University of Southern Queensland

Bayesian Prediction Distributions for Some Linear Models under Student-t Errors

A Dissertation submitted by

Azizur Rahman
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Abstract

This thesis investigates the prediction distributions of future response(s), conditional on a set of realized responses for some linear models having Student-t error distributions by the Bayesian approach under the uniform priors. The models considered in the thesis are the multiple regression model with multivariate-t errors and the multivariate simple as well as multiple regression models with matrix-T errors. For the multiple regression model, results reveal that the prediction distribution of a single future response and a set of future responses are a univariate and multivariate Student-t distributions respectively with appropriate location, scale and shape parameters. The shape parameter of these prediction distributions depend on the size of the realized responses vector and the dimension of the regression parameters' vector, but do not depend on the degrees of freedom of the error distribution. In the multivariate case, the distribution of a future responses matrix from the future model, conditional on observed responses matrix from the realized model for both the multivariate simple and multiple regression models is matrix-T distribution with appropriate location matrix, scale factors and shape parameter. The results for both of these models indicate that prediction distributions depend on the realized responses only through the sample regression matrix and the sample residual sum of squares and prod-

ucts matrix. The prediction distribution also depends on the design matrices of the realized as well as future models. The shape parameter of the prediction distribution of the future responses matrix depends on size of the realized sample and the number of regression parameters of the multivariate model. Furthermore, the prediction distributions are derived by the Bayesian method as multivariate-t and matrix-T are identical to those obtained under normal errors' distribution by the different statistical methods such as the classical, structural distribution and structural relations of the model approaches. This indicates not only the inference robustness with respect to departures from normal error to Student-t error distributions, but also indicates that the Bayesian approach with a uniform prior is competitive with other statistical methods in the derivation of prediction distribution.

Certification of Dissertation

I certify that the ideas, mathematical derivation of the formulas, findings and conclusions reported in this dissertation are the result of my own work, except where otherwise acknowledged. I also certify that the thesis is original and has not been previously submitted for any other award to any other university, except where otherwise acknowledged.

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Signature of Candidate Date

ENDORSEMENT

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Signature of Principal Supervisor Date

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Signature of Associate Supervisor Date

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