Secondary-Tertiary Transition: What Mathematics Skills Can and Should We Expect This Decade?

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Abstract

We report on the mathematics competencies of 206 Engineering and Science students commencing an algebra and calculus course at an Australian university in the first semester of 2006. To inform course design in the face of growing student diversity, skills were assessed via a pre-test covering six fundamental areas. These data were also compared with the 1997 to 2001 data. The findings revealed reasonable skills with arithmetic, fractions, and index laws but ongoing weaknesses in areas of algebra, functions, and trigonometry. These findings have important implications for planning in Australian universities. Implications for school curricula are also considered.

Introduction: The Australian Context

Secondary-tertiary transition and mathematics under-preparedness for tertiary studies have long been the focus of educational interest in Australia. Much was written on skills, foundation studies and related issues in the 1990's, especially in the context of the development of support structures in universities in Australia (Taylor, 1999). Recent interest in under-preparedness has also been reported internationally (Ulovec, 2006).

Widening tertiary entry policies in Australia generally, and the lowering of mathematics pre-requisites in many Engineering and Science programs, in particular, have had a dramatic effect on the mathematics skills of students commencing tertiary studies (Holton, 2002, Coutis, Cuthbert, & MacGillivray, 2002). In response, many Australian universities now offer what were mathematics foundation courses as full courses in Science and Engineering programs, to build basic competencies (Carmody, Godfrey, & Wood, 2006). While this flexibility has opened tertiary studies to more students, lower mathematics entry requirements have taken a serious toll on mathematics studies in Australia generally. Not only is it harder to persuade school students to do advanced mathematics subjects in Years 11 and 12, but accommodating school content in Science and Engineering degrees has reduced the study of higher level tertiary mathematics subjects.

These and other factors have contributed to the general downward spiral in commitment to studies in the mathematical sciences in Australia and elsewhere. Declining numbers of mathematics majors have resulted in Australian universities closing Mathematics Departments. In the recent National Strategic Review of Mathematical Sciences Research in Australia (Australian Academy of Science, 2006), international leaders reported that “Australia's distinguished tradition and capability in mathematics and statistics is on a truly perilous path”. Key findings were that Australian students are abandoning higher-level mathematics in favour of elementary mathematics, that not enough trained mathematics teachers are entering the high school system, and that many university courses such as engineering that should include a strong mathematics and statistics component, no longer do. Key recommendations included encouraging greater numbers of high school students to study intermediate and advanced mathematics, significantly increasing the number of university graduates with appropriate mathematical and statistical training, and ensuring that all mathematics teachers in Australian schools have appropriate training in the disciplines of mathematics and statistics to the highest international standards.

Against this background, declining numbers of tertiary mathematics teachers are endeavouring to support and retain students in their studies, and to provide courses appropriate for their needs. Faced with the challenge of assessing academic readiness quickly and efficiently, to counsel students and steer them into courses appropriate for their needs, there is a need to assess mathematics skills tests alongside other factors. Clear information on current entry-level skills is needed to inform support programs for under-prepared students, and to guide course and curriculum development at tertiary level. Empirical data provide information on the long effect of school studies on both school-leavers and mature-age students.

Skills Tests and Assumptions

Much of the early Australian literature on mathematics skills and skills-testing in secondary-tertiary transition and adult learning stems from specialists in the area of bridging and support (Taylor, 1999, Wood, 2002). However, diagnostic tests are now being used increasingly in mainstream first-year university mathematics and statistics courses, to identify, advise and support students who may be at risk of failing. In a recent report, University of Sydney academics Britton, Daners, and Stewart (2006) observed that many students are “not ready for the sophisticated level of mathematics at university”. While their main purpose in using a diagnostic skills test was to better inform students on their suitability for first-year university mathematics studies, they also found that combining diagnostic test results with school results gave a better predictor of students’ success in university courses than school results alone.

Expressing concern about first-year mathematics failure rates being “higher than in other discipline areas”, Sydney University of Technology academics Carmody, Godfrey, and Wood (2006, p 24) claimed that one reason for the high failure rate is the “differing mathematical backgrounds of students who enter university”. Their response was to administer a diagnostic skills test in the first week of the semester, and use the results to advise students on doing support studies or doing a foundation course to build skills. The diagnostic test was found to be useful in “alerting those students who were seriously under prepared for mathematics at university”.

Queensland University of Technology academics Coutis, Cuthbert, and MacGillivray (2002, p 97) reported the sharp increase in the diversity of academic preparedness as follows: “a substantial proportion of commencing students taking mathematically based university subjects do not have the prescribed assumed knowledge requirements”. Using diagnostic skills tests they identified students with weak mathematical background, and offered a range of support programs which they concluded were effective in bridging the gap between the students’ assumed and actual knowledge. Similarly, other reports on the effectiveness of interventions that attempt to address such gaps report positively on students’ participation and affective response. However, scanning the literature reveals no sustained objective research into the effects on learning and performance, and in fact, Wood (2002) claimed that short programs are not effective for what are termed "weak" students.

The emphasis in most Australian reports on the use of diagnostic tests has been on skills testing to inform student support and counselling. Certainly, there have been few attempts to compare the mathematics skills of students entering Science and Engineering in Australia now with the skills of those who entered a few years ago. Obvious reasons for this gap in the literature are that changes in student population and curriculum emphases in many university courses make comparisons difficult. However, clearly university programs must respond to these changes, and comparisons are valuable for informing both school and university curricula.

This paper describes the findings of a study that addresses this gap in the literature. We report on the core mathematics skills of students on entry to an Australian tertiary-level mathematics course in 2006, and compare these with the skills of students entering the same course five years earlier. And we consider the implications of the findings.

The Study and the Skills Test

The investigation targeted students entering Algebra & Calculus I at the University of Southern Queensland (USQ). The topics in this course are typical of those traditionally studied by Science and Engineering students on entry to their university studies: single-variable calculus, complex numbers, vectors, and matrices. With declining entry skills however, an increasing number of students now study a foundation mathematics course first, to develop skills that were previously established in school studies.

In the first week of their studies in 2006, Algebra & Calculus I students were encouraged to complete a diagnostic test covering six areas: basic numeracy and arithmetic, fractions and percentages, index laws and scientific notation, algebra, functions and graphs, and trigonometry. An existing test was used, to facilitate comparison with data from past years. Developed and administered by Janet Taylor and others in USQ's support division some years before, the test comprised 51 questions covering key skills academics had come to expect recent school-leavers to have on entry to Engineering and Science. This team also gathered the 1997-2001 data. Their contribution is noted with thanks. Evolving curricula and use of technology have made some questions on this test dated, but we retained all to capture maximum information and to facilitate comparison with earlier years. The findings of this study have been used to inform the development of a new test for subsequent stages of our work.

Of the 331 students enrolled initially, just over half were studying externally (52.6%). We administered the test electronically, but marked by hand. Submission was voluntary, but the response rate was good, 206 students (62.2%) completing the test. The majority (135) were engineering students, 54 were in science, 11 in education, and the remaining 6 in other faculties.

Analysis and Findings

Appendix A lists most of the questions on the test, and the success rates for each, in 2006 and the years 1997 to 2001. In this earlier period, data were only captured for on-campus Engineering students. Hence two sets of data are provided for 2006: the full group of 206 students, and the 75 on-campus Engineering students, a subgroup. Because of limited space, data for 13 questions are omitted: those on which performance was consistently high, over 80 or 90%, largely basic calculations and percentages.

Skills Data for 2006

The overall 2006 test results were disappointing. Converted to percentages, the mean and standard deviation of marks were calculated to be 62.7% and 20.0%, respectively. Sixty students (29.1%) scored less than 50% overall. Figure 1 shows the overall mark distribution for all 206 students.

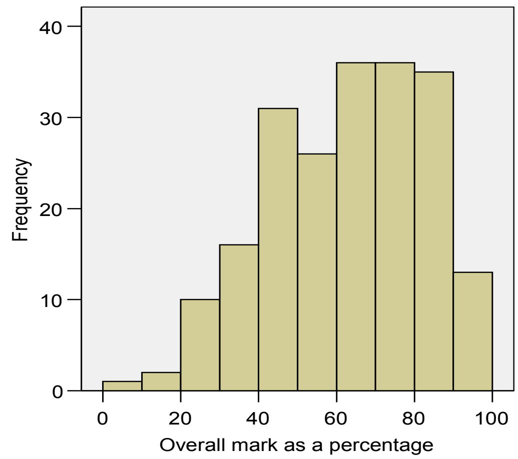

Figure 1: Distribution of test marks in 2006.

Of the six areas tested, questions on basic arithmetic, fractions, and the index laws were generally well answered. However, students' skills in the areas of algebra, functions, and trigonometry were cause for concern. Table 1 shows the percentage of students who scored less than 50% in each of these areas.

Table 1
Percentage of Students Scoring Less Than 50% in Each Area

Arithmetic	Fractions	Index Laws	Algebra	Functions	Trigonometry
1.0	1.0	10.7	48.5	37.4	44.2

See the boxplots in Figure 2 for more information on the spread of marks within each area. Algebra skills were very disappointing:

- 40% could not factorise the quadratic $6x^2 + x - 12$.
- 42% could not solve the quadratic equation $3x^2 + 4x - 8 = 0$.
- 43% could not rearrange the equation $y = (8t + 3)^3 + 4$.
- 44% could not expand $(x + 1)(-2x + 1)(x - 3)$.
- And 59% could not subtract two algebraic fractions.

Given current curriculum emphases, some success rates were expected to be low:

- Only 28.6% could solve a cubic equation.
- Only 21.8% could solve $|3x+3| < 6$.
- Only 15.4% could complete the square in a quadratic expression. Hence questions such as finding the centre and radius of a circle, given its equation, were poorly answered.
- Only 20% knew that $\sin 2\theta = 2\cos\theta\sin\theta$.

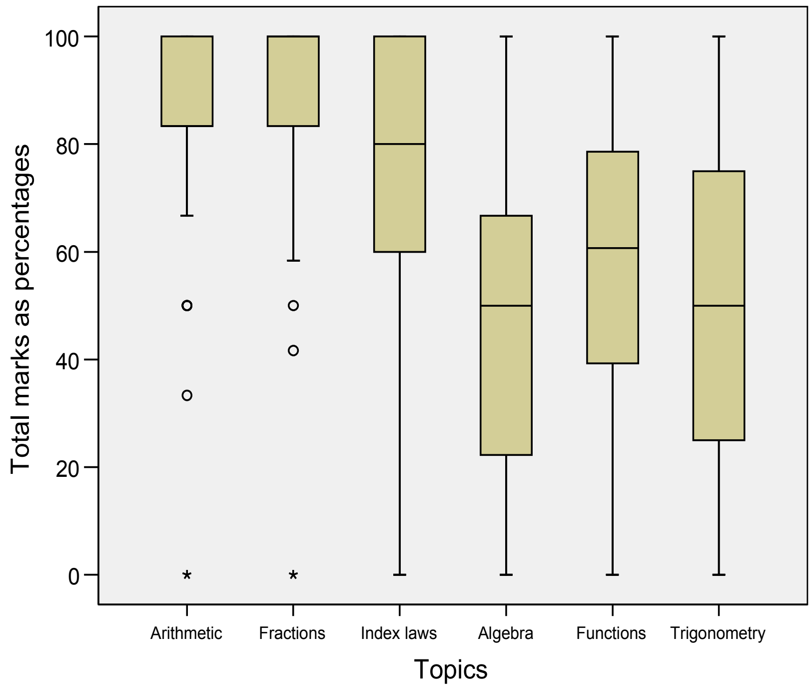

Figure 2: Boxplots showing the distributions of marks in these areas.

Graphing skills were also disappointing:

- 70% could draw the graph of a parabola, given its equation.
- But only 51% were able to sketch the graphs of sine and cosine functions.
- Only 34% could sketch $y = e^x$ and $y = \log_e x$.
- Only a quarter could find the domain and range of $g(x) = \sqrt{x-1}$.
- Less than a third could solve $x^2 - 1 = \sqrt{x}$ graphically.
- Similarly, only a third could sketch $y = \dfrac{1}{x-2}$.

Function notation skills were very limited. Given $f(x) = x^2 + 1$ and $g(x) = \sqrt{x-1}$.

- 64% could calculate $f(-1)$.
- But only 39% could find $f(x + \mathrm{h})$.
- And only 47% could find $f(g(x))$.

Straight line skills were mixed:

- 82% could find the equation given slope and y-intercept.
- But only 54% were able to find the equation of a line given 2 points.
- And 61% could write the equation of a line, given a simple graph.

Trigonometry skills were dismal:

- 68% knew the basic trigonometric identity $\cos^2\theta + \sin^2\theta = 1$.
- But when asked to find all angles between 0 and 2π that satisfy $\sin A = 0.4$, less than a third gave both angles. Using their calculators didn't help much either: only another 15% managed to use a calculator to give one angle correctly.
- Only around 44% could use the cosine rule to find one side of a triangle.

- Similarly, only about 45% could solve a simple word problem involving trigonometry.

Comparison with Previous Years

As noted above, skills data were only gathered for on-campus engineering students in the years 1997 to 2001. Therefore, for fairer comparison with the 2006 data, the skills of the subgroup of 75 on-campus engineering students in the 2006 class were compared with those of the 2000 and 2001 cohorts, comprising 86 and 71 students, respectively.

For these cohorts, no statistically significant differences were found in the six broad skills areas. However, differences were found for particular skills in algebra, functions and graphing, and trigonometry. These include a decline in ability to substitute $x + h$ into a given function $f(x)$, a trend continued in 2006. The success rate for sketching the basic trigonometric functions dropped from above 60% in the 1990's to below 50% in 2006. The ability to multiply out three given linear factors of a cubic polynomial was also disappointing, with success rates well below 50% in three out of the six years measured, and only 44% in 2006.

On the positive side, some skills showed improvement, but only one improved significantly to a success rate of over 50%: finding the equation of a straight line given the coordinates of two points. All others improved skills remained at low success rates, with increases generally from 10-20% to 30-40%. These include simplifying a fraction and writing it with no negative powers, determining the centre and radius of a circle, using a graph to find the solution to an equation, and using the cosine rule to find the side of a triangle. These general weaknesses are especially disappointing, given that 61 out of these 75 students had spent at least one semester in Foundation Mathematics, which covers these skills.

Further Analysis of the 2006 Data

T-tests were conducted on the following groups to assess differences in skills associated with the following factors:

- Mode of study (on campus versus external).
- Foundation Mathematics (studied versus not studied).
- Faculty (engineers versus non-engineers).
- Age-group (school-leavers versus older students).

Mode of study revealed the biggest differences, with externals (98 students) performing better in algebra than their on-campus counterparts (108 students) on four out of nine algebra questions (p-values ranging from 0.010 to 0.043). These include factorising a quadratic expression, subtracting two algebraic fractions, solving an inequality containing an absolute value, and completing the square. External students also performed better on two trigonometric questions, namely using the cosine rule ($p = 0.031$), and solving a real world problem ($p = 0.011$).

Foundation studies, faculty and age-group yielded no overall statistical differences in each of the six skills areas. However, differences were found for some specific questions. For example, non-engineers (71 students) performed better than engineers (135 students) on some tasks, including simplifying a fraction containing negative powers ($p = 0.020$), expanding three linear factors ($p = 0.047$), and substituting into a quadratic function ($p = 0.019$).

Students who did not do foundation mathematics (96) performed better than those who did (110 students), on the following tasks: solving a cubic equation, solving a system of linear equations, and recalling the trigonometric identity $\sin 2\theta = 2\cos\theta\sin\theta$. Note, however, that success rates for these three questions were low for both groups. For example, around 40% versus 25% success rate for expanding the cubic equation. Note too that since Engineering now recommends that its students do foundation mathematics studies, it can no longer be assumed that those who do not do foundation studies are those who come better prepared from school.

Data for age-groups were available for only 41 students. The school-leavers (14 students) performed better than the older students (27 students) on a number of tasks. The younger students were better with quadratic functions: describing its graph (p = 0.000), using the graph to predict y-values (p = 0.031), and finding the turning point (p = 0.041). They also performed better with fractions (p =0.003), finding the equation of a line given slope and y-intercept (p = 0.050), and sketching the sine and cosine functions (p = 0.018).

Discussion and Implications

The competencies of 206 students who completed a pre-test on entry to Algebra & Calculus I in 2006 were measured in six areas: basic numeracy and arithmetic, fractions and percentages, index laws and scientific notation, algebra, functions and graphs, and trigonometry. Data are reported for the 2006 cohort, and the 1997 to 2001 cohorts, as measured by the same test.

The 2006 findings revealed reasonable skills on arithmetic, fractions, and index law tasks, many of which could be done with the aid of a calculator. Of concern, however, are findings that reveal ongoing weak skills in areas of algebra, functions, and trigonometry. And these skills such as rearranging a straightforward equation, solving quadratic equations, finding the equation of a straight line, sketching sine and cosine, and finding angles from a sine value are fundamental for studies in calculus, vectors, and linear algebra.

Comparing the 2006 data with those of previous years, no significant differences were found in overall skills in each of the six areas described in this paper. There were differences in some specific skills, many related to functions and graphing, but the few that showed improvement remained at a low level. This was disappointing considering that the majority of the engineering students of 2006 had studied the foundation subject. Furthermore, the 2006 data revealed that students who had done the foundation studies performed significantly worse on two algebraic and one trigonometric task. It seems that these are not students who simply need some time to refresh these skills. More likely it is a warning that many have never engaged deeply enough with these fundamentals to internalise the concepts.

A significant 2006 finding was that the external students showed stronger algebraic skills overall than their on-campus counterparts in four out of nine algebra tasks. This may reflect a range of differences, including study habits. The differences between faculties were less pronounced, non-engineering students performing better than the engineers in just one algebra task and one function task. As expected, school leavers performed better than the older students on a few tasks, especially in the area of function and graphing. Nevertheless their skills levels were disappointing.

These findings have important implications for course and program planning in Australian universities. Algebra & Calculus I used to be the entry-level mathematics course for students in Engineering and Science, but declining levels of mathematical preparedness have resulted in many of these programs now placing students in foundation studies first. Enrolment in Foundation Mathematics at this university alone has risen by close to 6%, to around 900 students, the majority of these studying externally.

It is clear that in many Australian universities, foundation mathematics studies are now an essential part of the degree studies for increasing numbers of students. Should these students pay extra for these studies? Or should universities give credit points to students who enter having done advanced mathematics subjects at school? Either way, current tertiary entry-level skills tests are wish-lists; the reality is different. It is clear that tertiary teachers must radically re-examine the skills they assume their students have on entry to university mathematics courses, and tertiary programs and curricula need restructuring to respond appropriately. And it seems likely that non-foundation courses will need to sustain integrated and effective strategies to develop the core algebra, graphing, and trigonometry skills students need to facilitate even basic studies in calculus, vectors and linear algebra for higher studies in mathematics, sciences, and engineering.

The evolving nature of current tertiary mathematics studies raises questions about the implications for school mathematics curricula and assessment. If universities must respond to widening entry by incorporating current school content in tertiary courses, are school curricula freed from some content and constraints? Can focus be on depth in core skills and content, rather than breadth? We propose that the time is right for secondary-tertiary collaboration on the best path forward for Australian mathematics education at both levels.

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Appendix A. *Results of the Mathematics Testing of On Campus Bachelor of Engineering Students 1997-2001, 2006 (Right-most column shows the results for the whole class)*

Question	Percentage correct 1997 (n=65)	1998 (n=76)	1999 (n=70)	2000 (n=86)	2001 (n=71)	2006 (n=75)	2006 (n=206)
1 (e) Estimate $56 + 23 \times 9246 \div 125$ by using appropriate rounding	49.2	35.9	39.9	69.8	78.9	60.0	62.6
(f) Evaluate $\left\{20 - 3\left[3 + (12 / 4)^2\right]\right\}^3$	52.3	50.0	62.0	57.0	59.2	60.0	67.5
2 (f) Evaluate $\frac{1}{4} \div \frac{5}{6} + \frac{3}{4} - \frac{5}{2} \times \frac{4}{3}$ and express your answer as a fraction	66.2	74.4	73.2	77.9	59.2	72.0	71.8
3 (d) Express $\frac{16(a^2b^4)^{-1/2}}{b^{-3}}$ as a simple fraction involving no negative powers	30.8	20.5	19.7	18.6	21.1	36.0	39.3
4 (a) Factorize $6x^2 + x - 12$	52.3	50.0	25.4	47.7	47.9	54.7	60.2
(b) Expand $(x + 1)(-2x + 1)(x - 3)$	76.9	39.7	39.4	61.6	71.8	44.0	56.3
(c) Write this expression as a single fraction with no common factors $\frac{1}{x-3} - \frac{4}{x-2}$	40.0	33.3	32.4	32.6	35.2	32.0	40.8
(d) Make t the subject of the equation $y = (8t + 3)^3 + 4$	70.8	41.0	53.5	62.8	59.2	56.0	57.3
(e) Solve the quadratic equation for x, $3x^2 + 4x - 8 = 0$	61.5	51.3	33.8	50.0	53.5	58.7	58.3
(f) Solve the cubic equation for x, $x^3 - 4x^2 + x + 6 = 0$	21.5	14.1	8.5	16.3	22.5	25.3	28.6
(g) Solve for x, $\lvert 3x + 3 \rvert < 6$	20.0	9.0	15.5	16.3	11.3	16.0	21.8
(h) By completing the square, find the values of a and b where $x^2 + 3x + 1 = (x + a)^2 - b^2$	15.4	10.3	5.6	12.8	9.9	16.0	22.8
(i) Solve the following set of simultaneous equations $x + y + z = 0$, $x - 3y + 2z = 1$, $2x - y + z = -1$	35.4	35.9	28.2	27.9	32.4	32.0	39.3
5 (a) $f(x) = x^2 + 1$ and $g(x) = \sqrt{x-1}$ are given. (i) Calculate $f(-1)$	81.5	83.3	73.2	82.6	74.6	62.7	63.6
(ii) Find $f(x + \mathrm{h})$	61.5	57.7	63.4	64.0	47.9	29.3	38.8
(iii) Find $f(g(x))$	53.8	52.6	56.3	55.8	57.7	42.7	47.1
(iv) What are the domain and range of g?	15.4	6.4	19.7	20.9	19.7	25.3	25.2

(b) Write an equation for a straight line with slope of –4 and y-intercept of –3	73.8	85.9	73.2	73.3	78.9	80.0	82.5
(c) Find the equation of the straight line passing through the points (–3,1) and (–1,–2)	69.2	34.6	43.7	45.3	33.8	58.7	53.9
(d) Write an equation for the straight line below. (Sketch not shown here.)	58.5	53.8	52.1	66.3	54.9	54.7	60.7
(e) Sketch the graph of $y = -x/2 + 2$	70.8	53.8	46.5	66.3	60.6	68.0	72.3
(f) (i) Draw the graph of $y = x^2 + 7x + 6$	60.0	50.0	22.5	65.1	56.3	66.7	69.9
(ii) Use the graph drawn in (f) (i) to predict the y value when $x = -2.5$	58.5	44.9	19.7	68.6	67.6	72.0	69.9
(g) What is the turning point of the function drawn in (f)?	53.8	41.0	35.0	41.9	40.8	57.3	52.9
(h) Determine the centre and radius of the circle $x^2 + y^2 - 2x + 3y = 25$	13.8	3.8	5.6	9.3	12.7	30.7	24.3
(i) Sketch the graph of $y = 1/(x-2)$	33.8	41.0	22.5	44.2	40.8	26.7	34.5
(j) Indicate by a labelled sketch how you would graphically approximate the solution to the equation $x^2 - 1 = \sqrt{x}$	13.8	16.7	12.7	18.6	18.3	32.0	30.6
6 (a) Sketch a graph of $y = e^x$ and $y = \log_e x$	36.9	20.5	21.5	33.7	38.0	34.7	34.0
(b) Make x the subject of the equation $y = 3e^x + 2$	44.6	34.6	43.7	43.0	45.0	38.7	42.2
(c) Evaluate using the logarithmic rules (do not use your calculator) $\log_2 4 - \log_2 2 + \log_2 1$	40.0	25.6	35.2	33.7	39.0	44.0	48.1
7 (a) Convert $329°$ to radians	61.5	66.7	64.8	68.6	76.4	76.0	75.2
(b) Find all the angles between 0 and 2π radians that satisfy the equation sin A = 0.4	15.4	21.8	9.9	19.8	32.4	20.0	32.5
(c) In the triangle below find x. (Diagram not shown here.)	26.2	21.8	18.3	26.7	15.5	40.0	44.2
(d) On the same set of axes sketch and label the graphs of $y = \sin x$ and $y = \cos x$ for $-2\pi \le x \le 2\pi$	63.1	64.1	47.9	64.0	59.2	48.0	51.0
(e) Complete the following statements (i) $? + \cos^2\theta = 1$	75.4	78.2	12.7	73.3	71.8	72.0	68.0
(ii) $1 + ? = \sec^2\theta$	52.3	46.2	11.3	33.7	32.4	52.0	51.9
(iii) $\sin 2\theta =$	12.3	12.8	11.3	11.6	9.9	14.7	20.4
(f) A surveyor attempting to find the height of a vertical cliff makes the following observations: The angle of elevation from the ground to the top of the cliff is 30° at a certain distance away from the bottom of the cliff. But, the angle of elevation is 45° when 20m closer to the cliff. What is the height of the cliff?	38.5	24.4	14.1	29.1	32.4	40.0	45.1