Automated Rib Location and Optimization for Plate Structures

Y. C. Lam and Subramaniam Santhikumar

School of Mechanical and Production Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798.

ABSTRACT

For a given loading, the stiffness of a plate or shell structure can be increased significantly by the addition of ribs or stiffeners. Hitherto, the optimization techniques are mainly on the sizing of the ribs. The more important issue of identifying the optimum location of the ribs has received little attention. In this investigation, a methodology has been developed for the automatic determination of the optimum locations of the ribs for a given boundary conditions.

Keywords: Rib location, stiffener location, stiffness criteria, strain energy, thickness criteria.

INTRODUCTION

Structural optimization is used to improve an initial design by variation of its geometrical and material properties with regard to a set of prescribed objectives and constraints (Haftka and Grandhi 1986, Morris 1982 and Olhoff and Taylor 1983). In a competitive environment, manufacturers face the challenge to produce quality and cost effective products. Thus the optimization of material usage become an important consideration. The optimization process could adjust the thickness, shape and topology within the domain until an optimal design has been achieved (Tenek and Hagiwara 1994 and Vanderplaats 1993). Alternatively, the designer might already has a working design and would like to determine the optimal design parameter subject to the above behavioural constraints.

However, for plate or shell structures, without the introduction of ribs or stiffeners into the design, a simple sizing optimization will not produce much of a reduction in product weight. It would be advantageous to combine sizing optimization with a methodology of designing ribs in the structures. Research work on structural optimization of plates and shells are therefore focused on optimization of rib size and location (Chung and Lee 1997 and Stok and Mihelic 1996). Stock and Mihelic (1996) had identified the rib location by thickness distribution using optimization technique. Subsequently, they optimized the whole design with the rib in place by optimizing both the thicknesses of the plate and the rib. However, a major drawback of their approach is that only one set of rib location was determined.

For an optimum and practical design, we expect that a plate or shell structure would have a layout of ribs to take maximum advantage of rib structure. The consideration of rib layout is omitted in their research (Stok and Mihelic 1996). Chung and Lee (1997) have attempted to investigate rib layout, but they have not provided a logical reason for their choice of the locations of the ribs. For a pre-determined rib layout, they employed topology optimization technique to identify the size of the ribs. Hitherto a major aspect has been neglected, namely there is no serious attempt to determine the optimum layout of ribs. Quite often in the optimization of ribs, there is a size constraint during the optimization process.

Placing ribs in a plate structure cannot be done arbitrary as the locations of subsequent ribs have constraints due to manufacturing limitations. For example, there should be a clear gap in between two ribs. Similarly the size of the rib (height and width) should also be taken into consideration. Thus, the main thrust of this investigation is to determine the optimum rib locations subjected to these realistic constraints.

STRUCTURAL OPTIMIZATION

Optimization problems are dominantly solved using optimality criteria method. This method can deal with large number of design variables. As they depend on the behaviour and design condition of the structure, they can only deal with a limited set of constraints (Haftka and Grandhi 1986 and Lam *et al*. 2000). However for the nature of the problem investigated, it is a suitable method and it will be employed for the present investigation.

Optimality Criteria Methods

This method is utilized to identify the first rib location in the structure. Subsequent rib locations are obtained iteratively by placing the previous set of ribs in the design before the next iteration. Optimality criteria methods are essentially based on transforming the optimization problem into solving a nonlinear set of equations obtained using Kuhn-Tucker necessary conditions to the original design problem (Lam *et al*. 2000 and Morris 1982). Basically two criterions are generally employed in the optimum design of structures subjected to static loads. These are (i) stiffness criterion by achieving an uniform strain energy density distribution in a structure and (ii) stress criterion, which aims at achieving a more uniform stress distribution in the structure (Lam *et al*. 2000).

(I) Stiffness Optimality Criterion

Since the stiffness of a structure is often a major requirement in design, the objective of this criterion is to maximize the overall stiffness. This is the same as minimization of the total strain energy of the structure for a given maximum deflection.

At optimum, the ratio of strain energy density to specific weight is the same for all the elements. Thus the total strain energy of the structure, γ is given by (Lam *et al*. 2000)

$$\gamma = \frac{1}{2}\{d\}^T[K]\{d\} = \frac{1}{2}\sum_{i=1}^{n}\{d_i\}^T[k_i]\{d_i\} \qquad (1)$$

where $[K]$ and $[k_i]$ are the global and i^{th} element stiffness matrix respectively; $\{d\}$ and $\{d_i\}$ are the global and i^{th} nodal displacement vector respectively; and n is the total number of elements in the structure.

Strain energy density of the i^{th} element; U_i can be derived by dividing the strain energy of the element by its volume. Thus (Lam *et al*. 2000),

$$U_i = \frac{\frac{1}{2}\{d_i\}^T[k_i]\{d_i\}}{A_i t_i} \qquad (2)$$

where A_i and t_i are the surface area and thickness of the i^{th} element, respectively.

As a uniform strain energy density distribution is valid only at the optimum, resizing algorithms have to be employed in the optimization process (Lam *et al*. 2000). Thus, at the end of each iteration, elemental strain energy density, U_i is compared with the average strain energy density U_{av} which is computed by dividing the total strain energy γ by the total volume,

$$U_{av} = \frac{\gamma}{\sum_{i=1}^{n} A_i t_i}. \qquad (3)$$

To achieve the required uniform strain energy density distribution, element thickness is either increased or decreased in order for U_i to approach U_{av}. A recurrence relation can be written to modify the element thickness to satisfy the above requirements (Lam *et al*. 2000 and Stok and Mihelic 1996)

$$t_i^{k+1} = t_i^k \left(\frac{U_i}{U_{av}} \right)_k^{\frac{1}{r}} \qquad (4)$$

where $k+1$ and k are the number of the iterations, and r is the parameter determining the step size.

(II) Stress Optimality Criterion

Uniform stress distribution in the structure can lead to a fully stressed design. It is generally used in the optimization of discrete structures to improve its strength characteristics. Since yielding is the common failure criterion for isotropic materials, von-mises stress has been frequently used for this analysis. The von-mises stress can be calculated from

$$\sigma^{vm} = \frac{1}{\sqrt{2}} \left[\sigma_x^2 + \sigma_y^2 + (\sigma_x - \sigma_y)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \right]^{\frac{1}{2}}. \qquad (5)$$

To have a uniform stress distribution, element thickness of a highly stressed element should be increased. Similarly, the thickness should be decreased to lowly stressed elements. Element thickness is resized by comparing the von-mises stress of each element, σ_i^{vm} with the average of von-mises stresses across the whole plate structure. Similar to stiffness criterion, a resizing algorithm can be written as (Lam *et al*. 2000 and Stok and Mihelic 1996),

$$t_i^{k+1} = t_i^t \left(\frac{\sigma_i^{vm}}{\sigma_{av}^{vm}} \right)_k^{\frac{1}{r}} \qquad (6)$$

where σ_{av}^{vm} is the average von-mises stress across the whole structure.

RIB LOCATION OPTIMIZATION

For a plate or shell structure, an improvement can be obtained by either (i) reducing the amount of material used but with the same structural performance or (ii) with the same amount of material used but with better structural performance. To achieve this we could employ the structural optimization procedure detailed in the previous section by changing the thickness of the plate or shell continuously. However, such plate design with varying thickness is practically infeasible other than in very specific circumstances due to manufacturing difficulties (Chung and Lee 1997 and Maute and Ramm 1994). Thus, the optimization of plate thickness alone is not sufficient or desirable.

To overcome this manufacturing limitation and to retain the advantageous of a constant thickness plate but yet to improve the efficiency of the design, the plate structure has to be locally stiffened by ribs (Chung and Lee 1997 and Stok and Mihelic 1996). The problem is now being reduced to the determination of the optimum location of the ribs. This concept is the main thrust of this investigation, which aims to develop a methodology to automatically determine the rib locations.

To achieve the above task, the optimization process is carried out in two stages. Firstly, the plate with constant thickness will undergo the optimization process in order to obtain an optimized plate with varying thickness. Thereafter, from the above results the potential rib locations are identified. After putting in the ribs, this iterative procedure is repeated to determine all potential rib locations. At this last stage, constraint on distance between rib location is to be imposed.

For ease and clarity of explanation, this paper will only detail the procedure by employing the stiffness optimality criterion. With little modification, the method could easily be adapted for stress optimality criterion. The details of this procedure are as follow:

Identification of Rib Location

Stage 1. At this stage, the rib location is to be identified without consideration to its size. This is achieved by varying the thickness of the elements such that they will achieve uniform strain energy and by maintaining the total volume constant. For easy identification of the rib, the upper limit of the element thickness should be set much larger than the average thickness. The steps are as followed:

(i) Static structural analysis is carried out on the model.

(ii) Element strain energy density and average strain energy density for the structure are computed.

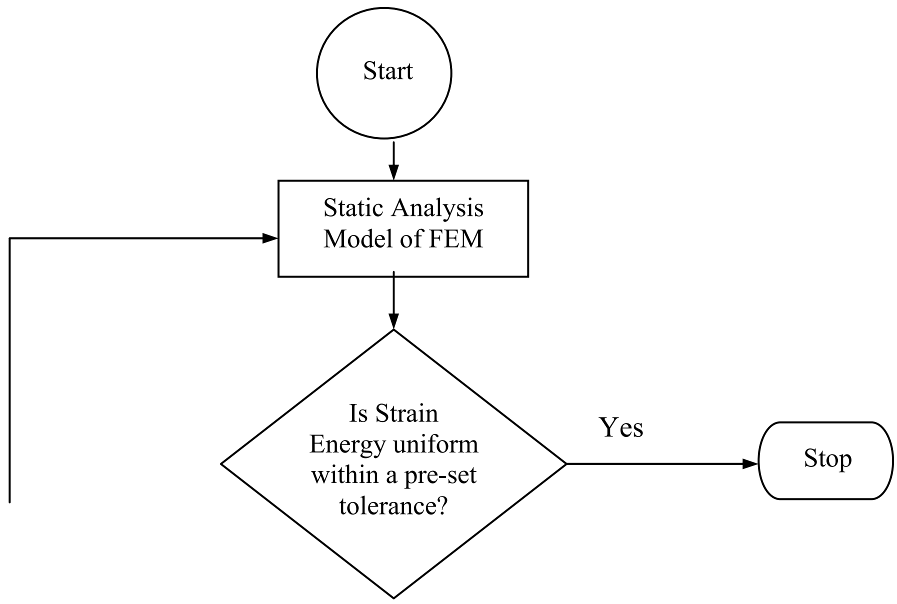
(iii) Stiffness criterion (4) is employed to compute the new element thickness.

(iv) If the element thickness is outside the pre-set lower and upper limits, the element thickness is set to the limiting value. The upper limit should be set at a value much larger than the average thickness of the model.

(v) Re-scale the whole model such that its total volume is kept constant.

(vi) The above iterative steps from (i) to (v) is repeated until the total strain energy converged to a prescribed value.

The above procedure for the stiffness criterion is summarized in the flow chart shown in Figure 1. The same procedure could be applied to stress criterion with some modifications.



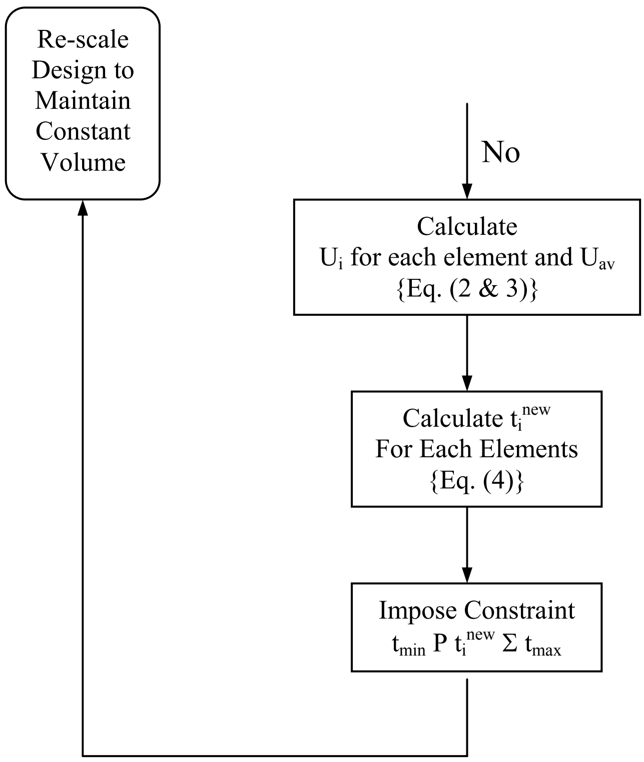


Figure 1: Flowchart for identification of rib location – Stage 1.

After the solution has converged, the rib location would be easily identified by elements with thickness value much higher than other elements.

Stage 2. By examining the solution from stage 1, rib could then be placed at location where the elements have very high thickness values. Constraints such as rib size (height and width), distance between two ribs is to be imposed at this stage. The steps are outlined below:

(i) Final thickness distribution of plate from Stage 1 is examined. Region where elements have thickness value higher than the specified thickness limit is identified as potential rib location.

(ii) Introduce a rib with a pre-defined width but with its height varies according to the original thickness distribution, but not exceeding the specified height limit, at the identified rib location.

(iii) Thus a new model with the addition of new ribs are created.

(iv) Distance constraints between neighboring ribs are imposed. This is achieved by imposing the requirement that the element thickness within a pre-set distance from the existing ribs is not to be changed from its initial value during the next iterations as described in Stage 1.

Stages 1 and 2 are to be repeated until there is no more potential rib location can be identified. The iteration can then be terminated and a design with an optimum rib location has now been identified.

The above procedure is summarized in Figure 2.

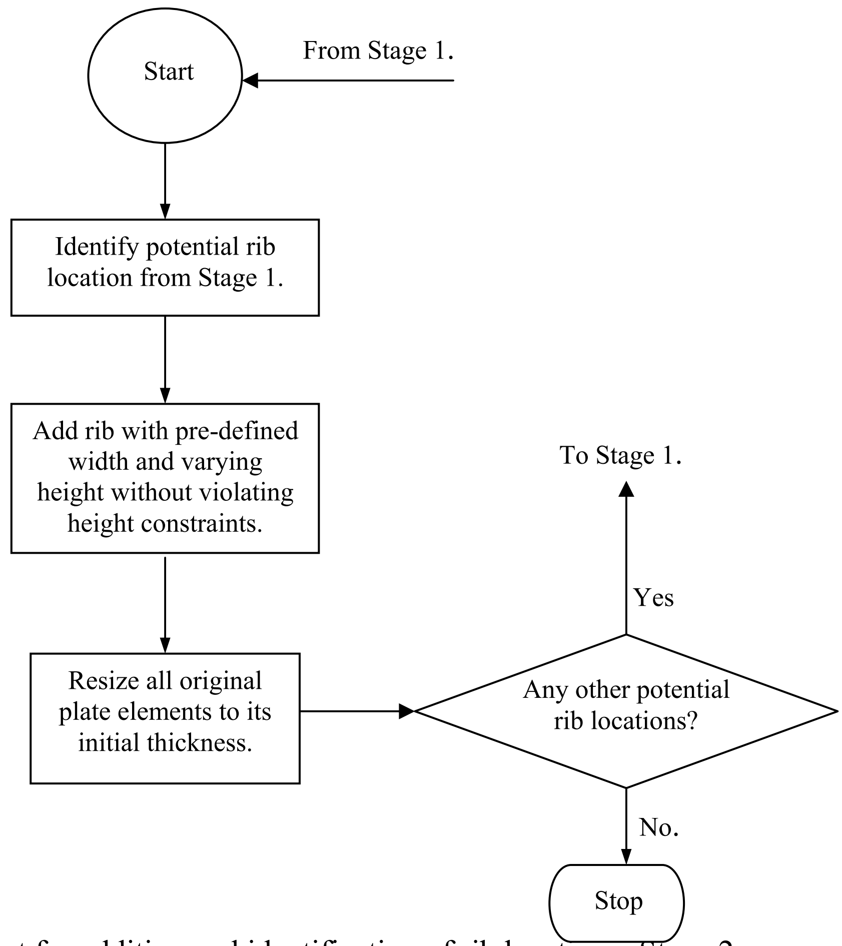


Figure 2: Flowchart for addition and identification of rib location – Stage 2.

EXAMPLES

Three examples are examined here to demonstrate the generality and reliability of the proposed automated rib location methodology. For all the three examples, 4 mm thick plate of 300 mm Δ 200 mm with a hole of 50 mm diameter in the middle is considered. One of the shortest edges is fixed. The first example has a point load of 10N acting at the middle of the free short edge, see Figure 3(a). Second example features two equals but opposite forces applying at the two corners of the free short edge, providing a torsional moment see Figure 4(a). The loading of the third example is the combination of the loading of the first and second examples.

Properties of the material for this analysis are given in Table 1. The predefined constraints are given in Table 2. As example 1 is symmetrical about the major axis, half of the model is analyzed by imposing symmetrical boundary conditions. For example 2, the geometrical model is symmetrical but the loading is antisymmetrical. Thus, half the model is analyzed by imposing antisymmetrical boundary condition. Since the example 3 has neither symmetrical loading nor symmetrical geometry especially after the addition of first rib, we have utilized full model for the analysis.

Material type	Plastic
E, Young's modulus	9.2 GPa
⇑, Poisson's ratio	0.365

Table 1: Material Properties.

<table>
<tr><td colspan="2">Initial plate thickness</td><td>4 mm</td></tr>
<tr><td colspan="2">Width of ribs</td><td>3 mm</td></tr>
<tr><td colspan="2">Maximum height of ribs</td><td>6 mm</td></tr>
<tr><td colspan="2">Minimum distance between two ribs</td><td>10 mm</td></tr>
<tr><td rowspan="2">Constraints used in Stage 1 only</td><td>t_{max} – maximum plate thickness</td><td>1000 mm</td></tr>
<tr><td>t_{min} – minimum plate thickness</td><td>0.5 mm</td></tr>
</table>

Table 2: Constraints employed during the optimization process.

FENAS finite element program of Moldflow version 1.1.0 is utilized for the structural stress analysis. Since Moldflow has introduce a specially formulated element called linear membrane triangular 3-node elements (LMT3) and they have recommended that LMT3 gives excellent performance and is efficient in regard to CPU time, we have employed LMT3 elements in our analysis.

RESULTS AND DISCUSSION

In our analysis, the final optimized model will have a plate thickness the same as the initial plate thickness, namely 4 mm. However, the optimized model will have added materials as ribs are added. To have a proper comparison, a uniform thickness model which has the same volume as the optimized model with rib is analysed. This equivalent model, when compared with the optimized model, will provide the proper basis at which the effectiveness of optimization can be judged. Table 3 shows the deflection of the initial model, model with rib at each stage and the corresponding equivalent model under all three loading conditions.

The result of example 1 is shown in Figure 3. The element thickness distribution after the first iteration, is shown in Figure 3(b). the potential rib location can be easily identified by this thickness distribution. The first rib is added as explained in stage 2, which is shown in Figure 3(c). This procedure is repeated till all the possible rib locations are identified and appropriate ribs are added. In this example, four rounds of optimization are required. Figure 3(d) shows the final optimized design with the layout of the ribs.

A maximum deflection of 13.56 mm is observed at the loading point in example 1 for the original model without any ribs. After the fourth round of optimization which leads to the final optimized design of plate with ribs (Figure 3(c)), the maximum deflection reduces to 6.44 mm at the loading point. An equivalent model with uniform thickness of 4.51 mm so that the total volume is equal to that of model in Figure 3(c) is analysed. Its maximum deflection is 9.47 mm. By comparing the optimized model with this equivalent model, a 32% reduction of the maximum deflection is achieved.

Results of the second example featured two opposing loads, which provide torsional moment are presented in Figure 4. Figure 4(b) shows the thickness distribution after the first stage of first iteration. Figure 4(c) shows the model with the first rib. The procedure is repeated and after the third round of optimization, all the possible rib locations are identified. The final design achieved is shown in Figure 4(d), after three successive rounds of optimization.

For example 2, the original model without ribs has a maximum deflection of 5.45 mm at the loading point. For the final optimized plate with ribs, shown in Figure 4(d), it has a maximum deflection of 3.69 mm as shown in Figure 4(e). The equivalent model with uniform thickness of 4.41 mm, has a maximum deflection of 3.99 mm as shown in Figure 4(f). Improvement in this example is only 7.5 % because a uniform thickness plate provides reasonable performance under torsional loading.

For example 3, we obtained the optimum rib layout after four rounds of successive optimization procedure. The final optimized plate with rib layout is shown in Figure 5(a), with a maximum deflection of 10.65 mm at one of the loading corners (Figure 5(b)). The equivalent model with uniform thickness has a maximum deflection of XXX (Figure 5(c)). Thus, through optimization, there is a XXX % reduction of deflection.

Table 3 summarizes the results of optimization for all three examples. It include percentage reduction in maximum deflection after each iteration. As ribs are added at each iteration, the thickness of the equivalent model has to be increased such that it has the same volume of materials as the optimized model. Figure 6 shows graphically the percentage reduction of deflection by the introduction of ribs over its equivalent plate with uniform thickness at the end of each round of optimization procedure. In general, at each round of optimization, better improvement is achieved for example 1 with a single vertical point loading. However in example 2, under torsional loading a better performance is observed only after the introduction of first rib and thereafter the percentage improvement is slightly decreasing, although the overall deflection is still decreasing with added ribs. Example 3, under combined loading, the percentage improvement is almost constant after first round of optimization. The results are not surprising as ribs are especially efficient for bending load, and less for torsional load.

	No. of Ribs	Equivalent model thickness (mm)	Maximum Deflection (mm)		Reduction in maximum deflection
			Model with rib	Equivalent model	
Example 1 Point Loading	0	4.00	13.56	13.56	0 %
	1	4.17	10.59	11.97	11.5 %
	2	4.30	8.50	10.92	22.2 %
	3	4.39	7.31	10.26	28.8 %
	4	4.51	6.44	9.47	32.0 %
Example 2 Torsion Loading	0	4.00	5.45	5.45	0 %
	1	4.21	4.22	4.68	9.8 %
	2	4.37	3.81	4.18	8.9 %
	3	4.41	3.69	3.99	7.5 %
Example 3 Combined Loading	0	4.00	18.56	18.56	0 %
	1	4.15	14.84	16.63	10.8 %
	2	4.26	12.87	15.37	16.3 %
	3	4.34	11.71	14.54	21.4 %
	4	4.44	10.65	13.60	21.7 %

Table 3: Results.

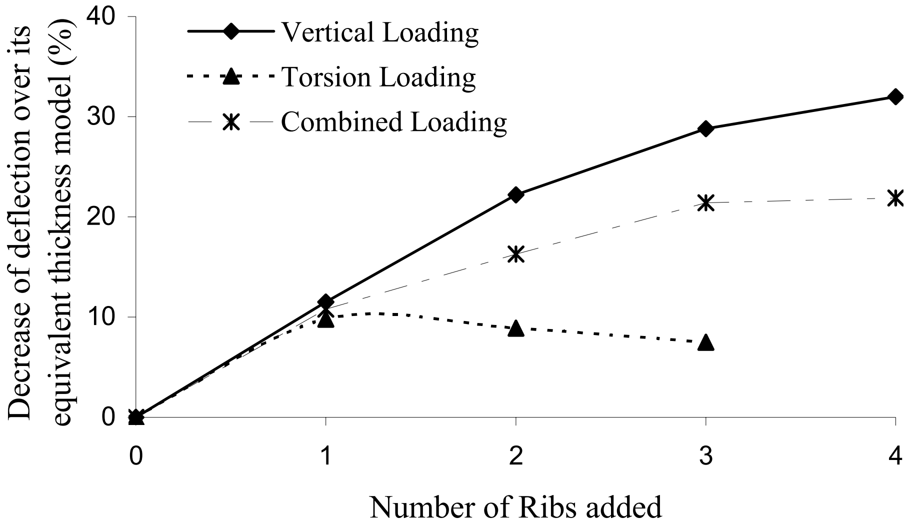


Figure 6: Comparision of decrease in maximum deflection of optimized plate with a model having uniform thickness and equivalent volume of material.

The case studies indicated that the proposed optimization methodology is capable to identify the optimum rib locations subjected to practical design constraints. The algorithm is simple, whilst yet effective.

CONCLUSIONS

A new methodology to design rib layout in a structure to increase the structural performance is proposed. This methodology avoids the difficulties of more conventional optimization algorithm for plate/shell structure which normally resulted in plates with varying thickness.

The examples investigated using this approach have resulted in a 32 %, 9.8 % and XX% for a plate with a hole subjected to bending, torsion and combined bending and torsional loads respectively. The proposed methodology can be easily implemented as a external loop to any standard finite element packages.

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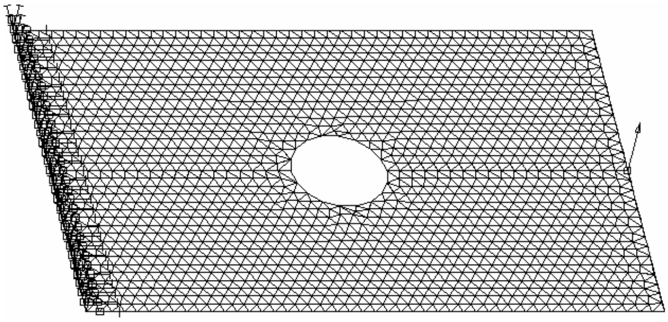
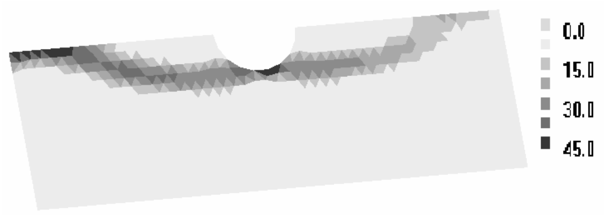
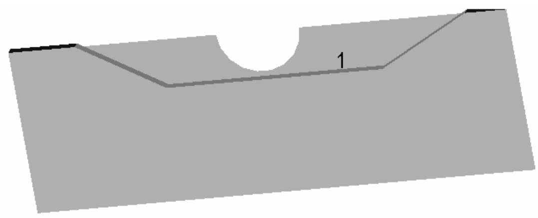


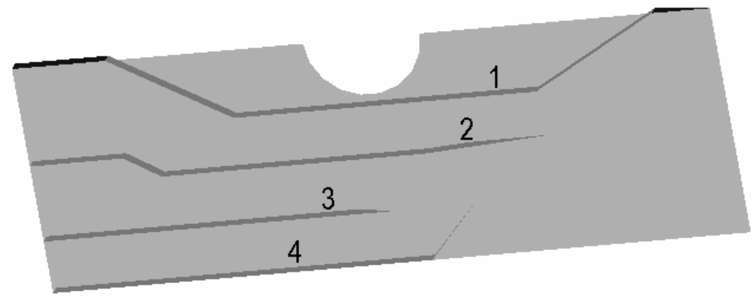
(a) Plate model of 4 mm thick with a hole in the middle with boundary conditions.



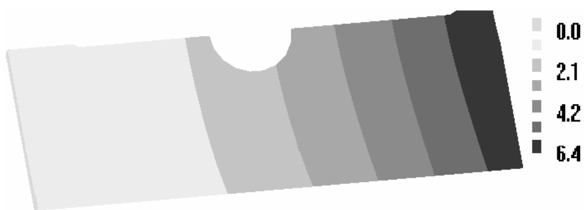


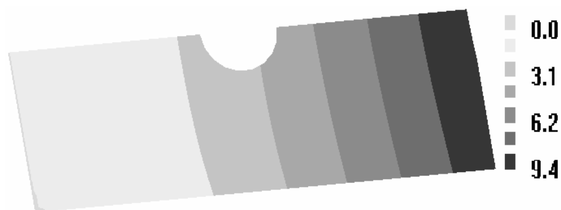
(b) Thickness distribution after stage 1 of optimization process.

(c) New model with first rib after first optimization.



(d) Final optimized plate of 4 mm thick, with three sets of ribs of 3 mm thick, after four rounds of optimization procedure.

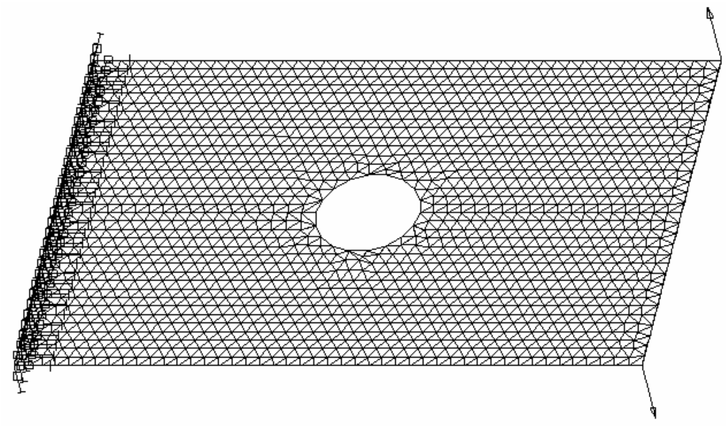




(e) Deflection of final optimized plate with ribs after three rounds of optimization procedure.

(f) Deflection of equivalent plate with uniform thickness.

Figure 3: Example 1, a rectangular plate with the shortest edge fixed and under point load acting at the middle of the free short edge.



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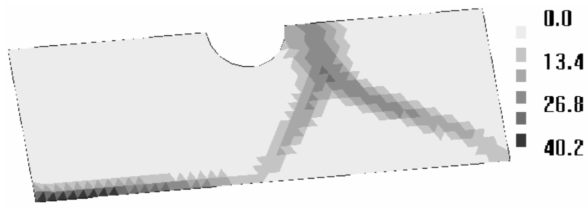
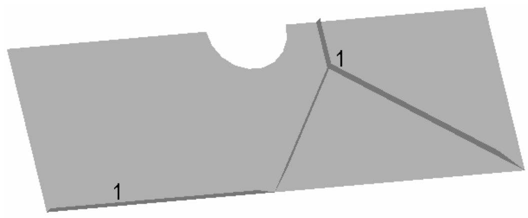
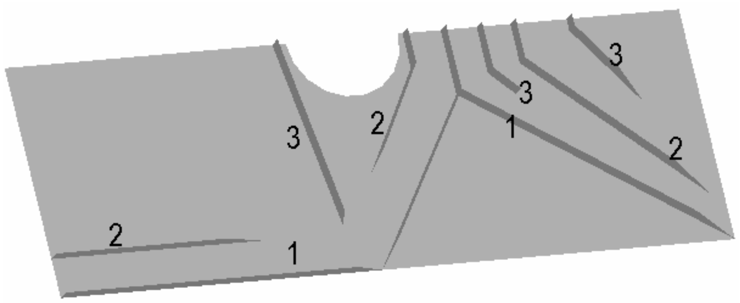


(b) Thickness distribution after stage 1 of optimization process.



(c) New model with first rib after first optimization.

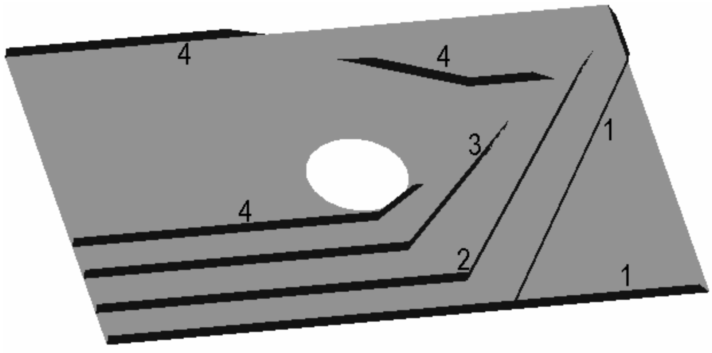


(d) Final optimized plate of 4 mm thick, with three sets of ribs of 3 mm thick, after three rounds of optimization procedure.

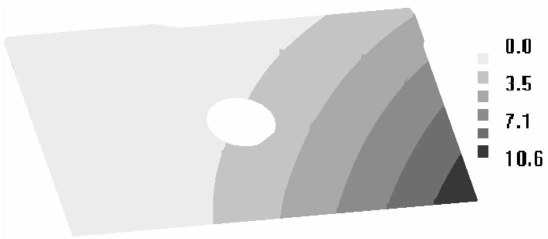
(e) Deflection of final optimized plate with ribs after three rounds of optimization procedure.

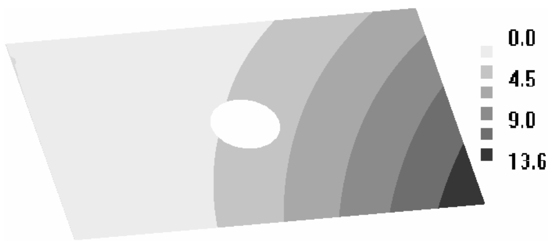
(f) Deflection of equivalent plate with uniform thickness.

Figure 4: Example 2, a rectangular plate with the shortest edge fixed and under two equal but opposite forces applying at the two corners of the free short edge, providing a torsional moment.



(a) Final optimized plate of 4 mm thick with four sets of ribs of 3 mm thick, after four rounds of optimization.





(b) Deflection of optimized plate with ribs after four rounds of optimization.

(c) Deflection of equivalent plate with a uniform thickness of 4.44 mm.

Figure 5: Example 3, a rectangular plate with the shortest edge fixed and under the combination of the loading of the first and second examples.