported a simulation approach using HSPICE to compute the transient and steady-state electrothermal behaviour of thermally isolated microheater[32]. However, these numerical methods are characterised by the large amount of computations and the tedious work of defining the model.

Analytical modelling of heat transport in MEMS microdevices has also been reported in a number of papers [33-37]. Although, the previous studies dealt with heat convection, heat conduction, and radiation in homogenous materials, these models were valid only for active area of the heater fabricated from a single material. The theoretical studies on the temperature distribution on segmented wires constructed from different materials have been rarely reported. On the other hand, miniature devices such as micro/nano wires can be a great asset for MEMS and Nano-Electro Mechanical Systems (NEMS), offering a broad range of applications [38-41]. In order to
integrate these materials into MEMS and NEMS, segments of them are often joined [42]. With the advantages of incorporating electrical, mechanical and thermal properties, segmented wires have been used in thermal flow sensors [43, 44]. Moreover, the primary objective of the heater of reducing the heat loss can be met by thermally isolating the segmented wire, allowing a higher operating temperature. Therefore, a preliminary study on a large-scale model could provide an insight on the thermal and electrical characteristics of segmented wires. Furthermore, a precise theoretical model of segmented wires is useful for determining parameters such as sensitivity, response time, is a valuable tool for parametric optimization of the device. The objective of the present work is to derive a steady-state analytical model for describing the temperature distribution in suspending segmented wires. The temperature distribution on the segmented wires was solved based on the equilibrium of the heating power to the heat losses caused by heat conduction along the wires, and heat convection through the wire-surface areas. Initial verifications were made between analytical and numerical data with good agreement. To verify the analytical model and to simplify the measurement, we employed a scaled up model, where nichrome served as the heating wire and copper as the conducting path. The agreement of the theoretical analysis, numerical simulation and experimental data demonstrate the potential of our segmented wire model for the development of thermal sensors and actuators.

2. Problem definition and mathematical formulation
We consider the one-dimensional problem of electric current induced Joule heating of a suspended bridge that dissipates heat by internal conduction and free convection in air. The length of the bridge is considered to be $2l$ with ends located at $-l$ and $+l$. The nichrome based heater element is $2l_o$ long and connected at each end by copper contact segments as shown in Fig.1. Upon current input through the copper wire, a steady-state temperature distribution along segment 1 (nichrome), $\theta_1(x)$ will develop with a maximum, $\theta_{\text{max}}(x = 0)$ at the middle of the bridge. The temperature distribution across segment 2 (copper) is represented by $\theta_2(x)$.

To develop expressions for the temperature distribution along the bridge, the following assumptions are made:

(i) Heat transfer occurs by conduction along the bridge and natural convection in air

(ii) Heat conduction and convection is significant in one dimension only. i.e. the temperature distribution is one-dimensional

(iii) The resistivity varies linearly as a function of temperature with temperature coefficient of resistance. It is represented by $\rho = \rho_o \left[ 1 + \xi (\theta - \theta_o) \right]$  

(iv) Thermal conductivity is independent of temperature

2.1 Steady-state temperature distribution along the resistive bridge

At the steady state, the supplied heating power is balanced by the heat loss through conduction and free convection. The rate of heat conduction in direction $x$ is proportional to the temperature gradient, which is the rate of change in temperature
with distance in this direction. On a rate basis, the general form of conservation of energy requirement is

\[ E_{\text{gen}} = E_{\text{cond}} + E_{\text{conv}} \]  

(1)

where the heat loss caused by conduction is described by

\[ E_{\text{cond}} = \kappa A_c \left[ (\theta(x) - \theta(x + \Delta x)) \right] \]  

(2)

Considering the thermal energy is generated in the bridge at the expense of electrical energy, the energy generation term will be positive and acts like a source. The generated thermal energy is expressed by

\[ E_{\text{gen}} = VI = I^2 R = I^2 \frac{\rho A c \Delta x}{A_c} = j^2 A_c \rho \Delta x \]  

(3)

The heat loss due to free convection is presented in the following equation:

\[ E_{\text{conv}} = h A_c (\theta - \theta_a) \]  

(4)

where \( A_c = \pi d \Delta x \) is the surface area of the portion \( \Delta x \). In the case of a horizontal cylinder, the heat transfer coefficient \( h \) in air is evaluated by the expression,

\[ h = \frac{\text{Nu} \kappa_{\text{air}}}{d} \]  

(5)

The Nusselt number (\( \text{Nu} \)) can be evaluated by finding the Rayleigh number \( Ra \), given by

\[ Ra = (\text{GrPr}) = \frac{g \beta_{\text{air}} \rho_{\text{air}}^2 C_{\text{p,air}}}{\mu_{\text{air}} \kappa_{\text{air}}} \]  

(6)

On substituting the corresponding values in equation (6) gives \( Ra = 15.932 \) which falls between \( 0.1 < Ra < 100 \) [45]. The correlation to evaluate the Nusselt number for the corresponding Rayleigh number is expressed by
\[ \text{Nu} = c \left( \text{Ra} \right)^m \]  

(7)

The constants \( c \) and \( m \) for the corresponding Rayleigh number is \( c = 1.02 \) and \( m = 0.148 \).

Accordingly, the heat transfer coefficient in air is calculated to be \( h = 31.562 \text{W/m}^2 \cdot \text{K} \).

Substituting equations (2), (3) and (4) in equation (1) and simplifying lead to

\[ \kappa A_c \left[ \frac{\dot{\theta}(x + \Delta x) - \dot{\theta}(x)}{\Delta x} \right] - h\pi d(\theta - \theta_o) = -j^2 A_c \rho \]

\[ \kappa A_c \frac{\text{d}^2 \theta(x)}{\text{d}x^2} - h\pi d(\theta - \theta_o) = -j^2 A_c \rho \]  

(8)

Following the assumption (iii), the equation (8) can be simplified further as

\[ \kappa A_c \frac{\text{d}^2 \theta(x)}{\text{d}x^2} - (h\pi d - j^2 A_c \rho_o \xi) \theta = j^2 A_c \rho_o \left( \xi \theta_o - 1 \right) - h\pi d \theta_o \]  

(9)

The above equation (9) represents the one-dimensional heat transfer equation along the wire. Therefore, the governing equations for the two segments are given by:

\[ \kappa_1 A_c \frac{\text{d}^2 \theta_1(x)}{\text{d}x^2} - (h\pi d - j^2 A_c \rho_{1,o} \xi_1) \theta_1 = j^2 A_c \rho_{1,o} \left( \xi_1 \theta_0 - 1 \right) - h\pi d \theta_o \]  

(10)

\[ \kappa_2 A_c \frac{\text{d}^2 \theta_2(x)}{\text{d}x^2} - (h\pi d - j^2 A_c \rho_{2,o} \xi_2) \theta_2 = j^2 A_c \rho_{2,o} \left( \xi_2 \theta_o - 1 \right) - h\pi d \theta_o \]  

(11)

Equations (10) and (11) represent a system of linear ordinary differential equations (ODE) of second order with temperature of two segments as a function of distance \( x \).

The constants resulting from the solution of these two equations are evaluated by imposing boundary and continuity conditions:

(i) Thermal gradient at the center of the bridge is zero, that is \( \frac{\partial \theta_1}{\partial x} = 0 \)

(ii) The end or contact temperatures at \(-l \) and \(+l \) are fixed at room temperature,

\[ \text{i.e., } \theta_{-l} = \theta_{+l} = \theta_o \]
(iii) The difference of heat flux across the interface, \( x = \pm l_o \) is zero, that is
\[
\kappa_1 \frac{\partial \theta_1}{\partial x}(x = l_o) = \kappa_2 \frac{\partial \theta_2}{\partial x}(x = l_o)
\]

(iv) The temperature across the interface \( x = \pm l_o \) is equal, that is
\[
\theta_1(x = l_o) = \theta_2(x = l_o)
\]

The solution for the temperature distribution in the heater element \( \theta_1(x) \) and in the end-contact segment \( \theta_2(x) \) take the following form
\[
\theta_1(x) = A \exp \left[ \sqrt{\frac{(h \pi d - j^2 A_{cr} \rho_{1,o} \xi_1)}{\kappa_1 A_{cr}}} x \right] + B \exp \left[ -\sqrt{\frac{(h \pi d - j^2 A_{cr} \rho_{1,o} \xi_1)}{\kappa_1 A_{cr}}} x \right] + \delta_1, 0 < x < l_o \quad (12)
\]
\[
\theta_2(x) = C \exp \left[ \sqrt{\frac{(h \pi d - j^2 A_{cr} \rho_{2,o} \xi_2)}{\kappa_2 A_{cr}}} x \right] + D \exp \left[ -\sqrt{\frac{(h \pi d - j^2 A_{cr} \rho_{2,o} \xi_2)}{\kappa_2 A_{cr}}} x \right] + \delta_2, l_o < x < l \quad (13)
\]

Where
\[
\delta_1 = \frac{j^2 A_{cr} \rho_{1,o} (\xi_1 \theta_0 - 1) - h \pi d \theta_0}{(j^2 A_{cr} \rho_{1,o} \xi_1 - h \pi d)} \quad \text{and} \quad \delta_2 = \frac{j^2 A_{cr} \rho_{2,o} (\xi_2 \theta_0 - 1) - h \pi d \theta_0}{(j^2 A_{cr} \rho_{2,o} \xi_2 - h \pi d)}
\]

For convenience, let us consider the constants in equation (12) and (13) as
\[
\gamma_1 = \sqrt{\frac{(h \pi d - j^2 A_{cr} \rho_{1,o} \xi_1)}{\kappa_1 A_{cr}}} \quad \text{and} \quad \gamma_2 = \sqrt{\frac{(h \pi d - j^2 A_{cr} \rho_{2,o} \xi_2)}{\kappa_2 A_{cr}}}
\]

The constants A, B, C, and D are solved using Cramer’s rule and are represented as follows:
The constants $A$, $C$ and $D$ depend on the geometry, the thermal conductivity, the convective heat transfer coefficient and the electrical resistivity of the wires. The
maximum point of the curve $\theta_{\text{max}}(x)$ which is also known as hot-spot location along the length of the bridge can be obtained by differentiating $\theta_1(x)$ with respect to $x$ and letting it to zero.

$$\frac{\partial \theta_1}{\partial x} = A\gamma_1 \exp(\gamma_1 x) + \delta_1 = 0, \quad -l_o < x < l_o$$

$x = 0$ is the only critical point for the curve in this region. The maximum temperature along the bridge is obtained by substituting $x = 0$ in equation (15):

$$\theta_{\text{max}}(x) = A\gamma_1 + \delta_1, \quad -l_o < x < l_o$$

### 2.2 Evaluation of electric potential distribution along the bridge

The voltage between any two end points is defined as

$$\Delta V = -\int E \cdot dx$$

(16)

The electric field can be written as, $E = \rho J$ where $\rho$ is the electrical resistivity of the material in $\Omega \cdot m$ and $J$ represents the current density in $A \cdot m^{-2}$:

$$\Delta V = -\int \rho J \cdot dx$$

(17)

Simplifying and integrating the above equation on both sides leads to,

$$V(x) = -\rho J x + C$$

(18)

The generalized equation (18) can be used to represent the electric potential distribution along the heater and end-contact segments as follows:

$$V_1(x) = -J \rho_1 x + C_1, \quad -l_o < x < l_o$$

(19)

$$V_2(x) = -J \rho_2 x + C_2, \quad -l < x < -l_o, \quad l_o < x < l$$

(20)
Where $C_1$ and $C_2$ are integration constants. The constants are solved by imposing boundary and continuity conditions which are stated below:

(i) The voltage drop at one end of the conductive segment is zero, i.e., $V_1(x = l) = 0$

(ii) The voltage drop across the interface $x^* = \pm \alpha$ between two segments is equal.

i.e., $V_1(x = \pm l_o) = V_2(x = \pm l_o)$

(iii) The voltage drop at the other end of conductive segment is twice the voltage drop at the centre of the bridge i.e., $V_2(x = -l) = 2V_1(x = 0)$

The electric potential distribution is given by,

$$V_1(x) = -J\rho_1x + J\rho_2(l - l_o) + J\rho_1l_o, -l_o < x < l_o$$  \hspace{1cm} (21)

$$V_2(x) = -J\rho_2x + J\rho_2l, l_o < x < l$$  \hspace{1cm} (22)

$$V_2(x) = -J\rho_2x + J\rho_2l + 2Jl_o(\rho_1 - \rho_2), -l < x < -l_o$$  \hspace{1cm} (23)

Equations (21), (22) and (23) represent the temperature independent solution for electric potential distribution along the bridge. Using the linear relationship between resistivity and temperature, the temperature dependent resistivity relationship for the two segments are represented using the above equation as follows:

$$V_1(x) = -J\left(\rho_{1,o} + \rho_{1,o}\xi_1\theta - \rho_{1,o}\xi_1\theta_0\right)x + J\left(\rho_{2,o} + \rho_{2,o}\xi_2\theta - \rho_{2,o}\xi_2\theta_0\right)(l - l_o) + J\rho_1l_o, -l_o < x < l_o$$  \hspace{1cm} (24)

$$V_2(x) = -J\left(\rho_{2,o} + \rho_{2,o}\xi_2\theta - \rho_{2,o}\xi_2\theta_0\right)x + J\left(\rho_{2,o} + \rho_{2,o}\xi_2\theta - \rho_{2,o}\xi_2\theta_0\right)l, l_o < x < l$$  \hspace{1cm} (25)

$$V_2(x) = -J\left(\rho_{2,o} + \rho_{2,o}\xi_2\theta - \rho_{2,o}\xi_2\theta_0\right)x + J\left(\rho_{2,o} + \rho_{2,o}\xi_2\theta - \rho_{2,o}\xi_2\theta_0\right)l + 2Jl_o\left(\rho_{1,o} + \rho_{1,o}\xi_1\theta_1 - \rho_{1,o}\xi_1\theta_0\right) - J\left(\rho_{2,o} + \rho_{2,o}\xi_2\theta - \rho_{2,o}\xi_2\theta_0\right), -l < x < -l_o$$  \hspace{1cm} (26)
Figure 2(a) shows the distribution of the electric potential along the segmented wire for various length ratios and for a constant current of 3 A. The solid lines represent the temperature independent potential distribution and the dashed lines represent the temperature dependent potential distribution. It is to be noted that the solid lines plot in Figure 2(a) were obtained using equation (21), (22) and (23) and the data in dashed lines were generated using equations (24), (25) and (26) respectively.

We observed that a concentration gradient of delocalised electrons is set up in the segmented wires circuit when a current is applied. Moreover, the concentration gradient is high on one end of the conducting path and zero on the other end. Furthermore, much potential energy is lost in the nichrome wire indicating the occurrence of more Joule-heating due to higher resistivity \( \rho_1 \gg \rho_2 \). Figure 2(b) shows the distribution of the electric potential along the segmented wire for varying currents and for a constant segment length ratio of \( \alpha = 0.25 \). We observed that higher the applied current, maximum is the electric potential for temperature dependent and independent cases correspondingly. In addition, the solid and dashed lines show almost the same potential for the rise in current, indicating that the change in resistivity due to temperature variation do not play a significant role to the electric potential.

2.3 Voltage and power characteristics of the bridge

Equation (17) can be integrated along the path from \( 0 < x^* < l_0 \) to determine the voltage across the heating segment. It is given by,

\[
\Delta V(x) = V_o - V_o = -\int_0^{l_0} \rho_1 \cdot J dx
\]
Integrating and simplifying the above equation leads to,

\[ V_o - V_x = \left\{ -J\rho_{1,0} \left[ I_o + \xi_1 \left( \frac{\text{Aexp}(\gamma_1 x)}{\gamma_1} - \frac{\text{Aexp}(-\gamma_1 x)}{\gamma_1} + \delta_1 I_o \right) - \xi_1 \theta_0 I_o \right] \right\}, 0 < x < I \]  

(27)

Similarly, equation (17) can also be used along the path \( I_o < x < I \) to determine the voltage across the conduction segment. It is given by,

\[ V \text{ or } V_x = \left\{ -J\rho_{2,0} \left[ I + \xi_2 \left( \frac{\text{Cexp}(\gamma_2 I)}{\gamma_2} - \frac{\text{Dexp}(-\gamma_2 I)}{\gamma_2} + \delta_2 I \right) - \xi_2 \theta_0 I \right] \right\}, I_o < x < I \]  

(28)

The Voltage Current (V-I) characteristics of the entire bridge can be obtained from equations (27) and (28) as follows:

\[ V = 2 \left[ (V_x - V_o) - (V_o - V) \right], -I < x < I \]  

(29)

The power dissipated in the bridge can be found as,

\[ P = (V_x) \cdot I \]  

(30)

2.4 Parametric Optimization

Beside the geometry, various material properties also affect the thermal behavior of a segmented wire. Material properties such as electrical resistivity and thermal conductivity play a major role in heat transfer in a Joule-heated segmented wire. Figure 3(a) shows the temperature plot along the bridge for different thermal conductivity values of \( \kappa_1 \) and a fixed \( \kappa_2 \). According to the Fourier’s law of heat conduction, a heater \( (\alpha = 0.15) \) with a higher thermal conductivity \( (\kappa_1 = 42 \text{W/m.k}) \) loses heat to the ambient air and subsequently results in a lower temperature at the center.
The inset of Figure 3(a) indicates the effect of $\kappa_1$ on the temperature distribution along the heater. In the same way, Figure 3(b) indicates the temperature distribution of the bridge for different $\kappa_2$ at a fixed $\kappa_1$ and 2A heating current. The inset in Figure 3(b) indicates that the fixed parameters contribute to a small temperature change as described in equations 12 and 13.

The assumption of linear temperature dependence of resistivity in our analytical model holds well over the range of ambient temperature to the maximum temperature as shown in Figure 4. The temperature is higher with higher resistivity $\rho_1$ and vice-versa as depicted in the inset of Figure 4(a). Furthermore, the temperature change due to the fixed resistivity ($\rho_2$) on the conducting wire is almost negligible. Likewise, Figure 4(b) shows that the change in temperature is small in conducting wires of different resistivities.

Other influencing parameters are wire diameter, heat transfer convection coefficient. Figure 5(a) shows the relationship between the wire diameter and the temperature distribution for a fixed segment length ratio ($\alpha = 0.15$) and a fixed 2A heating current. Increasing the diameter of the wires decreases the electric resistance and consequently lower heating power and lower temperature. Next, the influence of heat transfer convection coefficient on temperature is studied for various values of $h$ (20-40W/m²·K) at a fixed length ratio ($\alpha = 0.15$) and a constant current of 2A. Figure 5(b) shows that the temperature distribution is different for various values of $h$, and
the temperature at the center of the bridge is maximum for a small heat transfer coefficient \( h=20 \text{W/m}^2 \cdot \text{K} \).

3. Verification and discussions

3.1 Numerical Simulation

COMSOL Finite Element simulation (FEM) was used to first verify the analytical solutions. Figure 6 shows the analytical and simulation results of the temperature distribution along the heater and the conducting wires. Joule heating causes a high temperature at the centre of the heater, while the temperature decreases toward the Cu wires, reaching room temperature at the fixed ends. Increasing the applied current leads to an increase in the temperature at the center of the heater. Furthermore, the temperature of the heater exhibits a parabolic distribution, while that of the conducting wire shows a linear behaviour. Our assumption of neglecting radiation loss in the model holds well as the maximum temperature is only around 120 °C for the longest heating wire. Moreover, Figure 7 depicts the maximum temperature at center versus the current and indicates that the longest heater \( (\alpha = 0.35) \) results in the highest temperature. The analytical and numerical data agree well and suggest the accuracy of the proposed analytical model.

3.2 Temperature measurement

A millimetre-scale experimental setup was designed to verify the Joule heating model of segmented wires having a diameter of 1.2 mm each. The electrical resistivity of the nichrome (NiCr) and copper (Cu) wires are 1.05 \( \mu \Omega \text{m} \) and 0.016 \( \mu \Omega \text{m} \) respectively. The nichrome wire was cut into five different lengths 3 cm, 4 cm, 5 cm, 6 cm and 7 cm. Copper
wires were cut into suitable sizes so that the total length of the segmented wire is 20 cm. The two ends of the wire segments were joined by spot-welding through a high current pulse and the structure was suspended as a bridge between two copper blocks as shown in Fig. 8 (a).

The interfacial contact resistance plays an important role in metal-metal junctions and thus the value of contact resistance \( R_c \) was measured by the Transmission Line Method (TLM). This measurement technique involves making a series of metal-metal contacts separated by various distances. Probes were applied to pairs of contacts, and the resistance between them was measured by applying various current across the contacts and sensing the corresponding voltage. The resistance calculated is a combination of contact resistance of first contact and the contact resistance of second contact in the bridge. In doing so, a plot of resistance versus various contact separation (\( l \)) was obtained and shown in Fig. 8(c). The measured resistance was linearly fitted with a slope of 0.0109. The intercept of the line is 0.6 m\( \Omega \), which is twice the contact resistance. Therefore, the contribution of contact resistance in the bridge with the shortest and longest nichrome wire are only 1.7\% and 0.77\%, respectively. This negligible contact resistance strongly justifies the negligence of the contact resistance in our model.

Finally, the temperature measurements were carried out using IR thermal camera (GOBI-1513, Xenics IR solutions) shown in Fig. 8 (b). The infra-red detector used for this work is the bolometer Si: H Focal Plane Array (FPA) of 384\times288 pixels. The errors resulting from an IR thermographic temperature measurement include the errors induced by instrument. The two most commonly encountered errors in IR thermography
are spatial error and temperature error. The thermal image obtained from our IR camera resolution and the 0.2m long bridge. For this geometry, the relative spatial error and the relative temperature error are estimated to be 0.25% and 0.34%, respectively.

When applying a sufficiently large current, Joule heating caused the temperature at the NiCr wire to increase, while the temperature at the Cu sections remained relatively low in the steady state. It was found experimentally that the time taken to achieve steady-state temperature in the bridge was approximately 45 seconds. Once this state was achieved, the spatial temperature gradient along the bridge remained unchanged.

Figure 9 shows the analytical and experimental results of the temperature distribution of NiCr and Cu wires. Evidently, a high temperature was observed at the centre of the NiCr wire, while the temperature decreases toward the Cu wires, reaching room temperature at the fixed ends. It is also observed that increasing the applied current leads to an increase in the temperature in the middle of the NiCr wire. Furthermore, the temperature of the NiCr wire exhibits a parabolic distribution, while that of the Cu wire shows a linear behaviour. The experimental data validates the proposed analytical models.

Figure 10(a) and 10(b) show the analytical and experimental temperature profile for different segment length ratios with a constant current. The inset in Fig.10 (a) shows various wire geometries considered for the experiment. It is observed that longer segmented nichrome wire \( (\alpha = 0.35) \) exhibits higher temperature at the centre of the bridge due to low heat loss. On the other hand, 3cm nichrome wire \( (\alpha = 0.15) \) loses more heat leading to lower temperature in the middle of the bridge. This statement
suggests that longer nichrome wire receives more heating energy\(P=I^2R\) than the shorter wires under the same applying current. Experimental and analytical model agree well with, Fig. 10(a) and 10(b) respectively.

### 3.3 Voltage and power measurement

Figure 11(a) shows the voltage versus current (V-I) characteristics in air for different segment length ratios(\(\alpha\)). We observed that the resistance of NiCr linearly increases with increasing segment length ratio. Correspondingly, a longer heating segment results in a higher voltage drop. Since the area of the electrical contact between two segments is small, contact resistance could contribute to the total resistance. Furthermore, using a simple multimeter could add lead resistance to the resistance of the sample. Considering the small contact area and to primarily avoid lead resistance, four-point terminal sensing was used to measure the voltage across the bridge.

Figure 11(b) shows the power versus current characteristics from the analytical models and experimental measurements for various segment length ratios. For a fixed current of 3A, a factor of 13.1 times increase in resistance is observed between \(\alpha = 0.15\) and 0.2 due to increasing length of the nichrome segment. Furthermore, the same percentage increase in power is observed between the two configurations. Different wire lengths possess different resistance, and so different heating power. Figure 11(b) clearly shows the relationship between the geometrical configuration, electrical power and temperature. The analytical V-I and P-I curves were generated by equations (29) and (30) derived in section 2.3.
In a heater design, the maximum temperature rise should be lower than a specific range which helps to avoid the degradation of heater performance. Depending on the maximum current rating for the wire and the length, maximum temperature rise for the straight wires in open air are obtained. Figure 12 illustrates the relationship between the maximum temperature at the centre of the bridge and the applied current for various segment length ratios obtained experimentally. We observed that 7-cm nichrome wire with 18 standard wire gauge (SWG) reach a maximum temperature of 110 °C for 4A. Whereas, the 3-cm nichrome wire could rise up to 65 °C. This statement is well supported from experimental data shown in Fig.12

4. Conclusions

Though microheaters have been commonly used for decades, there was no accurate theoretical analysis for the thermal and electrical characteristics of the microheaters. Here, we derived an exact analytical solution for the steady-state temperature distribution in a Joule-heated segmented wire bridge. The model predicts the temperature distribution along the wire. The temperature distribution further was used to predict the temperature dependent electrical behaviour of the wire bridge. The analytical models were initially verified with numerical simulation and finally validated with a scaled-up wire model using infrared thermography measurement. An excellent agreement of results among each models demonstrate the potential of the proposed analytical technique. Therefore, the proposed model can be useful for the thermal design of Joule-heated thin, wire-like micro devices. Moreover, this heater model...
provides an insight into the model development for thermal sensors operating at high temperatures.

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Nomenclature

\( A, B, C, C_0 \) constants of ODE solution  

\( A_{cr} \)  cross-sectional area, \( m^2 \)  

\( A_{ls} \) lateral surface area, \( m^2 \)  

\( C_{p,\text{air}} \) specific heat capacity of air, J/kg K  

\( C_1, C_2 \) and \( D \) constants obtained in potential distribution solution  

\( d \) diameter of the wire, mm
Greek letters

\( \alpha \) segment length ratio, dimensionless

\( \beta_{\text{air}} \) thermal expansion coefficient of air, \( 1/\text{K} \)

\( \Delta x \) small incremental distance in the wire, cm

\( \delta_1 \) and \( \delta_2 \) particular integral solution (involving constants) for nichrome(NiCr) and copper(Cu) segments

\( \mu_{\text{air}} \) dynamic viscosity of air, Pa s

\( \rho_{\text{air}} \) density of air, kg/m\(^3\)

\( \rho_1 \) and \( \rho_2 \) electrical resistivity of nichrome (NiCr) and copper (Cu) wires, \( \Omega\text{m} \).

\( \kappa_1, \kappa_2 \) and \( \kappa_{\text{air}} \) thermal conductivity of nichrome(NiCr), copper(Cu) and air, W/m K
\( \xi_1 \) and \( \xi_2 \)  
\( \gamma_1 \) and \( \gamma_2 \) 
\( \theta_1(x) \) and \( \theta_2(x) \)  
\( \theta_o \) 

**temperature coefficient of resistance of nichrome and copper wires, \( 1/\text{K} \)**

**gamma values of nichrome and copper segment (involving constants)**

**temperature distribution along nichrome and copper segments, \( \text{C} \)**

**ambient temperature, \( \text{C} \)**

**Non-dimensional Numbers**

- **Gr**  
  Grashof number;

- **Nu**  
  Nusselt number;

- **Pr**  
  Prandtl number;

- **Ra**  
  Rayleigh number;

**Subscripts and superscripts**

- **cr**  
  cross section

- **cond**  
  conduction

- **conv**  
  convection

- **gen**  
  generation

- **l**  
  half-length of the bridge

- **ls**  
  lateral surface area

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