On Risk-Return Relationship: An application of GARCH(p,q)–M Model to Asia-Pacific Region

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Abstract: Despite the criticisms on the validity of the CAPM, finance researchers continue to adopt the model in trying to describe the relationship between risk and return. The introduction of the GARCH(p,q)-M model provides an avenue for testing the model within the time-varying variance framework. This study employs the same model to address the issue within ten selected Asia Pacific countries. The result, though not comprehensive, shows that the CAPM still holds in explaining the risk-return relationship in China and Malaysia. The significant positive risk parameter coefficient suggests a positive linear relationship which indicates that investors are compensated for assuming high risk. Judging by the significant finding in China and Malaysia, this study provides evidence that the conditional CAPM is a useful tool for decision making in investments and corporate finance.

Key Words: Stock market, risk-Return, generalised autoregressive conditional heteroscedasticity

1. Introduction

The Capital Asset Pricing Model (CAPM), credited to Sharpe (1964), provides an important foundation to understanding the relationship between risk and return which is considered to be an important subject in the field of investments as well as in corporate finance. Stemming from the Modern Portfolio Theory (MPT), proposed by Markowitz (1952), the CAPM is based on the assumption that individual investors will only hold mean-variance efficient portfolios. As such, for a rational investor who is considered to be risk averse, a portfolio of high returns is more attractive than the one with low returns if both were to carry the same amount of risk. Similarly, given a choice of two portfolios that yield the same rate of return, the one with the lower level of risk would be sought after. The CAPM argues that the expected rate of return for a security is influenced by the systematic risk which is the non-diversifiable component of the total risk. It postulates a positive linear relationship between risk and return since an investor would seek higher return in order to be compensated for assuming higher risk.

The most familiar expression of CAPM is $E(R_i) = R_f + \beta_i [E(R_m) - R_f]$ where $E(R_i)$ is the expected return on stock $i$, $R_f$ is the risk-free rate of return, $E(R_m)$ is the expected return on the market portfolio and $\beta_i$ is the measure of systematic risk. The model states that the expected return on a risky asset is made up of two components - the return on a risk-free asset and the risk premium which is proportional to the systematic risk of the asset. The $\beta$ coefficient measures the risk of stock $i$ relative to market risk. The model indicates a positive linear relationship between the expected return and the systematic risk since higher return is expected for assuming higher risk. This relationship holds for individual assets as well as for portfolio of assets. Since $\beta$ coefficient provides an estimate of risk for a particular stock or portfolio relative to the market, a $\beta$ coefficient that equals unity is said to depict the market portfolio. The CAPM claims that the market portfolio is a mean-variance efficient portfolio. Stocks with $\beta > 1$ are deemed to carry higher than market risk and vice-versa. Therefore, one can
anticipate a higher (lower) than the expected market rate of return for stocks with \( \beta \) exceeding (under) one.

The expression \( \text{E}(R_t) = R_n + \beta_i \left[ \text{E}(R_{mt}) - R_n \right] \) poses a measurement problem since it is based on the ex ante representation. To overcome the limitation, the transformed version based on the ex post representation was introduced under the assumptions that the capital markets are efficient and the rate of return on an asset is a fair game. The model is defined as \( R_n = R_n + \beta_i \left[ R_{mt} - R_n \right] + u_n \) where the error term is considered as white noise. This representation allows the use of observed data and helps solve the measurement issue since stock returns are measurable by taking the logarithmic rate of return for stocks with \( \beta \) exceeding (under) one.

The expression \( \text{E}(R_{it}) = R_{ft} + \alpha_i + \beta_i \left[ \text{E}(R_{mt}) - R_{ft} \right] \) is used to interpret the conditional CAPM. The conditional variance of the error term is non-normal and heteroscedastic. The Generalised Autoregressive Conditional Heteroscedastic-in-Mean (GARCH-M) model holds the advantage of handling time series data that fail to satisfy the basic assumption of classical linear regression model (CLRM). Within the premise of the original GARCH model, the conditional mean and variance of stock returns are assumed to be influenced by the past returns and volatility based on the available information at a particular point in time. The GARCH-M model, introduced by Engle, Lilien and Robins (1987), provides a new framework for studying the relationship between risk and return since the model explicitly links the conditional mean of returns. It specifies that \( R_n = \alpha_0 + \alpha_i R_{at-1} + \ldots + \alpha_n R_{t-n} + \beta_h h \), where \( h \) depicts the conditional variance, which is presented in the equation by the square root of time-varying variance. The inclusion of the conditional variance into the mean equation under GARCH-M depicts the resemblance with CAPM since it marks the presence of risk component in stock returns. The \( \beta \) coefficient can be interpreted as a risk aversion parameter which assumes a positive linear relationship between the conditional variance and returns.

This paper employs the GARCH-M model in examining the subject from the perspective of selected Asia Pacific countries. One peculiar feature of some of these stock markets, such as those of China, India and Indonesia, is that they are non-synchronously and thinly traded which make them different from the advanced markets in North America and Europe. In addition, some of the stock markets, like Hong Kong and Malaysia, are also speculation-driven. Nonetheless, they have attracted the attention of many international fund managers in the past to take advantage of the high returns and at the same time to diversify their portfolios. Given the growing concern over the effectiveness of portfolio diversification within the region, as a result of the financial crisis that hits the region, a clear understanding on the nature of risk and return within the respective stock market is indeed very useful. That provides a platform for this study to examine the risk-return relationship within the region. The outcome of this study will provide an update on the relationship that can be helpful for market practitioners in their decision making processes.

The organization of the paper is as follows; the following section presents the related literature on the subject. This is followed by the presentation of the data and methodology employed in the study. The next section discusses the results of this study. The last section offers the conclusion of the study.

2. Review of Literature

The ability of GARCH model (and its extensions) in exploring the relationship between risk and return has been validated in numerous studies. For instance, Brooks, Faff and McKenzie (2002) discover that the GARCH-based estimates of risk generate the lowest forecast error of all techniques. This finding echoes Asgharian and Hansson (2000) who find that the bivariate GARCH(1,1) process produces a more accurate measure of market beta than the beta estimated by OLS. Morelli (2003) believes that the GARCH model represents the CAPM better during periods of relative high volatility. In the same vein, Polasek and Ren (2001) find that the multivariate ARCH-M model is better than the VAR and VAR-GARCH models in modelling stock returns. Giannopoulos (1995) finds that the bivariate GARCH-M is able to capture the changes in the stocks’ systematic risk across time. This is an essential condition for modelling the conditional CAPM. In the non-synchronous and thinly trading environment, Solibakke (2002) finds that the ARMA-GARCH-M model cannot reject the conditional CAPM. Hansson and Hordahl (1998)
also find supporting evidence to suggest that the multivariate GARCH-M model is capable of modelling the conditional CAPM. Therefore, in general, the GARCH-M model seems like a credible tool to test the risk-return relationship within the premise of the conditional CAPM.

However, using the GARCH(1,1)-M model, Baillie and DeGennaro (1990) find very little evidence to statistically substantiate the significant relationship between stock returns and own volatility in the United States. This finding is supported by Theodossiou and Lee (1995) who also find no significant relationship between expected return and conditional volatility of returns in the United States as well as the other developed countries namely Australia, Belgium, Canada, France, Italy, Japan, Switzerland, the United Kingdom and Germany. On the contrary, Dean and Faff (2001) apply the EGARCH-(1,1)-M model and find evidence of a positive relationship between the market risk premium and its variance within the Australian equity market. Mougoue and Whyte (1996), who adopt the GARCH(1,1)-M specification, report a positive and statistically significant risk aversion component which implies the presence of a positive relationship. Thus, their result contradicts the findings of Theodossiou and Lee (1995).

Another contradicting finding is documented in China. Song, Liu and Romilly (1998) apply the GARCH-M models to the Shanghai and Shenzen Stock Exchanges in China. The two exchanges have much smaller capitalization and fewer listed companies when compared to well-developed financial markets. They discover significant positive risk premia on stock prices in both exchanges which suggests that higher risk results in higher return. This is consistent with the proposed theory. However, this finding contradicts that reported by Lee, Chen and Rui (2001) who find significant negative relationship in a study of the two Chinese stock exchanges. The discrepancies, therefore, could be attributed to the use of ex post data as well as the different time frame used in their respective studies. The latter covers a longer time span that includes the period of the regional financial turmoil in 1997. This finding, however, supports Girard, Rahman and Zaher (2001) who find that the risk premia is state-dependent. In their study on nine Asian capital markets and the U. S. before, during and after the Asian financial crisis they find that, despite the presence of a positive (albeit insignificant) relationship between risk premium and variance in all markets, the risk premia tend to be positive during the upstate and it becomes negative during the downstate.

Within the framework of an emerging stock market, Salman (2002) provides empirical evidence that supports the positive relationship between risk and return. Based on his study of the Istanbul Stock Exchange, he finds that the CAPM’s proposition is valid and he believes that both risk and return are integrated in the information provided to the market. Similarly, Omet, Khasawneh and Khasawneh (2002) also find a positive and significant association between risk and return in the Jordanian Securities Market. Koutmos, Negakis and Theodossiou (1993) also report a similar finding from the Athens Stock Exchange. They find that the estimated risk premium is positive and significant which implies that the returns are positively related to volatility. In a study across eight different industries in Taiwan, Chiang and Doong (1999) discover that the influence of conditional volatility on stock returns is mixed depending on the industry. Nonetheless, only the coefficients with negative signs are found to be significant. Therefore, the negative risk premium suggests that investors are penalized, not rewarded, for holding risky stocks.

The mixed results obtained from previous studies warrant this attempt to uncover the nature of the risk-return relationship in the Asia Pacific region. Given the different backgrounds of each market, it is anticipated that the nature of the risk-return relationship will vary from one country to another. This study seeks to clarify the matter by examining the risk-return relationship among selected Asia Pacific nations, within the premise of the conditional CAPM by using the GARCH-M model.

3. Data and Methodology

This study employs daily closing values for stock indices of ten exchanges in Asia Pacific countries, namely in Australia, China, Hong Kong, India, Indonesia, Japan, Malaysia, Singapore, South Korea and Taiwan. The stock market returns (expressed in percentages) are calculated as the log of the daily differences in the market index. The data series runs from the beginning of January 2000 to the end of February 2005. This period is chosen as it represents a stable market without the influence of the regional financial crises that hit some of the countries towards the end of 1990s. Moreover, the use of a more recent set of data provides fresh insight on the nature of the relationship between risk and return in the region. All series are expressed in their own
currency, eliminating any influence from exchange rate risk.

The study adopts the basic GARCH\( (p,q)\)-M model as proposed by Engle, Lilien and Robins (1987). The model specifies the conditional mean and variance of stock returns as follows:

\[
R_t = \mu + \gamma h_t + \varepsilon_t \\
\varepsilon_t^2 | \Omega_{t-1} \sim N(0, h_t^2) \\
h_t^2 = \beta_0 + \sum_{i=1}^{p} \beta_i h_{t-i}^2 + \sum_{j=1}^{q} \delta_j \varepsilon_{t-j}^2
\]

The coefficient \(\mu\) represents the conditional mean of stock returns at time \(t\), \(h_t\) is the standard deviation of the conditional variance which reflects the risk premium, \(\Omega_{t-1}\) marks the set of information available at time \(t-1\), \(\beta_0\) is a constant and the residual term is represented by \(\varepsilon_t\). The estimated parameters are \(\alpha, \beta, \delta, \gamma\). The conditional variance, \(h_t^2\), is assumed to be a function of the last period’s squared error as well as the last period’s conditional variance. The significant influence of conditional volatility on stock returns is captured by the estimated \(\gamma\) coefficient which depicts the relative risk aversion parameter. A significant and positive value of \(\gamma\) implies that investors were compensated by higher returns for bearing higher levels of risk. A significant negative coefficient indicates that investors were penalized for bearing risk. In this study, GARCH\((1,1)\)-M is estimated, taking into account Bollerslev’s (1986) claim that the chosen lag length for the past squared error and the conditional variance are sufficient to model stock returns.

4. Results

The descriptive summary for daily returns is presented in Table 1. All series fail to conform to the normal distribution since the Jarque-Bera statistics reject the null hypothesis of normality in all cases. With the exception of China, the skewness of the returns series for the other nine countries is found to be negative which suggests that the returns distribution of the shares traded on these exchanges has a higher probability of earning negative returns.

<table>
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<td>0.0218</td>
<td>0.0060</td>
<td>0.0157</td>
<td>0.0380</td>
<td>0.0173</td>
<td>0.0351</td>
<td>0.0067</td>
<td>0.0173</td>
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<td>0.0000</td>
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<td>0.0556</td>
<td>0.1104</td>
<td>0.0678</td>
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<td>0.0391</td>
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<td>2101.98</td>
<td>642.48</td>
<td>112.96</td>
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<td>1247.25</td>
<td>1461.23</td>
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Table 1: Descriptive Summary of Daily Returns

Notes: a, b, c = significant at 1%, 5% and 10%

\(Q(k)\) and \(Q^2(k)\) denote Ljung-Box (1978) test statistics for kth-order serial correlation of the return and squared return series under the null hypothesis of serial independence that follows the chi-squared distribution with \(k\) degree of freedom.
The presence of negative skewness also indicates that the distribution of stock returns is asymmetric. The kurtosis for each returns series is found to be greater than 3, which implies that the distribution of returns is non-normal with fat tails and sharp peaks. The Ljung-Box test statistics for the returns series suggest the presence of serial correlation in five of the countries, namely India, Indonesia, Malaysia, South Korea and Taiwan. Evidence of low-order autocorrelation is detected in India, Indonesia and Malaysia while higher-order autocorrelation is more prevalent in Singapore, South Korea and Taiwan. Perhaps the issues of thin and non-synchronous trading may have contributed to the presence of first-order autocorrelation in India, Indonesia and Malaysia. A further inspection on the squared return series also rejects the null hypothesis of homoscedasticity for all cases. The Ljung-Box test statistics detect traces of autocorrelation up to lag 30 for all stock returns series which indicates that the identically distributed hypothesis is rejected. It marks the presence of higher moment dependencies which implies the presence of conditional heteroscedasticity. Therefore the series cannot be modelled as white noise linear processes.

The presence of ARCH errors validates the use of GARCH-M model in modelling the volatility of all markets. Table 2 reports the results from the GARCH(1,1)-M estimates which test for the conditional CAPM. The residuals for all series are found to be non-normal as the Jarque-Bera statistics are found to be significant in all cases. Owing to this fact, the estimation of the standard error based on Bollerslev and Wooldridge (1992) is deemed appropriate since it provides robust estimates to accommodate the deviation from normal distribution. The Ljung-Box (1978) test statistics fail to reject the null hypotheses for the presence of serial correlation up to lag 30 in the residual of seven countries except for India, Indonesia and Malaysia. This raises the question of the validity of the specification for the returns series for the three countries. Nonetheless, the absence of serial correlation in the squared residuals implies that the conditional variance model is correctly specified. In addition, the ARCH effects are removed from the residual series as suggested by the ARCH-LM test statistics that were introduced by Engle (1982).

<table>
<thead>
<tr>
<th>Country</th>
<th>µ</th>
<th>γ</th>
<th>β₀</th>
<th>β₁</th>
<th>δ</th>
<th>J-B</th>
<th>Q(k)</th>
<th>Q²(k)</th>
<th>LM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.0290</td>
<td>0.0425</td>
<td>0.0109</td>
<td>0.0939</td>
<td>0.8852</td>
<td>634.79a</td>
<td>22.27</td>
<td>16.95</td>
<td>17.15</td>
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<tr>
<td>China</td>
<td>-0.3278</td>
<td>0.2833</td>
<td>0.0851</td>
<td>0.1483</td>
<td>0.8144</td>
<td>393.20a</td>
<td>27.68</td>
<td>26.82</td>
<td>34.49</td>
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<tr>
<td>Hong Kong</td>
<td>-0.0358</td>
<td>0.0509</td>
<td>0.0138</td>
<td>0.0486</td>
<td>0.9442</td>
<td>196.57a</td>
<td>23.53</td>
<td>20.13</td>
<td>29.36</td>
</tr>
<tr>
<td>India</td>
<td>0.1943</td>
<td>-0.0768</td>
<td>0.1055</td>
<td>0.1610</td>
<td>0.7993</td>
<td>135.89a</td>
<td>53.60a</td>
<td>23.33</td>
<td>22.82</td>
</tr>
<tr>
<td>Indonesia</td>
<td>0.1988</td>
<td>-0.0726</td>
<td>0.3088</td>
<td>0.1575</td>
<td>0.6977</td>
<td>350.42a</td>
<td>57.71a</td>
<td>28.10</td>
<td>24.31</td>
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<tr>
<td>Japan</td>
<td>-0.0129</td>
<td>0.0040</td>
<td>0.0492</td>
<td>0.0755</td>
<td>0.9045</td>
<td>87.17a</td>
<td>19.89</td>
<td>30.79</td>
<td>29.02</td>
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<td>Malaysia</td>
<td>-0.1934</td>
<td>0.2457</td>
<td>0.1332</td>
<td>0.2332</td>
<td>0.6510</td>
<td>1075.2a</td>
<td>97.10a</td>
<td>32.21</td>
<td>33.13</td>
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<tr>
<td>Singapore</td>
<td>0.0669</td>
<td>-0.0361</td>
<td>0.0247</td>
<td>0.0995</td>
<td>0.8913</td>
<td>1127.1a</td>
<td>34.55</td>
<td>12.16</td>
<td>13.31</td>
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<tr>
<td>South Korea</td>
<td>0.2255</td>
<td>-0.0778</td>
<td>0.0516</td>
<td>0.0746</td>
<td>0.9159</td>
<td>645.32a</td>
<td>35.35</td>
<td>14.93</td>
<td>16.02</td>
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<tr>
<td>Taiwan</td>
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<td>0.0237</td>
<td>0.0656</td>
<td>0.9284</td>
<td>63.84a</td>
<td>21.36</td>
<td>31.69</td>
<td>30.16</td>
</tr>
</tbody>
</table>

Note: a, b and c mark significance at 1%, 5% and 10% respectively.
The risk aversion parameter, coefficient \( \gamma \), is found to be significant in only two countries, i.e. China and Malaysia. In both cases, the coefficient has a positive value which renders support to the positive linear relationship between risk and return. Consistent with the CAPM, this finding indicates that investors in China and Malaysia are compensated for bearing high risk. The level of risk premium is almost the same in the two countries judging by the relatively similar value of coefficient \( \gamma \). The result, however, is not comprehensive. The coefficients \( \gamma \) for the other eight countries are not significant. In addition, five of the eight countries produce negative coefficients which contradict the proposed relationship. This finding is not surprising since Chou (1988) warns that the positive relationship will only prevail within the \textit{ex ante} scenario whereas this study employs \textit{ex post} data.

It is also worth highlighting that the volatility of returns is persistent in all countries. The coefficient \( \beta_1 \) and \( \delta \) are significant in all cases and the total value is high, ranging from 0.85 in Indonesia to 0.99 in Hong Kong, Singapore South Korea and Taiwan. As such, the impacts of shocks on the stock returns seem to fade away at a slow rate in the Asia Pacific stock market. The clustering of volatility is more prevalent in Indonesia and Malaysia judging by the higher value of coefficient \( \beta_1 \) compared to the other countries. Coupled with the shorter period of half-life volatility (i.e. calculated based on \( \log(0.5)/\log(\beta_1 \) and \( \delta \)) for the two countries, 4.43 and 5.63 days respectively, the findings may reflect the speculative nature of the market participants who tend to act on speculative news. On the other hand, Taiwan and China register the longest half-life period with 115.18 and 95.92 days respectively. It seems that the stock market in the two countries takes a longer time to recuperate from the jitters brought about by the volatility in the market.

5. Conclusion

Using the GARCH(1,1)-M model, the study seeks to determine the relationship between risk and return within the framework of the conditional CAPM. The result fails to produce convincing evidence to fully support the positive linear relationship in the Asia Pacific stock markets as postulated by the CAPM. Only China and Malaysia provide evidence that is coherent with the model. In both cases, investors are rewarded for bearing additional risk. The conditional CAPM model, therefore, may be useful for market practitioners in determining the expected rate of return as well as the cost of capital in the two countries. Perhaps, GARCH(1,1)-M is not adequate to capture the risk-return relationship within the premise of the CAPM in the other countries. But the significant evidence found in China and Malaysia goes to prove the merit of the CAPM in describing the risk-return relationship in some emerging stock markets.

References


