The Impact of Problem-based Learning on Pre-service Teachers’ Development and Application of Their Mathematics Pedagogical Content Knowledge

A Thesis submitted by

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**Abstract**

The main aim of this study was to investigate the effectiveness of problem-based learning (PBL) compared to a traditional teacher-led instructional approach on pre-service teachers’ mathematics pedagogical content knowledge (PCK) and their ability to apply that knowledge.

Predictors of teacher effectiveness in relation to student achievement are based on defined attributes, for example: (a) the ability to use a range of evidence-based teaching strategies (Dean, Hubbell, Pitler, & Stone, 2012; Hattie, 2009), such as those encompassed in the PBL framework as described by Hmelo-Silver and Barrows (2015) and Savery (2015); (b) the ability of a teacher to enact the PCK he or she possesses (Hattie, 2009; Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005; Shulman, 1986; Tatro & Senk, 2011) and (c) the level of confidence, and/or self-efficacy, a teacher maintains in his or her ability to teach (Bandura, 1977; Schunk, 1991; Tschannen-Moran & Hoy, 2001). Teachers who have high self-efficacy may also maintain the expectation that they can influence positive academic outcomes for their students (Bandura, 1986; Enochs, Smith, & Huinker, 2000). This type of cognitive belief is identified by Bandura (1977) as outcome expectancy.

Although studies on PBL are numerous, an inspection of the literature did not reveal any which examined the specific characteristics noted above in pre-service teachers when PBL was introduced to the teaching context. As a result, this study aimed to investigate the impact specifically that closed loop PBL had in a tertiary mathematics education subject, compared to using a traditional teacher-led approach, on pre-service teachers’ mathematics PCK, their ability to enact their PCK, their self-efficacy for teaching mathematics and their mathematics teaching outcome expectancy. To measure pre-service teachers’ knowledge of mathematics PCK and their self-beliefs for teaching mathematics, a Mathematics Pedagogical Content Knowledge Instrument (MPCKI) and a Mathematics Teaching Efficacy Belief Instrument (MTEBI) were developed and used in a pilot study. The instruments were administered pre-intervention and post-intervention to both a control group (n=15), who received traditional instruction, and a treatment group (n=15) who were instructed using the closed loop PBL teaching method (the intervention). The outcomes of the pilot study provided insight into the need for
additional data to be collected from pre-service teachers’ end-of-semester exam questions (N=37) to measure specifically their ability to enact their mathematics PCK. Qualitative data were also collected in the form of semi-structured interview responses. The interviews, which were conducted at the end of the intervention, asked questions of the treatment group participants (n=17) relating to the impact PBL had on the development of their mathematics PCK, the ability to enact their PCK and their teaching beliefs.

The analysis of the results of the MPCKI, which was designed as a multiple-choice survey instrument, did not demonstrate a significant difference between the two groups’ mathematics PCK. However, the analysis of the pre-service teachers’ constructed responses to the end-of-semester exam mathematics PCK-specific questions did demonstrate a significant difference between the two groups’ ability to enact their mathematics PCK. Furthermore, the qualitative data collected from the interview responses from the PBL treatment group indicated a unanimous satisfaction with being taught by the PBL method and a unanimous affirmative response when asked if they felt the closed loop PBL teaching method was more effective than traditional instruction in developing their mathematics PCK and their ability to enact their PCK. The results of the MTEBI analysis did not demonstrate a significant difference between the two groups’ self-efficacy for teaching and teaching outcome expectancy. However, an analysis of the interview transcripts from the PBL group of pre-service teachers revealed a new sense of confidence and teacher effectiveness, which, they felt will positively impact on their students’ academic success.

It was hypothesised that the closed loop PBL method is a more effective pedagogical approach in teacher education compared to traditional teacher-led instruction for mathematics education. As the aim of teacher education is to enhance graduate teachers’ abilities to enact their mathematics PCK, simply put to be better mathematics teachers, it was concluded that closed loop PBL is a useful pedagogical strategy to afford them the confidence and skills to enact their personal mathematics PCK. Although the findings returned mixed results, most of the evidence supports closed loop PBL’s potential as a more effective pedagogical approach for developing pre-service teachers’ ability to enact mathematics PCK.
Certification of Thesis

This thesis is entirely the work of David A Martin except where otherwise acknowledged. The work is original and has not been previously submitted for any other award, except where acknowledged.

Student and supervisors’ signatures of endorsement are held at USQ.

Professor Romina Jamieson-Proctor
Principal PhD Supervisor

Professor Peter Albion
Associate PhD Supervisor
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Introduction

1.1 Overview

The main aim of this study was to investigate the effectiveness of problem-based learning when compared to a traditional teaching approach in a tertiary mathematics teacher education subject. Specifically, the study investigated the closed loop problem-based teaching method’s impact on pre-service teachers’ mathematics pedagogical content knowledge (PCK), their ability to enact their PCK, self-efficacy for teaching mathematics and their mathematics teaching outcome expectancy. This chapter presents the background to the research problem, followed by the researcher’s strategy to address the research problem. Background is provided by description of the researcher’s experiences and prior studies which were influential in the conceptualisation of this study. Concluding the chapter is the structure of the thesis and a glossary of key terminology used throughout.

1.2 The Research Problem and Aim of the Study

Concerns regarding Australian students’ mathematical achievements continue to be raised in the public and government sectors (Department of Education Science and Training (DEST), 2007; Ministerial Council on Education Early Childhood Development and Youth Affairs, 2008; Teacher Education Ministerial Advisory Group, 2014). Results of the Trends in International Mathematics and Science Study (TIMSS) reveal an unsatisfactory performance in mathematics among Year 4 students (National Center for Education Statistics, 2009, 2012). The average mathematics achievement score of 516 for Year 4 students has remained the same since 2003, but their international ranking has dropped from 14th place in 2007 to 19th place in 2011. This statistic suggests that other countries have improved their Year 4 maths scores while Australian Year 4 students’ performance has remained unchanged. Between 2009 and 2012, in the Programme for International Student Assessment (PISA), Australia fell from 15th to 19th place among participating Organisation for Economic Cooperation and Development (OECD) countries in mathematics (OECD, 2013). Nationally, the proportion of Australian students failing to meet minimum proficiency standards in mathematics has increased (Teacher Education Ministerial Advisory Group, 2014). This declining performance in national and international testing has raised concern and debate about the quality of Australia’s teaching workforce (Teacher Education Ministerial Advisory Group, 2014).
Recognising the importance of mathematics, the Council of Australian Governments (2008) commissioned a report which states the quality and commitment of Australia’s mathematics teaching workforce, at all levels of education, are critical if its citizens expect to improve the nation’s workforce participation and productivity. Under a $550 million funding initiative, the five-year (2009-2013) Improving Teacher Quality National Partnership agreement provided incentives for Australian state governments. The funding supported a range of reforms including initiatives aimed at maintaining the quality of Australia’s teaching workforce and improving students’ overall levels of numeracy (Australian Curriculum, Assessment and Reporting Authority, 2011). Funding these initiatives was justified since research suggests that being taught by effective teachers is one of the main factors impacting student achievement (Darling-Hammond & Rothman, 2011; Hattie, 2009, 2012).

Subsequently, one of the key findings of the Teacher Education Ministerial Advisory Group (TEMAG) (2014) was evidence of poor practice in a number of initial teacher education programs. An area of concern is that higher education providers working with pre-service teachers were using pedagogical practices which were not informed by research and not “based on evidence linked to impact on student learning outcomes” (Teacher Education Ministerial Advisory Group, 2014, p. 15).

Many researchers point to the depth of understanding of, and interaction between, content knowledge, pedagogical knowledge and pedagogical content knowledge as being positively linked with teaching performance. Highly effective teachers possess strong pedagogical content knowledge that enables them to improvise and alter teaching strategies in response to different classroom situations. (Teacher Education Ministerial Advisory Group, 2014, p. 18)

Pedagogical content knowledge (PCK) is defined as “the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (Shulman, 1987, p. 8). The Advisory Group to the Australian Education Minister states, “the difference between expert teachers and pre-service teachers is this depth of pedagogical content knowledge” (Teacher Education Ministerial Advisory Group, 2014, p. 18). The Advisory Group’s findings indicate that not all graduating pre-service teachers possess adequate PCK to teach effectively. This
suggestion proposed by the Advisory Group is that possessing strong PCK is viewed as being linked with high teacher performance. Thus, higher education providers should use and model evidence-based practices which will effectively develop pre-service teachers’ PCK and their ability to enact their PCK. The Advisory Group presented 38 recommendations which they believe will provide the structural change needed to strengthen initial teacher education in Australia. Recommendation 6 of the TEMAG report requires initial accreditation of teacher education programs to be linked to tertiary providers demonstrating that their programs use evidence-based pedagogical approaches (Teacher Education Ministerial Advisory Group, 2014). Recommendation 14 to the Education Minister requires that “higher education providers deliver…a range of pedagogical approaches that enable pre-service teachers to make a positive impact on the learning of all students” (2014, p. xiii).

Thus, the research problem that this study addresses is the inference from several recent studies that not all pre-service teachers are graduating with the mathematics PCK necessary to teach effectively. This research study aims therefore to identify evidence-based pedagogical practices which are more effective in developing the mathematics PCK of pre-service teachers and their ability to enact their PCK. In the context of teaching mathematics, enacting would be defined as analysing or evaluating a student’s mathematical solutions or arguments as well as providing appropriate feedback, and the ability to guide classroom discourse as well as to explain or represent mathematical discourse or procedures (Döhrmann, Kaiser, & Blömeke, 2012).

1.3 Addressing the Research Problem

The approach to addressing the research problem was to examine research undertaken in the domain of evidence-based pedagogical practices looking for shared elements of effective teaching. The premise being that locating shared elements of effective teaching in the literature would reveal potential evidence-based pedagogical practices higher education providers could use to more effectively develop pre-service teachers’ mathematics PCK. Initially, two studies (Lingard et al., 2001; Marzano, 2001) were examined by the researcher.
Firstly, Marzano, an international leader in research on effective teaching, proposes that one mark of effective teachers is the ability to use an array of evidence-based instructional strategies and, by integrating these strategies into their instruction, teachers are better equipped to enhance student learning (Dean et al., 2012; Marzano, 2001). Marzano identified 22 instructional strategies used by effective teachers (Marzano, 2001; Marzano Research Laboratory, 2015). The results of the examination revealed that five of the evidence-based teaching strategies studied by Marzano’s research team align closely with the principles and characteristics of PBL: cooperative learning, cues and questions, complex cognitive tasks, providing feedback, and setting goals and objectives (Hmelo-Silver & Barrows, 2006; Marzano, 2001; Marzano Research Laboratory, 2015; Savery, 2015).

Secondly, the ‘Productive Pedagogies’ model (Lingard, Hayes, & Mills, 2003; Mulcahy, 2005) was examined as a teaching framework capable of responding to the Ministerial Advisory Group’s recommendations. The term productive pedagogies was conceptualised from the Queensland School Reform Longitudinal Study (QSRLS) (Lingard et al., 2001). The Productive Pedagogies model is a framework of 20 classroom teaching and learning practices students experience which form the strands of effective teaching that support enhanced student learning outcomes (Lingard et al., 2003; Mulcahy, 2005). “Problem-based learning qualifies as ‘productive pedagogy’ [because] it demands social inquiry where social means that learning is an interpersonal, constructive process and inquiry connotes active, student driven learning” (Mulcahy, 2005, p. 319). Amongst the other 19 productive pedagogies, seven share characteristics with PBL. These pedagogies are: Connectedness to the World, which promotes the use of schoolwork that has a resemblance or connection to real-life contexts; Background knowledge, which looks to connect with students’ prior knowledge; Higher-order Thinking, which embeds a critical analysis component; Student Control, which proposes student-directedness; Engagement, which places the research and self-discovery squarely in the hands of the students and; Social Support and Group Identity, which requires a cooperative learning component in which students work together throughout the curriculum. In both studies the PBL pedagogical approach was found to encompass many of the elements which support effective teaching.
Additionally, meta-analyses, meta-syntheses and reviews of PBL studies concluded that traditional instruction, which is the predominant pedagogical approach used in tertiary education, may not be the most effective instructional approach in developing practical application and critical thinking skills (see for example, Albanese & Dast, 2014; Strobel & van Barneveld, 2009; Walker & Leary, 2009). In this regard, these studies and reviews of studies have reported PBL may be a more effective pedagogical practice at the tertiary level. According to the findings from a meta-analysis of PBL studies, PBL leads to favourable outcomes “when assessment is at the application level” and when the intervention uses the full closed loop approach” (Walker and Leary, 2009, p. 28). Although the finding was based on a small sample size of closed loop PBL studies conducted in medical education, Walker and Leary concluded that similar learning outcomes would be expected based on the type of PBL implementation used in other disciplines. It is for these reasons that I elected to investigate PBL and its effectiveness as a pedagogical approach in teacher education.

Closed loop PBL is one of six variations in Barrows’ (1986) PBL taxonomy. Barrows (1986, 1996) suggests that the six teaching variations in his PBL taxonomy have degrees of impact on key educational objectives, for example, structuring knowledge for use in clinical contexts and developing an effective clinical reasoning process. In such a context medical students are presented with the symptoms of a sick person. The problem posed to the medical students is to achieve for that sick person a relatively healthy state, and many factors go into determining the best treatment for the patient to achieve the goal of good health. In order to solve this problem, the students must possess a fairly deep understanding of human physiology and disease states. The patient’s history and genetics also need to be considered. After diagnosing the illness, the medical students must provide a solution or treatment. Of all six variations of his PBL taxonomy, Barrows states the closed loop variation is best positioned to enhance these educational objectives (Barrows, 1986). These educational objectives for medical students, acquiring the necessary skills to diagnose and heal effectively, correspond with the educational objectives that pre-service teachers are required to achieve – to acquire the necessary skills to diagnose and teach effectively. Relating the above analogy to the context of this study, pre-service teachers need to possess a deep understanding of mathematics content, curriculum and assessment to determine the cognitive demands of a task on their students. Next, the pre-service teachers need to diagnose the students’ mathematical solutions or arguments and
identify any learning difficulties and misconceptions they exhibit as a result of engaging in the task. Lastly, the pre-service teachers should be able to appropriately respond to the misconceptions and learning difficulties by formulating responses into representations which make it comprehensible to the students (Shulman, 1986; Tatto et al., 2008).

Based on the research presented, a closer look into the effectiveness of closed loop PBL to effectively enhance PCK seemed warranted. Specifically, the purpose of this research was to compare how closed loop PBL and traditional teacher-led instruction impact pre-service teachers’ mathematics PCK and their ability to enact their PCK throughout a semester-long undergraduate mathematics education subject. The research may also contribute a response to TEMAG regarding Recommendation 6 and Recommendation 14 (Teacher Education Ministerial Advisory Group, 2014). Nevertheless, the Education Minister’s concerns did not provide the initial stimulus for this study. Prior to commencing this study, the researcher conducted two smaller studies using a PBL approach in two separate mathematics teacher education subjects to test its impact on pre-service teachers’ content knowledge and their conceptual understanding of place value. These two prior experimental studies (Martin, 2012; Martin & Jamieson-Proctor, 2010), which were influential in the conceptualisation of this study, are described in the following section.

1.4 Background to the Conceptualisation of the Study

Observations from my past experiences as an educator in the United States (USA) influenced the conceptualisation of the two prior studies as well as this study. Two positions I held, in particular, provided these experiences: a middle school mathematics teacher in the public school system and a casual mathematics curriculum and pedagogy lecturer at a regional university in Florida.

As a novice teacher teaching middle school mathematics to groups of students from low socio-economic backgrounds, I was concerned that the year-six students were not engaging with their class work when experiencing what was viewed as sound traditional teaching practice. Through trial and error I discovered that providing students with meaningful, real-world problems to solve collaboratively with their classmates increased their level of engagement and understanding compared to using predominantly teacher-
led traditional pedagogies. Increased academic success and a decrease in the number of discipline reports written that school year provided anecdotal evidence of the positive impact of this social constructivist teaching approach.

When my casual lecturing career commenced in a Florida university, I found that some students entering the initial teacher education program did not possess an adequate level of mathematics content knowledge (MCK) and the university which employed me was aware of this inadequacy. Consequently, part of my teaching duties as a lecturer was to assist the pre-service teachers to enhance their MCK, in addition to developing their mathematics PCK. To meet both obligations within the time frame allotted, I delivered my instruction using a traditional teacher-led didactic approach. Based on my observations and the results of the pre-service teachers’ assessments, I recognised that the students were not reaching a deep level of understanding in many content areas of mathematics nor in their mathematics PCK. I concluded that to assist pre-service teachers to achieve deeper understanding in these areas a different pedagogical approach was required. Subsequently, I actively sought to enhance the pre-service teachers’ learning and engagement in their mathematics education subject through the use of a constructivist, problem-based learning approach. For example, the first assignment was redesigned so that the pre-service teachers were required to develop and present a representative lesson plan, based on their favoured teaching grade-level, to their peers. Feedback from the pre-service teachers confirmed that they believed designing and presenting a lesson plan was a valuable assignment in terms of developing that specific content, practising their teaching skills and building teacher self-confidence. These observations, experiences and outcomes as an educator in America, while not formally researched, cemented this researcher’s conviction that constructing one’s own knowledge through the use of meaningful, real-world problems as the stimulus for learning was a more effective pedagogical approach than using a didactic, teacher-led instructional approach.

As a full-time Education lecturer in an Australian university, this conviction led to action research studies (Martin, 2012; Martin & Jamieson-Proctor, 2010). Both studies were based on mathematics curriculum and pedagogy subjects which I have taught since 2009 in Australia and both studies were conducted with Bachelor of Education pre-service teachers at a regional Queensland university. The two studies were conceptualised based
on the difficulties pre-service teachers had in developing their content knowledge, particularly with respect to place value, but also with other concepts that underpin number and algebra (Copeland, 2010; MacDonald, 2008; Stacey et al., 2001). Based on my earlier successes, rather than using traditional instruction I chose to use PBL as the pedagogical approach to assist pre-service teachers to achieve a greater understanding of place value, as well as to assist them to develop the PCK required to effectively teach the concepts, skills and strategies related to place value. The findings suggest the PBL pedagogical approach adopted may have contributed to an increase in the number of pre-service teachers who appeared to have achieved a multi-structural understanding of the place value concept.

The conclusion reached is that a social constructivist, PBL approach may be more effective than a traditional teacher-led instructional approach to assist pre-service teachers to develop the PCK required to effectively teach the concepts, skills and strategies related to place value. I also theorised, based on the pre-service teachers’ higher level of engagement while working collaboratively on their tutorial tasks, that PBL shows promise in assisting them to develop the PCK required to effectively teach. On reflection, the outcomes of the two studies served as a reminder of the responsibility of initial teacher preparation programs, which is to use evidence-based pedagogies to assist pre-service teachers to develop their content knowledge and then progress onto the critical dimension of learning how to teach the content (Ball & McDiarmid, 1990; Ball, Thames, & Phelps, 2008; Shulman, 1987).

To review, my teaching experiences and early research contributed to my deeper understanding of the PBL teaching method. My observations, findings and commitment to using the PBL method were the catalyst for the present study. However, the study was also informed by my subsequent exploration of the literature with respect to the impact of closed loop PBL on the development of pre-service teachers’ mathematics PCK.

### 1.5 Chapter Summary

Characteristics of an effective teacher include possession of sound PCK and the ability to enact the PCK utilising an array of evidence-based instructional strategies (Dean et al., 2012; Marzano Research Laboratory, 2015; Teacher Education Ministerial Advisory
Moreover, effective teaching begins with effective teacher preparation (National Research Council, 2010; Office for Standards in Education, 2005; Tatto & Senk, 2011; Teacher Education Ministerial Advisory Group, 2014). However, static results in Australian students’ performance is now pointing to evidence of poor practice in a number of Australian initial teacher education programs (Teacher Education Ministerial Advisory Group, 2014). PBL appears to be a promising pedagogy for (a) addressing the research problem that pre-service teachers are not graduating with adequate mathematics PCK to teach effectively and (b) responding to the Education Ministerial Advisory Group’s recommendations on how “initial teacher education in Australia could be improved to better prepare new teachers with the practical skills needed for the classroom” (2014, p. v). Consequently, when considering using PBL for developing sound PCK in pre-service teachers, “it is of considerable importance that questions about what forms of PBL produce which outcomes for which students in what circumstances are rigorously investigated” (Newman, 2003, p. 5).

The following chapter reports on examination of the literature about the essential elements of effective teaching, the importance PCK holds as an essential element of effective teaching, the construct of PCK, and the gap in the literature regarding closed loop PBL’s impact on developing mathematics PCK in pre-service teachers and their ability to enact their PCK.

### 1.6 Structure of the Thesis

The thesis is presented in six chapters according to the following outline. Chapter 1 introduces the background to the research problem. Chapter 1 also describes the importance of two earlier studies which were influential in the conceptualisation of this study.

Chapter 2 reviews the literature relevant to the underpinnings of a teacher’s pedagogical content knowledge, the notion of self-efficacy for teaching and the social constructivist PBL method. Additionally, the chapter examines inter-connections between PBL and self-efficacy for teaching, and, their impact on pre-service teachers’ mathematics PCK and/or their ability to enact their PCK. Based on the literature review, the research questions are presented.
Chapter 3 describes the researcher’s epistemological stance and conceptual framework for the research. Next, the pilot study is described which was designed to formulate and refine the PBL teaching intervention program and validate the Mathematics Pedagogical Content Knowledge Instrument (MPCKI) and the Mathematics Teaching Efficacy Belief Instrument (MTEBI) that were used to collect data to answer the research questions in the main study.

Chapter 4 describes the main study’s methodology. It describes the mixed methods approach and the data collection and analysis methods used to answer the research questions. Lastly, the quality of the research and the study’s ethical considerations are addressed.

Chapter 5 presents the results derived from the analysis of both the quantitative and qualitative data sets to answer each of the research questions.

In the context of the research questions, Chapter 6 presents the discussion and the limitations of the research undertaken, along with conclusions drawn from the study and recommendations for further research.

**1.7 Glossary of Terms and Acronyms used throughout**

*Content Knowledge* - refers to the number and organisation of concepts, skills, strategies, and subject matter - related to the content being taught that resides within the teacher (Shulman, 1986).

*Pedagogical Knowledge* - is the knowledge of teaching practice. It represents the teacher’s deep knowledge about the “broad principles and strategies of classroom management and organisation that appear to transcend subject matter” (Shulman, 1987, p. 8).

It includes knowledge about techniques or methods used in the classroom; the nature of the target audience; and strategies for evaluating student understanding. A teacher with deep pedagogical knowledge understands how students construct knowledge and acquire skills and how they develop habits of mind and positive dispositions toward learning. (Koehler & Mishra, 2009, p. 64)
Pedagogical Content Knowledge (PCK) – is a teacher’s special form of professional understanding used to formulate the content knowledge and his or her concepts into the most powerful representations, analogies, illustrations, explanations and demonstrations which make them comprehensible to students (Shulman, 1986). A year later, Shulman defined PCK as “the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (Shulman, 1987, p. 8).

Mathematics Pedagogical Content Knowledge – primarily refers to the interaction of mathematical content knowledge and pedagogical content knowledge necessary for effective mathematics instruction.

Mathematics Pedagogical Content Knowledge Instrument (MPCKI) – is a survey instrument developed by this researcher for this study, based on existing instruments, to measure pre-service teachers’ mathematics pedagogical content knowledge.

Self-efficacy – refers to Albert Bandura’s theory that those who believe they can perform well will be more likely to attempt and persist at tasks (Bandura, 1989).

Outcome Expectancy – is the belief that specific behaviours result in desired outcomes.

Teaching Outcome Expectancy – is the belief that effective teaching will have a positive effect on student achievement (Enochs et al., 2000).

Self-efficacy for Teaching – is defined as a belief in one’s ability to teach effectively (Enochs et al., 2000).

Mathematics Teaching Efficacy Belief Instrument (MTEBI) – is an established instrument used in this study that was specifically designed to measure pre-service teachers’ mathematics teaching efficacy and mathematics teaching outcome expectancy (Enochs et al., 2000; Huinker & Enochs, 1995; Huinker & Madison, 1997).
**Personal Mathematics Teaching Efficacy** – is one of the two constructs measured by the MTEBI and refers to the belief in one’s ability to effectively teach mathematics.

**Mathematics Teaching Outcome Expectancy** – is the second of the two constructs measured by the MTEBI and refers to the belief that effective mathematics teaching will have a positive effect on students’ mathematical achievement (Enochs et al., 2000).

**Evidence-based Teaching Practice** – “Teaching practices or strategies that are based on research and data that are considered reliable and valid, and that can be used to support a particular idea, conclusion or decision” (Teacher Education Ministerial Advisory Group, 2014, p. 95).

**Problem-based Learning (PBL)** – is a student-centred instructional method in which students work in small groups to solve real-world problems they are likely to face in their future careers. Of particular interest to education,

PBL is an educational approach in which complex problems serve as the context and the stimulus for learning. In PBL classes, students work in teams to solve one or more complex and compelling “real world” problems. They develop skills in collecting, evaluating, and synthesising resources as they first define and then propose a solution to a multi-faceted problem. (Major & Palmer, 2001, p. 1)

**Closed loop PBL** – is the most student-centred variation of PBL in Barrows (1986) taxonomy of problem-based learning methods. Closed loop PBL is an extension of Barrows’ problem-based variation from his taxonomy but also contains a reiterative component.

After an episode of self-directed study is completed, students are asked to evaluate the information resources, and then return to the problem as it was presented originally. On the basis of what they learned in self-directed learning, students reanalysed the problem to see how they might have better reasoned their way through it and gained a better understanding. (Barrows, 1986)
2 Literature Review

2.1 Overview

As outlined in Chapter 1, the main aim of this study is to evaluate closed loop PBL as an evidence-based pedagogical practice successful in more effectively developing pre-service teachers’ mathematics PCK and their ability to enact that PCK compared to a traditional teacher-led instructional approach. Chapter 2 presents the literature relevant to PBL as an evidenced-based pedagogical practice. Its role in improving practising teachers’ PCK, their ability to enact their PCK and their resultant self-efficacy for teaching and teaching outcome expectancy is reviewed. Subsequently, the paucity of the literature related to studies using the closed loop variation of PBL for developing these same attributes in pre-service teachers is highlighted.

This chapter is organised into four main sections. The literature review begins by describing several attributes which are integral to educators considered to be effective teachers, and, how those attributes contribute to this study’s investigation. Next, categories of knowledge which underpin teachers’ PCK are unpacked and discussed. The second section of the chapter describes self-efficacy (Bandura, 1994) and its applicable categories regarding a teacher’s self-efficacy and teaching outcome expectancy. The third section of Chapter 2 introduces and defines PBL, and describes its impact on tertiary educational outcomes, justifying its use as the teaching intervention in this study. The chapter concludes with a synthesis of the literature which reveals a gap in the research regarding the impact of closed loop PBL on pre-service teachers’ (a) knowledge of mathematics PCK and/or ability to enact their PCK, (b) self-efficacy for teaching mathematics and (c) mathematics teaching outcome expectancy.

2.2 Attributes of Effective Teachers

A large body of research confirms that during the school day “what teachers know and can do is the most important influence on what [and how much] students learn” (Darling-Hammond, 2000, p. 6; Dean et al., 2012; Hattie, 2012; Ijeh & Onwu, 2013; Miller, 2003). For example, meta-analyses on the importance of effective teaching “reveals a 39 percentage-point difference in student achievement between students with ‘most effective’ teachers and ‘least effective’ teachers” (Miller, 2003, p. 2). Furthermore, a synthesis of over 50,000 studies found several major sources of influence which impacted
on student achievement (Hattie, 2003, 2012). What the students themselves bring to the learning context was identified as the number one source of influence on their achievement, accounting for 50% of the variance. The variable ‘teacher’ accounts for approximately 30% of the variance (2003, 2012). Although the influences students bring to the classroom are greater than what the teacher brings, it was the effect-sizes of these influences which determined that it was the effective teacher who has the greatest recognisable and meaningful effect on student learning.

An effect size is the difference between two means (e.g., treatment minus control) divided by the pooled standard deviation. The result of this formula describes the relative increase or decrease in achievement of the experimental group (Marzano, 2001). Basically, it is a measure of the magnitude of the experimental effect (Thalheimer & Cook, 2002) as conceptualised by Cohen (1992). In terms of educational outcomes, $d = 0.20$ is considered a small effect size, $d = 0.40$, a medium effect size and $d = 0.60$ a large effect when measuring student achievement related to the major contributors to learning (Hattie, 2009). Thus, an effect size of 0.40 sets a level where the effects of the treatment enhance achievement in such a way that an observable difference can be made (Hattie, 2009, 2012). Table 1 identifies 11 of the 14 highest sources of influence on student learning which were not only above the effect-size mean of .40, but also were directly related to the teacher. Hattie concludes from these data that “the greatest source of variance… relates to teachers” (2012, p. 15).

<table>
<thead>
<tr>
<th>Influence</th>
<th>Effect Size</th>
<th>Source of Influence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feedback</td>
<td>1.13</td>
<td>Teacher</td>
</tr>
<tr>
<td>Students’ prior cognitive ability</td>
<td>1.04</td>
<td>Student</td>
</tr>
<tr>
<td>Instructional quality</td>
<td>1.00</td>
<td>Teacher</td>
</tr>
<tr>
<td>Direct Instruction</td>
<td>.82</td>
<td>Teacher</td>
</tr>
<tr>
<td>Remediation/feedback</td>
<td>.65</td>
<td>Teacher</td>
</tr>
<tr>
<td>Students’ disposition to learn</td>
<td>.61</td>
<td>Student</td>
</tr>
<tr>
<td>Class environment</td>
<td>.56</td>
<td>Teacher</td>
</tr>
<tr>
<td>Challenge of goals</td>
<td>.52</td>
<td>Teacher</td>
</tr>
<tr>
<td>Peer tutoring</td>
<td>.50</td>
<td>Teacher</td>
</tr>
<tr>
<td>Mastery learning</td>
<td>.50</td>
<td>Teacher</td>
</tr>
<tr>
<td>Parent involvement</td>
<td>.46</td>
<td>Home</td>
</tr>
<tr>
<td>Homework</td>
<td>.43</td>
<td>Teacher</td>
</tr>
<tr>
<td>Teacher style</td>
<td>.42</td>
<td>Teacher</td>
</tr>
<tr>
<td>Questioning</td>
<td>.41</td>
<td>Teacher</td>
</tr>
</tbody>
</table>

Table 1
Sources of Variance on Student Achievement (modified from Hattie, 2003, p.4)
Another meta-analysis of 38 independent studies which involved over 300 teachers identified nine categories of instructional strategies which have the highest probability of improving student achievement (Haystead & Marzano, 2009). The findings revealed that, on average, when particular instructional strategies were used, there was a 16% learning gain between students’ pre-test and post-test scores. Table 2 identifies the nine instructional strategies. Based on the findings, Marzano reported that all nine categories of instructional strategies have mean effect sizes ranging from .59 to 1.61. Translating the effect sizes into percentile gains, Marzano reported gains ranging from +22 to +45, respectively. Of considerable interest to this study is that four of the “notable nine” strategies (summarising, cooperative learning, setting objectives and providing feedback) (Dean et al., 2012) each align with the principles and characteristics of the social constructivist PBL teaching method (Hmelo-Silver & Barrows, 2006; Savery & Duffy, 1995). This is an important point which will be discussed in a later section as a rationale for using closed loop PBL as the intervention with the treatment group in this study.

Table 2
Categories of Instructional Strategies that affect Student Achievement (Marzano, 2001, p. 7)

<table>
<thead>
<tr>
<th>Category</th>
<th>Mean Effect Size</th>
<th>Percentile Gain</th>
<th>No. of Effect Sizes</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identifying similarities and differences</td>
<td>1.61</td>
<td>45</td>
<td>31</td>
<td>.31</td>
</tr>
<tr>
<td>Summarising and note taking</td>
<td>1.00</td>
<td>34</td>
<td>179</td>
<td>.50</td>
</tr>
<tr>
<td>Reinforcing effort and providing recognition</td>
<td>.80</td>
<td>29</td>
<td>21</td>
<td>.35</td>
</tr>
<tr>
<td>Homework and practice</td>
<td>.77</td>
<td>28</td>
<td>134</td>
<td>.36</td>
</tr>
<tr>
<td>Non-linguistic recommendations</td>
<td>.75</td>
<td>27</td>
<td>246</td>
<td>.40</td>
</tr>
<tr>
<td>Cooperative learning</td>
<td>.73</td>
<td>27</td>
<td>122</td>
<td>.40</td>
</tr>
<tr>
<td>Setting objectives and providing feedback</td>
<td>.61</td>
<td>23</td>
<td>408</td>
<td>.28</td>
</tr>
<tr>
<td>Generating and testing hypotheses</td>
<td>.61</td>
<td>23</td>
<td>63</td>
<td>.79</td>
</tr>
<tr>
<td>Cues, questions, and advance organisers</td>
<td>.59</td>
<td>22</td>
<td>1,251</td>
<td>.26</td>
</tr>
</tbody>
</table>
Other attributes of effective teachers were identified in Shulman’s (1986) seminal work. These include possessing content knowledge specific to the subject being taught and the ability to organise and deliver that content knowledge to their students in an age appropriate, comprehensive manner using their pedagogical knowledge. When used in unison, content knowledge and pedagogical knowledge form the domain of pedagogical content knowledge (PCK); with PCK arguably the most influential on student achievement (Ball et al., 2008; Shulman, 1986). This unique form of teacher professional knowledge is described as recognising how to organise curriculum, content, pedagogy, and knowledge of students’ understanding in a form which can be used for decision-making in the classroom in specific situations (Weizman et al., 2008).

Since pre-service teachers’ mathematics PCK and their ability to enact their PCK will both be measured in this study, establishing the critical elements which transform into PCK is essential if a valid measure of the construct is to be achieved (Hill et al., 2008). Therefore, it is warranted that PCK’s conceptual formation and structure be explored. The research should begin by “taking care to elaborate the theoretical or empirical basis for the construct, delineate the boundaries of the construct, and specify how it relates to other similar constructs” (2008, p. 378). The next section reviews the literature and unpacks content knowledge and pedagogical knowledge in terms of their relationship to PCK. The domain of PCK will be examined alongside the researchers who progressed the field of study beginning with Shulman (1986). As a result of the literature reviewed, the researcher’s model for representing and then measuring mathematics PCK for use in this study is presented.

### 2.3 Content Knowledge

According to Shulman (1986), content knowledge refers to the amount and organisation of concepts, skills, strategies, and subject matter related to the content being taught that a teacher possesses. A year later, he built upon his definition identifying content knowledge as, “the knowledge, understanding, skill, and disposition that are to be learned by school children” (Shulman, 1987, p. 8). Basically, Shulman inferred teachers should possess not only a depth of understanding with respect to the discipline area, but also be able to clarify alternative explanations of the same concept or principle. They should be capable of explaining why a particular proposition seems warranted, or conversely, why that
proposition breaks the rules. To clarify these ideas, consider the following example. The context is that of a teacher following a primary mathematics curriculum designed to be taught as a set of facts and algorithms to be memorised. The teacher is guiding a revision lesson about negative and positive numbers and then asks the students to calculate negative 4 by negative 2. A student raises her hand and correctly answers “positive 8.” The student then asks, “I know what the rule is, but can you tell me why a negative by a negative is positive?” In this situation, the teacher should be able to demonstrate a multi-structural understanding of the principle, and provide an explanation or deliver a conceptual representation, possibly using concrete materials. The underlying belief being that a teacher of mathematics must not only possess the ability to answer a student’s question and know that it is so, “the teacher must further understand why it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in its justification can be weakened and even denied” (Shulman, 1986, p. 9). It is a distinction between knowing mathematics content and using it in the context of teaching mathematics (Chick, 2003).

The distinction between teachers possessing content knowledge and using it in ways which make it comprehensible to students was progressed by Ball (2000). Ball used an example by which students are asked to, “write down a string of 8s. Insert some plus signs at various places so that the resulting sum is 1,000” (2000, p. 242). In this situation, teachers should be able to address the problem’s worthiness and reach considerations such as: Is this assessment task appropriate for my students in terms of the learning objective? Are there other important concepts or processes involved in the problem? Are the solutions provided by the students correct? If the problem is too easy, how can it be adapted to make it more challenging? Ball suggests this type of knowledge, reasoning and planning reveals how much one core task involves intertwining content knowledge and pedagogy in teaching. It requires possessing a deep understanding of the content, but also significant mathematical reasoning in the context of teaching and how, as a result, this knowledge base can affect student learning.

Hill, Rowan and Ball (2005) and Ball, Thames and Phelps (2008) continued investigating what knowledge teachers of mathematics need to carry out their work. Their studies led to the conclusion that how teachers hold mathematical knowledge may be as important to their effectiveness as how much they hold. Hill et al. (2005) found that teachers’
mathematical content knowledge for teaching positively predicted gains in mathematical achievement in primary school students. To clarify the distinction, consider the work of chefs who need to be able to double fractions when following a recipe which requires twice the amount of ingredients the recipe calls for. However, they are not required to explain the reason why when you add $1/3$ and $1/3$ you do not add the denominators. Based on that premise, it is possible for a teacher to possess content knowledge but lack the content knowledge for teaching that enables them to come up with mathematical representations that clarify meaning and guide thinking. Ball et al. (2008) found two discernible subdomains of content knowledge within mathematics PCK; common content knowledge and specialised content knowledge. This multi-dimensional view of teachers’ content knowledge in relation to their PCK suggests that: (a) together, they transform into “topic-specific knowledge for teaching a particular subject” (Abell, 2008, p. 1413) and (b) teachers’ content knowledge and their knowledge for teaching appear to interact in determining teacher effectiveness (Darling-Hammond, 2006; Hill et al., 2008).

These are important distinctions when characterising PCK because effective instruction begins with possessing both the content knowledge and the ability to contextually teach it (Hill et al., 2008). This researcher adopts the same view. Both influence, and are interconnected with, PCK (Buschang, Chung, Delacruz, & Baker, 2012; Chick & Beswick, 2013; Friedrichsen et al., 2009; Kleickmann et al., 2013). Instruction then progresses from that point using pedagogical knowledge (Shulman, 1986, 1987), another critical attribute of a teacher’s knowledge base. Generally, this pedagogical and organisational teacher skill is used to effectively deliver content in a well-managed classroom. Again following a chronological order, beginning with Lee Shulman’s 1986 work, the following section reviews the conceptualisations of pedagogical knowledge and its status as a critical element to a teacher’s PCK.

2.4 Pedagogical Knowledge
Pedagogical knowledge “goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching” (Shulman, 1986, p. 9). A year later Shulman modified his conceptualisation of pedagogical knowledge to those “broad principles and strategies of classroom management and organisation that appear to
transcend subject matter” (Shulman, 1987, p. 8), and the organisation and delivery of the content knowledge, during instruction, in ways that allow it to be developed by students.

Conceptualisations of pedagogical knowledge by other researchers followed with several elements consistent across descriptions. For instance, pedagogical knowledge was conceptualised as encompassing “knowledge of theories of learning and general principles of instruction, an understanding of the various philosophies of education, general knowledge about learners, and knowledge of the principles and techniques of classroom management” (Grossman & Richert, 1988, p. 54). Building on the work of Grossman and Richert (1988), and Grossman (1995), Magnusson, Krajcik, and Borko (1999), conceptualised pedagogical knowledge with four main components (Figure 1).

![Figure 1: Facets of Pedagogical Knowledge (modified from Magnusson et al., 1999, p. 98)](image)

In the same year, Carlsen (1999), structured pedagogical knowledge into three components namely, learners and learning, classroom management, and general curriculum and instruction. In summary, each of these conceptualisations constituted a general agreement on the basic nature of pedagogical knowledge. In each description, pedagogical knowledge is generally comprised of a combination of (a) classroom management, (b) knowledge of teaching practice and (c) knowledge of the learner and learning. In terms of classroom management, teachers manage classrooms effectively through the ability to address more than one classroom event at a time or by demonstrating withitness in identifying and resolving problems in a timely manner, and by setting clear expectations for behaviour, academic work standards and classroom procedures (Morine-Dershimer & Kent, 1999). Knowledge of teaching practice and knowledge of the learner and learning was identified as knowledge of how to represent ideas to students that
bridges their prior and current knowledge and the content they are to learn (Ball & Wilson, 1990).

Following the work on conceptualising and describing pedagogical knowledge this century, pedagogical knowledge continued to be mentioned mostly as references to Shulman’s (1986, 1987) work by researchers as a prelude to their studies. Eventually, while not enduring a similar amount of scrutiny and debate as the domain of content knowledge, a comparable development in the province of pedagogical knowledge did emerge. König, Blömeke, Paine, Schmidt, and Hsieh (2011) conceptualised pedagogical knowledge into four dimensions in the context of pre-service teachers who are preparing to graduate. Firstly, these pre-service teachers should be able to prepare, structure and evaluate lessons. Secondly, they should be able to motivate and support students as well as manage the classroom. Thirdly, the pre-service teachers should be able to deal with mixed ability learning groups in the classroom. Lastly, they should be able to effectively assess students. The first three of these dimensions align with the previously presented consensus. Assessment was added “because assessing students is an essential teacher task and particularly relevant with respect to student achievement” (2011, p. 190).

Synthesising the literature presented, pedagogical knowledge is considered methods of classroom management as well as teaching practice which: (a) enables teachers to prepare, structure and evaluate lessons; (b) design differentiated instruction for heterogeneous classroom groups; (c) motivate and support students and (d) assess students. Encompassing such teacher attributes, pedagogical knowledge has been accepted in the literature and educational documents as a core element of PCK (Australian Institute for Teaching and School Leadership [AITSL], 2014; Department for Education, 2012; Grossman, 1995; König et al., 2011; National Commission on Teaching and America’s Future, 2010; Tatto et al., 2008).

The Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto et al., 2008) is of interest to this study because the TEDS-M test assesses pre-service teachers’ PCK and the ability to enact their PCK. Additionally, the test assesses pedagogical knowledge as its own domain and as a component of PCK. The TEDS-M project is an international comparative study which examined the level and depth of the mathematics and related teaching knowledge attained by pre-service teachers from the 18 countries
who participated in the testing. The items of the TEDS-M contribute to one of two dimensions, MCK or mathematics PCK (Beswick & Goos, 2012). It should be noted that three of the participating countries, the United States, Germany and Taiwan participated in the option for measuring pre-service teachers’ general pedagogical knowledge (König et al., 2011). The decision to measure the constructs in this manner was based on the need to categorise, using cross-national comparisons of initial teacher education programs, what knowledge for mathematics teaching is required of pre-service teachers once they are considered ‘ready to teach’. The judgement reached by the international members of the TEDS-M project was that MCK be categorised as part of general education and mathematics PCK and pedagogical knowledge categorised together as part of professional training. As a result, most pedagogical knowledge testing was conducted in conjunction with testing mathematics PCK. Hence, many test items that were considered mathematics PCK were set in a pedagogical context (teaching and learning). Essentially, the TEDS-M test measured pre-service teachers’ mathematics pedagogical knowledge as a component of teachers’ PCK (Döhrmann et al., 2012). Likewise, many MCK items have a pedagogical aspect set in a teaching context since the content must be communicated by the teacher to make it comprehensible to students. Such a structure in the TEDS-M test questions indicates that if mathematics PCK is to be measured accurately in this study, the survey items used to measure PCK and the ability to enact PCK would need to consist of content knowledge questions written in a pedagogical context and pedagogical knowledge questions written in a content context (Chick, Baker, Pham, & Cheng, 2006). This integration of content knowledge, pedagogical knowledge and contextual knowledge which transforms to create PCK forms the foundation of this researcher’s theoretical framework for PCK, the context for the following section and also the basis for developing the questions for assessing PCK in this study.

### 2.5 Pedagogical Content Knowledge

Shulman (1986) uses Shaw’s (1903) vilifying quote “He who can, does. He who cannot, teaches” to point out accusations of incompetence within the teaching workforce. Shulman explains the claim suggests teachers lack the professional tools necessary to be an effective teacher. One of the professional tools of teachers he identified as essential for effective teaching is PCK. In an attempt to describe the interconnection between subject matter knowledge and pedagogy, Shulman (1986) defined PCK as a special
domain of teacher knowledge formed from the amalgamation of content and teachers’ special form of professional understanding which they use to formulate concepts and content into representations which make it comprehensible to students. Shulman (1987) also characterised PCK as one of seven categories of teachers’ knowledge base. The other six categories were: content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational ends, purposes and values. However, he considered PCK to be of special interest because:

It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (Shulman, 1987, p. 8)

Shulman called for the need to explore PCK as the inherent interconnection between content knowledge and pedagogical knowledge. In the research that followed, PCK was sometimes conceptualised as being separate from content knowledge, but mostly it was recognised as being either an integration of or transformation from teachers’ content knowledge and their pedagogical knowledge (Cochran, King, & DeRuiter, 1991; Gess-Newsome, 1999; Grossman, 1990; Marks, 1990). For example, Grossman (1990) blended the traditionally separated knowledge bases of content and pedagogy, identifying PCK as a unique domain of teacher knowledge. Marks (1990) indicated that PCK, by its nature, contains elements of both content knowledge and pedagogical knowledge. In such a structure, the teacher must first examine the content for its composition and significance requiring a process of interpretation. The interpretations are then transformed as necessary to make it comprehensible and compelling in a particular context (to a particular group of learners in a particular subject area) by adopting pedagogically useful representations of the content. Pedagogical knowledge can be seen in the form of teachers’ use of questioning strategies, knowledge of assessment or their knowledge of students’ learning processes. Consider a teacher who must determine the correctness of different answers given by students when demonstrating 100% using a geoboard (a board covered with a square grid with pins at the corners of the squares to which students attach rubber bands to create 2D shapes). Firstly, the teacher must not only determine any errors in those responses, but also the nature of those errors. Next, the teacher would have to strategically choose the type and timing of responses to use with the students based on
his or her knowledge of them. Marks identified this step as the “appropriate instantiation of a broadly applicable idea in a particular context” (1990, p. 7). Ultimately, Marks (1990, 1991) characterised PCK as a synthesis of a balance of varying degrees of content knowledge and pedagogical knowledge which are dependent on contextual knowledge demands placed on the teacher such as the grade level, the objective and the level of student ability.

Synthesising the literature regarding PCK as being either an integration of or transformation from teachers’ content knowledge and their pedagogical knowledge, PCK was categorised in two distinct models (Figure 2) - the Integrative model (circular Venn diagram) and the Transformative model (Gess-Newsome, 1999). Under the Integrative model PCK does not exist as a separate category of knowledge, but encompasses the intersection of content knowledge, pedagogical knowledge and contextual knowledge that are combined by the teacher during the course of instruction. In the Transformative model, these three knowledge categories are synthesised and transformed into PCK to form a new knowledge category.

Figure 2: Two Models of Teacher Knowledge (Gess-Newsome, 1999, p. 12)  
= PCK - knowledge needed for classroom teaching

Gess-Newsome (1999) used an analogy from chemistry and the difference between mixtures and compounds to illustrate the difference between the two models. When two substances are mixed (integrated), the parent ingredients retain their physical and chemical properties and can be separated by physical means. The integrative model can be likened to a teacher who draws upon the three categories of knowledge during his/her
classroom practice, but during that process the three categories of knowledge do not lose their distinct characteristics. In contrast, the parent ingredients in a compound do lose their chemical properties. Their initial properties can no longer be detected because they are transformed into a new substance. Likewise, in the Transformative model, content knowledge, pedagogical knowledge and contextual knowledge “are inextricably combined into a new form of knowledge” (Gess-Newsome, 1999, p. 11), “such that it is a transformation of the bodies of knowledge arising from their interaction” (Lee & Tan, 2010, p. 6). Magnusson et al. (1999) conceptualised PCK similarly. However, as Figure 3 illustrates, their conceptualisation shows the influences on PCK are not only the domains of content knowledge, pedagogical knowledge and contextual knowledge, but each is intertwined with teachers’ beliefs.

![Figure 3: A Model of Relationships among the Domains of Teacher Knowledge (Magnusson et al., 1999, p. 98)]
Research that followed endorses Magnusson et al.’s (1999) conceptualisation of PCK (Ball et al., 2008; Beswick & Callingham, 2011; Hill et al., 2005; Tato et al., 2008; Verloop, Van Driel, & Meijer, 2001; Zeidler, 2002). In support of the Transformative model, Zeidler (2002) concluded that it does not take a stretch of the imagination to accept that the ability to transform complex ideas into concepts that students can grasp requires a level of both content knowledge and pedagogical knowledge. However, Zeidler (2002) did accept that it is still necessary to establish a common reference point. In her view, content knowledge refers to the teacher’s quantity, quality, and organisation of information, conceptualisations, and underlying constructs in their area of expertise. Pedagogical knowledge relates to a teacher’s general knowledge of instructional pedagogy such as classroom management, pacing, questioning strategies, handling of routines and transitions. PCK pertains to a teacher’s ability to convey the underlying content and its constructs, in their area of expertise, in a manner that makes it comprehensible. Zeidler (2002) also emphasised that the influence of teachers’ instructional belief systems drives their decisions.

The TEDS-M study is of interest again because the developers determined that attitudes and beliefs are factors affecting teacher education programs (Tatto et al., 2008). Specifically, the framework comprises both professional knowledge and affective-motivational characteristics (beliefs, professional motivation, and self-regulation) as criteria for determining teachers’ effectiveness (Döhrmann et al., 2012). Beliefs are so closely intertwined in the context of practice that the constructs should be included in any conceptualisation of teacher knowledge (Beswick, Callingham, & Watson, 2012; Beswick & Goos, 2012; Tato et al., 2011). Other evidence in the literature also indicates that teachers’ beliefs may account for individual differences in their effectiveness to teach (Gibson & Dembo, 1984; Sandholtz & Ringstaff, 2014; Tschannen-Moran & Hoy, 2001). The literature specific to this study suggests pre-service teachers’ teaching beliefs may also influence the development of their PCK (Briley, 2012; Huinker & Madison, 1997; Pendergast, Garvis, & Keogh, 2011; Tato et al., 2011; van Dinther, Dochy, & Segers, 2011). This researcher accepts the view that teachers’ instructional belief systems drive their decisions, and are therefore held to be important for guiding the enacting of teachers’ mathematics PCK (Döhrmann et al., 2012; Zeidler, 2002). This accepted conceptualisation indicates that the domains of content knowledge and pedagogical
knowledge, when used in different contexts, each influence PCK and cannot be separated from teachers’ beliefs (Fennema & Franke, 1992).

Much of the research on beliefs and motivation stems from Bandura’s self-efficacy (1977) and social cognitive (1986) theory. Bandura’s work has provided a framework for studying self-efficacy for teaching and teaching outcome expectancy (Coladarci, 1992). Both are constructs identified as having an impact on pre-service teachers’ effectiveness as a result of their university coursework and practicum placements (for example, Huinker & Madison, 1997; Moody & DuCloux, 2015; Swars, Smith, Smith, & Hart, 2009). Other studies link pre-service teachers’ levels of self-efficacy for teaching with teaching effectiveness when instructed using PBL (Dunlap, 2005; Schmude, Serow, & Tobias, 2011; Shin et al., 2010). Therefore, prior to ascertaining a theoretical framework for PCK to be used in this PBL study, Bandura’s two classes of expectations, efficacy expectations and outcome expectations (Bandura, 1977) and the literature on efficacy and outcome expectations of teachers will be explored.

### 2.6 Self-efficacy and Outcome Expectancy

Although self-efficacy and outcome expectancy are fundamentally linked, Bandura (1977) separated the two constructs when he conceptualised his self-efficacy theory. “An outcome expectation is defined as a person’s estimate that a given behaviour will lead to certain outcomes. An efficacy expectation is the conviction that one can successfully execute the behaviour required to produce the outcomes” (Bandura, 1977, p. 193). Bandura’s argument was that individuals can believe that certain actions or behaviours will lead to certain outcomes. However, if they do not have faith in their own abilities to perform the necessary activities to produce those outcomes, they will not persist or even possess the motivation to execute the behaviours (Bandura, 1994, 2006b). Figure 4 illustrates the link and distinction.

*Figure 4: Distinction between Self-efficacy and Outcome Expectations (Bandura, 1977, p. 193)*
The social and cognitive elements of Bandura’s social cognitive theory work in concert with his self-efficacy theory and together they collectively influence human action, adaption and change (Bandura, 1989, 2001). “In other words, the social part acknowledges the social origins of much of human thought and action…whereas the cognitive portion recognises the influential contribution of thought processes to human motivation, attitudes, and action” (Stajkovic & Luthans, 1998, p. 63). Bandura posits that individuals strengthen, or lose, their sense of efficacy based on information they receive cognitively from four sources (a) their performance accomplishments, (b) vicarious experiences, (c) verbal persuasions they receive and (d) physiological states (Bandura, 1977, 1986). Bandura hypothesised these four influences on personal efficacy determine whether a particular behaviour or task will be initiated, how much effort will be expended on that behaviour or task, and how long the effort will be sustained. In essence, successful achievements, witnessing others perform necessarily threatening activities (vicarious experiences) with positive consequences, and motivational encouragement raise efficacy levels. Individuals also cognitively obtain efficacy information from physiological indicators, for example, increased heart rate or sweating caused from nervousness. These types of physical symptoms might be self-construed as skills that are lacking (Schunk, 1991) and therefore may increase avoidance behaviour (Bandura, 1977). Self-efficacy is not the only influence on behaviour. “Outcome expectations… are important because individuals are not motivated to act in ways they believe will result in negative outcomes” (Schunk, 1991, p. 209). They will give up trying, or not even attempt a task, “because they expect their behaviour to have no effect on an unresponsive environment…” (Bandura, 1977, p. 205).

### 2.6.1 Self-efficacy and Outcome Expectancy for Teaching

Bandura’s social learning theory combined with his social cognitive theory has also been applied to education (Sandholtz & Ringstaff, 2014; Tschannen-Moran & Hoy, 2001; van Dinther et al., 2011). In terms of teaching and teacher education, “personal teaching [self] efficacy is defined as a belief in one’s ability to teach effectively and teaching outcome expectancy as the belief that effective teaching will have a positive effect on student learning” (Enochs et al., 2000, p. 195). In practising and pre-service teachers, a strong sense of self-efficacy for teaching indicates confidence in teaching abilities which may account for individual differences in teacher effectiveness, and subsequent differences in

Gibson and Dembo (1984) reported that practising teachers who possess a high perception of self-efficacy for teaching devoted more instructional time to academic learning and provided their students with more sustained assistance, believing that student learning can be positively influenced. Woolfolk and Hoy (1990) found that teachers with a high sense of instructional efficacy are more likely to utilise self-directed learning techniques with their students. Relevant to this study, the development of effective self-directed learning skills is also one of the core goals of PBL (Barrows, 1986; Hmelo-Silver & Barrows, 2015; Savery, 2015).

Several studies report that higher education students’ self-efficacy can be positively affected in different capacities by different factors during university coursework (Huinker & Madison, 1997; Moody & DuCloux, 2015; Pendergast et al., 2011; Tatoo et al., 2008; van Dinther et al., 2011). Huinker and Madison’s (1997) study investigated whether science and mathematics education methods subjects would influence pre-service teachers’ self-efficacy and teaching outcome expectancy in regards to teaching these two subjects. Their hypothesis is that, “the more positive the impact on pre-service elementary teachers’ efficacy during their teacher preparation programs, the more likely it is that these individuals will engage in effective teaching behaviours in the future” (Huinker & Madison, 1997, p. 109). For most of the semester the students met for three hours on Monday for science and three hours on Tuesday for mathematics, and instruction was guided by a constructivist philosophy which encouraged collaboration among the pre-service teachers. The results of the study indicated pre-service teachers’ degrees of self-efficacy for teaching and teaching outcome expectancy were significantly impacted by science and mathematics education subjects delivered using a constructivist approach. Huinker and Madison concluded from their study that “improving science and mathematics teaching efficacy will ultimately improve instruction and student achievement…” (Huinker & Madison, 1997, p. 109). Applying Huinker and Madison’s findings to this study, when a pre-service teacher believes a specific teaching behaviour will result in a desired academic outcome, he or she is likely to believe that possessing sound PCK will enhance their ability to increase the academic achievement of their students (Enochs et al., 2000; Huinker & Madison, 1997).
A more recent review of 39 empirical studies measured factors shown to affect the self-efficacy of higher education students (van Dinther et al., 2011). These 39 studies were divided into three categories. Five measured self-efficacy at one moment in time during the experiment. Twelve studies were categorised based on whether the researchers used an intervention program with underlying theories different from Bandura’s self-efficacy theory. The other 22 were classified by whether the researchers investigated the effects of intervention programs on students’ self-efficacy, whether the studies did or did not use a control group and which were based on Bandura’s social cognitive theory. The findings revealed that 85% of the intervention programs which were based on Bandura’s social cognitive theory, and used a control group, were effective in raising self-efficacy. Of further interest to this study is that mastery experiences from performance accomplishments were stated as the most powerful source in creating a strong sense of efficacy. “With regard to this source nearly every study stresses the relevance of providing students with practical experiences, i.e. students performing a task while applying knowledge and skills within demanding situations” (van Dinther et al., 2011, p. 104). In the context of this study, the aim of teacher education is to graduate pre-service teachers who are able to demonstrate their ability to enact their mathematics PCK, which is the objective of the mathematics subject but also of PBL. Taking into account PBL’s characteristics of using practical experiences when designing the PBL problems which drive students’ learning, consideration should be given to the notion that using a PBL instructional approach may also positively impact a teachers’ self-efficacy for teaching and teaching outcome expectancy. Hence, self-efficacy and teaching outcome expectancy are considered additional dependent variables which may affect the dependent variable PCK and should be controlled or considered factors to be measured in an investigation of the impact of PBL on pre-service teachers’ mathematics PCK.

In such a PBL study, pre-service teachers, designing solutions as performance tasks to the type of problems they will encounter in their future careers, undertake the role of the teacher. These roles help lend authenticity to the tasks because the pre-service teachers must not only collaborate to solve the tasks, but they then deliver the solution in the form of a ‘lesson’ in front of their peers in a simulated classroom context. These real-world activities allow the pre-service teachers to practise their future profession while participating in a community of practice, potentially raising their confidence for teaching
those specific lessons (Albanese & Mitchell, 1993; Dunlap, 2005). In terms of vicarious experiences, a PBL intervention can provide pre-service teachers with the opportunities to strengthen their sense of teaching efficacy by observing how their colleagues design and deliver lessons in a safe and supportive environment. According to Bandura’s theory, engaging individuals in these types of vicarious experiences develops a ‘can do’ attitude and that they too can meet the challenge of the task if they intensify and/or persist in their efforts (Bandura, 1977). As Bandura has suggested, the most effective way of creating a strong sense of efficacy is through performance accomplishments, vicarious experiences and, verbal encouragement or persuasion (Bandura, 1994), such as those which a well-constructed PBL intervention can provide (Dunlap, 2005).

In review, Bandura’s social-cognitive, self-efficacy theory suggests that a person’s level of self-efficacy determines how much effort will be expended on a task and how long the effort will be sustained (Bandura, 1977). When individuals are provided with: (a) the ability to succeed at solving problems of professional value; (b) opportunities to witness how their peers approach and successfully solve problems; (c) verbal encouragement that they possess the capability to successfully perform given tasks; and (d) reinforcement of their outcome expectations from their performances, self-efficacy is hypothesised to impact on the choices and direction of most behaviours (Bandura, 1989; Schunk, 1991). Similarly, if pre-service teachers, learning using the PBL method, experience mastery from delivering solutions to real-world situations they will encounter in their teaching career, and experience positive feedback, their self-efficacy for teaching and teaching outcome expectancy may be enhanced (Bandura, 2006b; Dunlap, 2005; Schmude et al., 2011). It is therefore posited by this researcher that pre-service teachers’ self-efficacy for teaching and their teaching outcome expectancy may be positively impacted by PBL experiences within their education subjects. Furthermore, it is hypothesised that pre-service teachers with high self-efficacy and outcome expectancy for teaching would likely believe that possessing mathematics PCK is useful and will impact positively on their future students’ mathematics academic achievement; which, in turn may motivate them to pursue that knowledge (Biggs, 1989; Huinker & Madison, 1997). As a result, self-efficacy for teaching mathematics and mathematics teaching outcome expectancy will also be investigated in this study, and how these two constructs will be measured is described in detail in the methodology chapter.
The descriptions presented of PCK by Shulman (1986, 1987), Grossman (1990), Marks (1990) and Gess-Newsome (1999) provided the initial sources for this researcher’s conceptualisation of PCK as its own domain - a transformation from teachers’ content knowledge and their pedagogical knowledge. However, holding the view that teachers’ beliefs are intertwined in the context of practice, Magnusson et al.’s (1999) framework of PCK (Figure 3), shown as transformed knowledge for teaching drawn from content knowledge, pedagogical knowledge and knowledge and beliefs about context, will be used as the theoretical framework for PCK in this study. Having identified the theoretical framework for PCK, the next section of the chapter examines the literature which led to the process of developing the survey questions used in this study for measuring pre-service teachers’ mathematics PCK and their ability to enact their PCK.

2.7 Assessing PCK and the Ability to Enact PCK

Most relevant to this study is the TEDS-M project. It was the first cross-national study to bring together international experts in mathematics education, research, curriculum, instruction and assessment to develop assessment items for measuring pre-service teachers’ mathematics PCK and the enacting of PCK (Tatto et al., 2008). Therefore, the test items from the TEDS-M will be examined for use as a model for the items to be used for measuring mathematics PCK and enacting PCK in this study.

One of the first challenges facing the TEDS-M project developers was defining PCK in an international context. This cross-national effort to develop a conceptual framework for PCK for the TEDS-M highlights the importance that PCK holds for determining what experts in the field consider essential elements of effective teaching and how to measure the different sub-domains of mathematics PCK (Tatto et al., 2012). To tackle this challenge, expert representatives from the participating countries met to develop and approve a classification for mathematics PCK. The result produced two internationally accepted sub-domains of PCK: (a) *curricular knowledge and knowledge of planning for mathematics teaching and learning* and (b) *enacting mathematics for teaching and learning* (Tatto et al., 2008, p. 40). Figure 5 illustrates one of the multiple-choice test items dedicated to the sub-domain of enacting mathematics PCK (Australian Council for Educational Research for the TEDS-M International Study Centre, 2011).
[Amy] is building a sequence of geometric figures with toothpicks by following the pattern shown below. Each new figure has one extra triangle. Variable \( t \) denotes the position of a figure in the sequence.

\[
\begin{align*}
\text{\( t = 1 \)} & \quad \text{\( t = 2 \)} & \quad \text{\( t = 3 \)} \\
\end{align*}
\]

In finding a mathematical description of the pattern, [Amy] explains her thinking by saying:

I use three sticks for each triangle.

Then I see that I am counting one stick twice for each triangle, except the last one, so I have to remove those.

Variable \( n \) represents the total number of toothpicks used in a figure. Which of the equations below best represent [Amy’s] statement in algebraic notation?

\[
\begin{align*}
\text{A.} & \quad n = 2t + 1 \\
\text{B.} & \quad n = 2(t + 1) - 1 \\
\text{C.} & \quad n = 3t - (t - 1) \\
\end{align*}
\]

*Figure 5: TEDS-M Example Item Measuring Mathematics PCK in the Sub-domain of ‘Enacting’ (ACER 2011, p. 5)*

Instruments from other studies using different formats for measuring pre-service teachers’ mathematics PCK for teaching and learning have emerged, such as short answer questionnaires (Cheang, Yeo, Chan, & Lim-Teo, 2007), observations and interviews (Chick, 2007; McCray & Chen, 2012) and testing pre-service teachers’ content knowledge and PCK using multiple-choice formats (Callingham et al., 2011). The above studies, as well as the TEDS-M, were examined by the researcher and several have laid the foundation for this study’s research design which aims to investigate both pre-service teachers’ PCK and their ability to enact their PCK. These studies have also been the source of specific items for measuring mathematics PCK (Callingham et al., 2011;
Summarising the chapter thus far, the majority of researchers who have progressed Shulman’s (1986) work agree that PCK is the transformation from teachers’ content knowledge and pedagogical knowledge, with the ability to use content knowledge in a pedagogical context and pedagogical knowledge in a content context (Chick et al., 2006; Grossman, 1990; Loughran, Berry, & Mulhall, 2012).

The combination of the rich knowledge of pedagogy and content together, each shaping and interacting with the other so that what is taught, and how it is constructed is purposefully created to ensure that particular content is better understood by students in a given context, because of the way the teaching has been organised, planned, analysed and presented. (Loughran et al., 2012, pp. 7-8)

Effective teaching, through sound PCK, requires knowing and understanding the subject matter, knowing students as learners and skilfully being able to choose from, and use, a variety of evidence-based pedagogical strategies (National Council of Teachers of Mathematics, 2014). Thus, in the context of this study’s main aim (to identify evidence-based pedagogical practices which are more effective in developing the mathematics PCK of pre-service teachers and their ability to enact their PCK) it was important to conceptualise PCK as an initial step in the process of accurately measuring the construct.

Possessing the ability to enact PCK is also indicative of effective teaching and student academic gains (Chick et al., 2006; Hattie, 2009; Hill et al., 2008; Tattoo et al., 2008; Teacher Education Ministerial Advisory Group, 2014). In the context of this study, enacting PCK was identified as a process demonstrated when a teacher, as a result of analysing and interpreting students’ solutions or arguments, organises and represents content knowledge in a wide array of methods which allow it to be accurately developed by students within the classroom (Döhrmann et al., 2012). This includes providing appropriate feedback and the ability to guide classroom discourse.

In light of the research problem (that not all pre-service teachers are graduating with the mathematics PCK necessary to teach effectively) the literature presented conveys the potential and significance of the present study. As a result, this thesis adopts the following positions from the literature:
• Effective teachers possess sound PCK and the ability to enact their PCK (Tatto et al., 2008; Teacher Education Ministerial Advisory Group, 2014).

• As an attribute of effective teaching, teachers should possess the knowledge of the strategies most likely to be successful in reorganising the understanding of their students (Shulman, 1986).

• In order to teach mathematics effectively teachers must combine their multi-structural or relational understanding of mathematics with the recognised knowledge of their students as learners, and then adeptly select teaching strategies for the delivery of their lessons (Chick et al., 2006).

• Self-efficacy and outcome expectancy for teaching drives academic motivation and instructional decisions (Gibson & Dembo, 1984; Huinker & Madison, 1997; Pendergast et al., 2011; Tatto et al., 2008). Therefore, teachers who believe that enacting sound PCK will impact positively on their students’ academic achievement may motivate them to pursue that knowledge.

• Teachers can positively impact student achievement using evidence-based instructional strategies (Dean et al., 2012; Marzano, 2001), such as those which underpin PBL.

• PBL is considered as one of the 20 classroom teaching and learning practices which form the productive pedagogies framework which support effective teaching (Lingard et al., 2001). Amongst the other 19, seven are attributes of PBL.

• PBL is an effective pedagogical practice at the tertiary level to develop practical application and critical thinking skills compared to traditional instruction (Albanese & Dast, 2014; Leary, 2012; McPhee, 2002; Strobel & van Barneveld, 2009; Walker & Leary, 2009).

The following section examines the literature which links the goals and objectives of PBL with this study’s main aim. Specifically, closed loop PBL is explored as the pedagogy hypothesised in this study to best develop pre-service teachers’ mathematics PCK and their ability to enact their PCK compared to traditional teacher-led instruction.

2.8 Problem-based Learning

This section defines problem-based learning (PBL), explains its theoretical foundation, structure and processes and describes its origin. The section provides a synthesis of the
research findings of PBL’s impact on learning outcomes of tertiary students. Barrows’ (1986) taxonomy of PBL is then introduced. An explanation follows of which variation of Barrows’ PBL taxonomy is to be used to underpin the teaching intervention in this study because it is considered most effective for addressing the research problem. Next, a description of the specific elements which contribute to a PBL curriculum’s effectiveness is presented. Lastly, the criteria which should be considered when developing PBL type problems for students to solve in a PBL environment is described. The section concludes with a summary of the literature which provides the rationale for PBL’s use in the study’s research design.

PBL is considered consistent with the principles of effective instruction arising from constructivist, sociocultural and cognitive development foundations. It is thought of as one of the best exemplars of a social constructivist, student-centred learning approach (Hmelo-Silver & Barrows, 2006, 2015; Hmelo-Silver & Ederbach, 2012; Savery & Duffy, 1995). Social constructivism has developed from the educational theories of Dewey (Hickman, 2009), the cognitive constructivists Piaget (1977) and Bruner (1961), and Vygotsky’s sociocultural theory (Cole, John-Steiner, Scribner, & Souberman, 1978).

Dewey’s theories of experience and inquiry suggest that the teacher-student relationship in the ideal classroom should be structured similarly to the relationship between an apprentice and an expert. The apprentice learns to expand their capacities by solving problems through exploration and trial and error with guidance from the expert, rather than from an instructor who simply provides the knowledge to the learner (Garrison, Neubert, & Reich, 2012). As stated in his pedagogic creed, Dewey believed that the school must represent real life (Archambault, 1964). The ideal teacher-student relationship in a PBL classroom is strikingly similar to the expert-apprentice relationship. A PBL teaching approach provides students with real-world problems to solve in a self-directed learning environment with the teacher as facilitator, guiding the reasoning processes of the students only when necessary (McCaughan, 2015).

In terms of cognitive development, PBL is underpinned by the information processing theory, whereby prior knowledge is activated at the start of the PBL process (Hmelo-Silver & Ederbach, 2012). When provided with a complex, open-ended PBL problem to solve, cognitive processes are initiated when the students begin to identify what they
already know, and more importantly, what they do not know. The information processing principle also guides the self-directed learning process. It becomes the students’ responsibility to determine the fundamental essence of the problem and define the gaps in their knowledge and pursue and secure that knowledge. Constructivist, self-directed learning supports the process whereby information is provided to students in motivating and exciting ways that leads them to discover the knowledge for themselves (Bruner, 1961, 2006; Olsen, 2014). The overall goal is that cognitive development will be enhanced when students are required to actively engage in the learning process, which is built around solving real-world, meaningful problems.

Vygotsky agreed that cognitive development was a constructive process, but believed the construction of knowledge is essentially a social activity (Mercer, 2007). The foundation of Vygotsky’s sociocultural theory suggests the acquisition of knowledge develops as a result of using cultural tools such as artefacts, language and social interaction (Grandin, 2006; Hmelo-Silver & Ederbach, 2012). Thus, those cultural tools, which form each person’s realm of understanding, are the basis of an individual’s higher order processes and knowledge development (Grandin, 2006). The depth and extent of the knowledge developed is dependent on the culturally authentic resources and experiences made available to the learner and the type of language interaction with the people around the learner (Mercer, 2007). Vygotsky believed these learning experiences needed to occur socially and collaboratively within each student’s zone of proximal development (ZPD). Central to ZPD is the distinction between the learner’s problem-solving ability when working alone compared to the learner’s problem-solving ability when coached by a more experienced person (Daniels, 2002). The extent of learning that takes place is based on the tutelage the learner receives through scaffolding. Scaffolding is a form of guidance, task structuring and hints learners receive from teachers, peers, or other adults to move them to the next level of understanding without providing the solutions (Hmelo-Silver & Ederbach, 2012; Powell & Kalina, 2009). This form of guidance and task structuring is considered a key aspect of PBL because for students it represents a significant shift in learning since most are not necessarily skilled at problem solving (Jonassen, 2011). Jonassen suggests providing initial cognitive scaffolding which moves students from working in a teacher-led instructional environment, using well-structured problems as the impetus for learning, to a student-centred, PBL instructional approach which utilises more complex, open-ended problems. Once the groups are working successfully in a student-
centred environment, PBL calls for scaffolding in the form of skilled questioning of the students’ thought processes by the PBL facilitator. Socratic dialogue of this type serves to properly sequence the facilitator’s questions in a manner which will most efficiently enable students to solve a problem using their own thinking and intuitions (Garlikov, 2011; van der Linden & Renshaw, 2004). Furthermore, effective scaffolding of this type by the PBL facilitator guides the learners’ reasoning process; thus, reducing the potential that they will deviate from the intended learning outcomes while in pursuit of a solution to the task they are in the process of developing (Bruner, 1978).

To define the key components of PBL, students work together collaboratively in small groups to analyse, research and find solutions to complex, open-ended, real-world problems which have many potential solutions. The teacher facilitates the learning process by challenging the students’ thinking through asking key, higher order questions which probe deeply into what students know or do not know. The teacher, as the facilitator, has a responsibility to avoid transferring his or her own knowledge when guiding the students, instead, attempting to provoke thought and provide direction. Next, the students determine what they need to learn to solve the problem. This may require research, discussion and re-analysis of the problem. The resulting information the students assemble is analysed and then synthesised by the group into new coherent forms of understanding required to solve the set problems. The process is usually completed with a tangible solution to the problem in the form of a presentation (Barrows, 2002). As a social constructivist pedagogical approach, PBL is a “premier example of a student centred learning environment as students co-construct knowledge through productive discourse practices” (Hmelo-Silver & Barrows, 2015, p. 71).

2.8.1 The Effectiveness of PBL in Higher Education

Pioneered at McMaster University in the 1960s, problem-based learning was designed to prepare new doctors to think critically and solve complex medical problems. It was developed due to low enrolments and a dissatisfaction in the medical school’s use of a traditional instructor-led model of teaching (Barrows, 1986, 1994). PBL has since become regarded as a pedagogy which offers a great deal for other professional practices (Gijbels, Dochy, Van den Bossche, & Segers, 2005), including schools of education (De Simone, 2008; McPhee, 2002; Savery & Duffy, 1995; Strobel & van Barneveld, 2009).
Meta-analyses and syntheses of meta-analyses which report on PBL’s effectiveness in the field of clinical education exist (Albanese & Mitchell, 1993; Colliver, 2000; Newman, 2003; Strobel & van Barneveld, 2009; Vernon & Blake, 1993). For example, Strobel and van Barneveld’s (2009) meta-synthesis of eight meta-analyses found PBL produces mixed results depending on whether the studies measured student and faculty satisfaction, long-term or short-term knowledge assessment, performance or skill-based assessment, or the mixed knowledge and skill category. In the category of student and faculty satisfaction, the findings were favourable for PBL. Studies examining the effects of PBL on knowledge, specifically on the acquisition of short-term subject content knowledge returned mixed results, but tended to favour traditional instruction. However, knowledge assessment related to long-term knowledge retention (twelve weeks to two years) consistently favoured PBL. Additionally, the performance or skill-based assessment category which assessed the skills, understanding and application of medical knowledge favoured PBL. The performance assessment category corresponds with assessing the skill of enacting PCK, such as when a teacher interprets students’ misconceptions and then reorganises the topic using a wide array of methods which allow it to be appropriately developed by students. Lastly, the mixed knowledge and skill category, which required both knowledge and skill for performance, also favoured PBL.

Other meta-analyses have gone beyond medical education and included studies across disciplines reporting the effect sizes on identified factors considered to be in line with the main goals of PBL (Dochy, Segers, Van den Bossche, & Gijbels, 2003; Gijbels et al., 2005; Leary, 2012; Walker & Leary, 2009). Dochy, Segers, Van den Bossche, and Gijbels (2003) performed a meta-analysis which investigated the effects of PBL in relation to two categories of learning, acquisition of knowledge and the application of skills. The meta-analysis found that PBL had a negative effect in regards to the acquisition of knowledge ($d = -0.776$) compared to a traditional learning environment. Conversely, the findings indicate a robust positive effect size ($d = 0.658$) from PBL on the application of skills by the students. The researchers reported the results were statistically significant; thus, the students instructed using PBL were better at enacting their knowledge and skills. In teacher education studies, similar results were also found by Leary (2012) and Walker and Leary (2009). Both reviews indicated that PBL is shown to produce modest, positive statistical gains in understanding, thinking, problem solving and mental skills as compared to traditional lecture-based instruction. The reasons for the mixed findings from
the above reviews may be due to several factors which are relevant if PBL is to be considered as the teaching intervention used in this study to address the research problem.

Firstly, the inconclusiveness in the reviews may be due to pooling studies which did not define the specific type of PBL used in each study (Newman, 2003; Walker & Leary, 2009). Newman likens the analogy to “taking apples and oranges and averaging such measures as their weight, sizes, flavours and shelf lives” (2003, p. 29). In response, specific elements of Barrows’ (1986) PBL taxonomy will be presented in the following section allowing for the classification of the specific variations of PBL and identification of the specific variation used in this study.

Secondly, if students cannot accurately identify the learning objectives from the problem itself, their academic efforts during the PBL process may be inadvertently diverted from addressing the intended learning outcomes. Essentially, when the structure of the problem becomes the concern, the students’ acquisition of the intended knowledge could be degraded (Hung, 2011). Hence, following the description of Barrows’ (1986) taxonomy of PBL, the review of the literature will examine Jonassen’s (2000) typology of problems and describe the types and structures of the problems that would best be used in a PBL program (Jonassen & Hung, 2008).

### 2.8.2 Barrows’ (1986) Taxonomy of PBL

To suitably describe the features of PBL, Barrows’ (1986) taxonomy of problem-based learning allows for the understanding and appreciation of using real-world problems to direct the learning process. Barrows is considered “a pioneer in the development of problem-based learning and assessment in medical education” (Hmelo-Silver, 2011, p. 6). His often cited taxonomy is illustrated along a continuum in Figure 6.

![Figure 6: Taxonomy of Problem-based Learning (modified from Barrows, 1986, p. 483)](image-url)
Barrows developed his taxonomy to address the deviating structures of problem-based learning different professions were using based on the needs of their discipline, the educational method employed and the skills of the instructors. Barrows suggested that a PBL method has the potential to address four educational goals in medical education: (a) structuring of knowledge for use in clinical contexts, (b) developing an effective clinical reasoning process, (c) developing effective self-directed learning skills and (d) an increased motivation for learning. Barrows conceptualised that his PBL taxonomy provided an awareness of these variations and educational objectives “to help teachers choose a problem-based method most appropriate for their students” (Barrows, 1986, p. 481).

As shown at one end of the continuum, lecture-based cases are teacher directed and begin with information provided by the instructor during a lecture. Then cases, or snippets of cases, are presented to the students to provide relevance to the lecture material. Although some group work hypothesising and diagnosis may still be required with this method, no inquiry or case-building skills are needed.

Next, case-based lectures use essentially the same format as lecture-based cases except that students are provided with the case vignettes prior to the lecture. Following on the continuum is the case method approach where students are provided with an entire case to study and research. A class discussion follows which is directed by the students and facilitated by the teacher. It is at this stage along the continuum that a sense of student-directed learning is noted. In the modified case-based method, often used in medical schools, more of the students’ reasoning skills are challenged, but cueing and restricted inquiry prevent the full implementation of the reasoning process or self-directed learning. Implementing the ‘problem-based’ approach the teacher, as a facilitator, activates the students’ prior knowledge. The facilitator then presents the students with a real-world problem which allows for free inquiry and teacher-guided exploration and evaluation of the problem.

At the far right of the continuum is the closed loop (reiterative) problem-based approach. This variation of PBL is an extension of the problem-based method with the addition that once students complete their self-directed learning they are asked to evaluate their research, processes and solution(s) to the problem. They are then asked to return to the
original problem to reflect on how they might have improved their research and reasoning processes on the basis of what they learned during their self-directed learning, thus ‘closing the loop’ (Albanese, 2010; Barrows, 1986; Walker & Leary, 2009). The advantage of the closed loop PBL method is that it further addresses the students’ clinical reasoning processes, their structuring of knowledge for use in clinical contexts, and their development of effective self-directed learning skills. These steps require them “to go beyond the acquisition and discussion of new knowledge in a way that allows them to see its value and to evaluate actively their prior knowledge and problem-solving skills” (Barrows, 1986, p. 484). The closed loop variation of PBL encompasses all of the below characteristics and is the most student-centred in Barrows’ taxonomy.

In general, the characteristics of PBL challenge students to

- take responsibility for their learning;
- use free inquiry in their approach to solving a complex, real-world problem one would expect to encounter in their future profession;
- apply prior knowledge and understanding to the problem;
- analyse the problem, its context and consider possible solutions;
- discuss strategies and conduct research with others in small groups;
- review the proposed process and solution(s);
- create a tangible solution to the problem in the form of a presentation or product; and
- reflect on the problem, process and their results (Newman, 2005; Savery, 2015).

Along with the above characteristics there are specific design elements which contribute to a PBL curriculum’s effectiveness which need to be identified. The curriculum has to be thought out in terms of objectives, choice of problems, scheduling of time and the development of resources. First, the PBL curriculum design requires the creation of open-ended problems, which in the context of this study are relevant to mathematics teachers. Also, closed loop PBL requires the creation of reflective questions which students complete at the end of each succession of the PBL process to evaluate their reasoning, choice of resources and research methods used while solving the problem(s). At the other end of Barrows’ taxonomy, the lecture-based method requires the least effort for PBL designers and no special teaching skills or materials (Barrows, 1986). The other variations of PBL fall in between in regards to their overall complexity so that “the methods with the greatest educational potential are also the more difficult to mount” (1986, p. 485).
Since Barrows introduced his PBL taxonomy in 1986, there have been few studies which have specifically tested closed loop PBL. Until 1998, the Southern Illinois University's School of Medicine claimed that no study had looked specifically at the closed loop PBL approach (Distlehorst & Robbs, 1998). In 2009, although based on a small quantity of evidence, one meta-analysis reported that PBL does much better, in terms of educational goals, when the closed loop variation of PBL is used (Walker & Leary, 2009). Although closed loop PBL did appear to improve medical students’ learning outcomes ($d = 0.54$) assessed at the concept, principle, and application level, “the fact that PBL does so much better when it uses the closed loop problem based approach provides support for Barrows’ claims about potential benefits in terms of education goals” (2009, p. 23). Additionally, when the inclusion of 47 outcomes from studies outside the fields of medical education and allied health were added to the data set for analysis, PBL students in teacher education studies tended to do better than their lecture-based counterparts (Walker & Leary, 2009). Walker and Leary concluded that it seems logical to expect that the type of PBL implementation used in other disciplines might also play a role in learning outcomes. Walker and Leary’s findings lend support for an investigation using closed loop PBL as the teaching intervention with the treatment group to address the research problem in this study. Walker and Leary (2009) further concluded that along with specifying which variation of PBL a researcher intends to use in their study, one other important design element to consider, to optimise and maximise the effects of PBL, is its key component, the problems themselves.

### 2.8.3 The Problems - a Key Feature of a PBL Program

 Appropriately designed problems, used in meaningful real-world contexts, aimed at the intended learning outcome(s) play a key role in determining the success of a PBL program (Jonassen & Hung, 2015). Appropriately designed problems motivate the learner, and calibrates and targets the learners’ thought processes and research efforts. Without careful consideration to the structure and type of problems created by the PBL designer, the intentions for cultivating students’ applicable mindset of self-directed learning may be weakened (Hung, 2011).

Utilising the appropriate type of problem to provide the students with appropriate contexts as well as the unique characteristics of that type of problem is critical for ensuring the effectiveness of PBL instruction, and in turn, optimising PBL students’ learning outcomes. (Hung, 2011, p. 547)
While there is a large focus on using real-world problems as the foundation of PBL, the underlying nature of the problems should also be considered, such as their structure, difficulty and context based on the nature of the learners (Jonassen, 2011). Jonassen and Hung (2008) challenge PBL researchers to consider problem type and difficulty when designing a PBL program. To suitably identify the type and difficulty of the problems in the context of this PBL study, the typology of problems as conceptualised by Jonassen (2000) is considered useful.

Jonassen’s (2000) typology of problems (Figure 7) consists of 10 classes of problems categorised along a continuum. The problems are assigned to these classes based on the problem’s level of difficulty, complexity and structuredness, in relation to the dominant type of problems employed for a particular context. In a later study Jonassen and Hung (2008) investigated Jonassen’s typology of problems to determine which types of problems are best suited to a PBL program. After considering three types, diagnose-solution, decision-making and situated cases/policy problems, they hypothesised, based on nine dimensions of difficulty, that “decision-making problems should be used as the problem focus of PBL” (Jonassen & Hung, 2008, p. 21).

![Figure 7: Typology of Problem Types (modified from Jonassen, 2000, p. 74-75)](image)

Decision-making problems characteristically have several competing alternatives. Thus, arriving at solutions to this type of problem require diagnosis, negotiation and design. Basically, decision-making problems “typically involve selecting a single option from a set of alternatives based on a set of criteria” (Jonassen, 2000, p. 77); for example, which argument would be most effective to plead my case in court? In the context of the present
study, an example is envisaged as “What lesson activity will best resolve my students’ misconceptions regarding what any number raised to the zero power equals?” Hence, decision-making problems are positioned as more well-structured than ill-structured according to Jonassen’s (2000) typology. This type of problem has a limited number of solutions, but, “the number of factors to be considered in deciding among those solutions,” and the implications of each decision, can be very complex (Jonassen, 2011, p. 98). Problems of this nature require pre-service teachers to decide, based on set of criteria, which lesson plan activity is best used with which mathematics resources, in which order, and using which teaching strategy, to address difficulties children experience with particular mathematics concepts and skills. In the present study, the set of criteria directing the pre-service teachers’ decision-making is based on ensuring that the learning outcomes of the mathematics education subject are achieved (developing mathematics PCK in pre-service teachers and their ability to enact it), which also addresses the research problem, but more importantly, the main aim of teacher education more broadly.

To review, as a pedagogical intervention, PBL originated in medical education in a bid to better educate medical students (Albanese & Dast, 2014; Albanese & Mitchell, 1993; Barrows, 1986) using mainly diagnosis-solution problems (Jonassen & Hung, 2008). Teacher education and K-12 education have since adapted the approach (Choy & O'Grady, 2012; McPhee, 2002; Murray-Harvey & Askell-Williams, 2005; Savery, 2015; Weizman et al., 2008). More than 300 undergraduate institutions were reporting the use of PBL in their subjects by the year 2000 (Samford University, 2000). Consequently, PBL has been so broadly adopted by institutions using variations or degrees of structure of PBL, based on the needs of their discipline, that “the meaning of the term problem-based learning has become clouded and confused” (Barrows, 1994, p. vi). Most studies and meta-analyses revealed that students exposed to a PBL treatment gained slightly less knowledge, but remembered more of the acquired skills than their peers taught using conventional instruction (Dochy et al., 2003). Furthermore, problem difficulty or problem design has not been adequately reported in PBL research (Jonassen & Hung, 2015). Basically, researchers are not identifying which specific variation of PBL they used in their study or the type and difficulty of the problems used within the context of the educational environment. These variables make it difficult to conduct reliable comparative studies or meta-analyses which produce clear findings as the data ends up
comparing ‘apples to oranges’ (Newman, 2003). Hence, the findings to date on the effectiveness of PBL are mixed and “existing overviews of the field do not provide high quality evidence with which to provide robust answers to questions about the effectiveness of PBL” (2003, p. 5).

As stated in several studies (Barrows, 1986; Newman, 2003; Walker & Leary, 2009), it is suggested that future researchers embarking on a study using PBL should specify which variation of PBL they intend to use, and which degree of student or teacher directedness they would utilise based on their discipline, the content, the context and the intended learning outcomes (Walker & Leary, 2009). Furthermore, there may be a correlation between PBL’s mixed reviews and problem types. Therefore, in this study, Barrows’ (1986) closed loop variation of PBL (the most student-centred of all six alternatives) will be used to create the pedagogical intervention for the pre-service teacher participants, to determine its impact on their mathematics PCK. A control group will be instructed using a traditional teacher-led instructional approach where pertinent information is provided by the lecturer prior to the same open-ended, real-world problems being presented to the students. In this way both participant groups will receive identical problems as their tutorial tasks throughout the study.

Further, Jonassen’s (2000) typology of problems guided the researcher’s choice to use decision-making problems as the design for this study’s PBL-type problems, in the form of teaching scenarios, which are based on real-world, open-ended problems the students will likely face in their future teaching. “Scenario construction can be used to support or assess the ability to make meaningful decisions” (Jonassen, 2012, p. 353). It was envisaged that the pre-service teachers involved in this study, following their diagnosis of the problems and negotiation of research undertaken to find solutions, would design and implement lesson plans based on a combination of tactical teaching strategies and approaches which support the aim of the lesson (Walker & Leary, 2009). Furthermore, choosing these types of problems addresses the concerns that (a) the nature of the problem they are being asked to solve aligns with the education domain and nature of the learner and (b) the problems’ designs do not potentially distract students from the attainment of the subject’s intended learning outcomes (Hung, 2011; Jonassen, 2011).
Therefore, to address the research problem presented in this study, the researcher hypothesises the closed loop variation of PBL is a pedagogy which may enhance pre-service teachers’ mathematics PCK and/or the ability for them to enact their PCK to a larger degree than that of a control group of pre-service teachers instructed using a traditional teacher-led instructional approach. While the researcher’s hypothesis suggests that closed loop PBL is the factor impacting pre-service teacher’s knowledge and application of mathematics PCK in this study, an extended review of the literature revealed other variables which may impede the researcher’s ability to accurately draw conclusions in relation to the hypothesis. Specifically, when an instructional intervention such as PBL is implemented with real students in a real classroom, rather than in controlled conditions, numerous variables could potentially affect students’ final learning outcomes (Hung, 2011). Consequently, if the pre-service teachers in this study: (a) experience mastery from delivering solutions to meaningful, real-world situations they will encounter in their future career in a safe and supportive environment; (b) are provided positive feedback and (c) develop teacher competence vicariously, by observing their colleagues design and deliver lessons, their self-efficacy for teaching and teaching outcome expectancy may be enhanced by these closed loop PBL experiences (Bandura, 2006b; Dunlap, 2005; Schmude et al., 2011). Hence, it is further hypothesised that pre-service teachers with high self-efficacy and outcome expectancy for teaching would likely believe that being able to enact sound mathematics PCK will impact positively on their future students’ mathematics academic achievement; which, in turn may motivate them to pursue that knowledge more vigorously (Biggs, 1989; Huinker & Madison, 1997). In response to the researcher’s hypotheses, the following section summarises his experiences and the research which led to the study’s specific research questions.

2.9 Research Problem and Research Questions

Personal observations as a teacher and pre-service educator in the USA led this researcher to using a constructivist, student-centred instructional approach to develop students’ content knowledge related to place value. As a mathematics education lecturer in Australia, a continued commitment to using PBL led to conducting action research of the efficacy of PBL (Martin, 2012; Martin & Jamieson-Proctor, 2010). The pilot and present study emerged from the literature review undertaken with respect to the research problem and closed loop PBL’s potential impact on pre-service teachers’ mathematics PCK, their
ability to enact their PCK and their teaching self-efficacy and outcome expectancy. Based on the literature search concerning the research problem, and the gaps found in the literature, the overarching research question for this study is:

What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ mathematics PCK and/or their ability to enact their PCK?

As the literature also suggests, self-efficacy for teaching and teaching outcome expectancy may have an impact on pre-service teachers’ developing PCK, this study will also investigate pre-service teachers’ self-efficacy for teaching and teaching outcome expectancy through the following two research questions:

What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ self-efficacy for teaching mathematics?

What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ mathematics teaching outcome expectancy?

The following section concludes this chapter by summarising the literature which led to the study’s research aim and the researcher’s rationale for the study’s research design.

2.10 Chapter Summary

It was the primary function of the chapter’s first section to present the literature which establishes PCK as a core attribute of effective teachers formed from a synthesis of content knowledge, content knowledge for teaching, contextual and pedagogical knowledge intertwined with teachers’ beliefs (Briley, 2012; Magnusson et al., 1999; Pendergast et al., 2011; Shulman, 1986, 1987; Tatto et al., 2011).

The second section of the chapter described Bandura’s social cognitive and self-efficacy theories in terms of teaching and learning (Bandura, 1977, 1989, 1994). Literature was presented which indicated pre-service teachers’ self-efficacy for teaching and teaching
outcome expectancy may also affect their developing mathematics PCK and/or their ability to enact their PCK (for example, Huinker & Madison, 1997; Moody & DuCloux, 2015; Shin et al., 2010). Based on the presented literature it was determined that these two factors will also be investigated in this study.

The third section of the chapter introduced PBL, followed by a synthesis of the research findings of PBL’s effectiveness and its impact on participants’ cognitive development and learning outcomes. The six variations of PBL in Barrows’ (1986) PBL taxonomy were described, followed by which variation is considered the most student-centred and best positioned to enhance tertiary students’ (a) clinical reasoning process, (b) ability to structure knowledge for use in clinical contexts, (c) effective self-directed learning skills, and (d) motivation for learning. Closed loop PBL was situated best to develop all four objectives (Barrows, 1986). Lastly, this section described the importance that needs to be placed on the design of problems in a PBL program. Based on Jonassen’s (2000) typology, the problems’ type and complexity are to be designed for authenticity in the appropriate context and for the purpose of moving the students towards self-discovery of the intended learning outcomes. In a successful PBL program these factors need be considered when developing the problems the pre-service teachers will be required to solve in a PBL classroom context appropriate for pre-service education. In this way, as a real-world or contextualised problem is presented, for which a solution is not immediately apparent, the students become the owners of the work and assume responsibility for their own learning. The problem type chosen for this study, decision-making problems (Jonassen, 2000), will require the pre-service teachers to decide, based on a set of criteria, how to design lesson plans which resolve obstacles children experience with particular mathematics concepts and skills.

In closing, PBL’s efficacy in the medical education field is well established in the literature. Various studies can also be found on PBL’s effectiveness at the primary and secondary school level. The literature on PBL then begins to diminish as the search for PBL’s effectiveness targets specific content areas such as mathematics in the secondary school system, and to an even greater degree, studies on the effectiveness of closed loop PBL at the tertiary level in faculties of education is almost non-existent. Despite the use of PBL by so many academics, the empirical studies on its effectiveness are mixed. Barrows’ taxonomy was often cited in studies related to PBL, yet few have reported their
interventions in terms of the specific variation of PBL being used. It seems vital that focused research be undertaken to determine what forms of PBL produce which outcomes for which students in what circumstances (Newman, 2003; Walker & Leary, 2009). Fundamentally, “we need to move beyond claims that proclaim something as ‘evidence based’ to robust models for interpreting truly evidence-based studies” (Dean et al., 2012, p. vi). The absence in the literature of research determining PBL’s impact, specifically closed loop PBL, on pre-service teachers’ mathematics PCK, their ability to enact that PCK, their self-efficacy for teaching mathematics and their mathematics teaching outcome expectancy, using an appropriate type and design of problem, makes this study unique in its ability to contribute new knowledge to the field of PCK development in tertiary mathematics education. The following chapter will provide a detailed description of a pilot study which was used to validate the data collection instruments and test the feasibility of the study’s research design.
3 The Pilot Study

3.1 Overview

In the previous chapter the literature pertaining to the study’s aim and three research questions was presented. This chapter commences with a description of the researcher’s epistemological stance and overall conceptual framework underpinning the research. Following this, the chapter describes the PBL teaching intervention developed in the first instance for the pilot study and adapted, based on the results of the pilot, for the main study, and the rationale behind the decisions that were made. Next, the chapter describes the three aims of the pilot study which were to: (1) evaluate whether the research design was feasible, (2) provide an opportunity to implement and refine the closed loop PBL pedagogical intervention in preparation for the main study and (3) allow for the development and validity testing of the data collection instruments.

The pilot study took place over the course of one semester with two different cohorts of pre-service teachers studying the same mathematics education subject. With one cohort (the on-campus students), aims 1 and 2 were addressed (n=30). In order to demonstrate the statistical validity and reliability of the measurement instruments a larger cohort was required, so aim 3 was addressed with the online pre-service teachers (n=238). This chapter concludes with a summary on the outcomes of the pilot study, which were used to guide the final structure and methodological instruments and processes implemented in the main study the following year. The following chapter deals with the methodological differences between the pilot and the main study; and presents the information specific to the main study whilst avoiding duplication of material presented in this chapter detailing the pilot.

3.2 The Underpinning Conceptual Framework for the Research

This researcher’s ontological and epistemological stance aligns with Guba and Lincoln’s (1989) and Denzin and Lincoln’s (2013) descriptions of the constructivist paradigm.

Ontologically, [constructivism] denies the existence of an objective reality, asserting instead that realities are social constructions of the mind, and that there exists as many such constructions as there are individuals. (Guba & Lincoln, 1989, pp. 43-44)
The constructivist paradigm assumes a relativist ontology (there are multiple realities), a subjective epistemology (knower and respondent co-create understandings), and a naturalistic (in a natural world) set of methodological procedures. (Denzin & Lincoln, 2013, p. 27)

Social constructivism stems from Dewey’s view that humans learn and make sense of their world when interacting with their environments as active participants (Guba & Lincoln, 1989; Marra, Jonassen, Palmer, & Luft, 2014; Savery & Duffy, 1995; Torp & Sage, 2002) and Vygotsky’s Sociocultural Theory which stresses the importance of using social collaboration to aid learning (Eggen & Kauchak, 2012). Amalgamation of these two theories demonstrates that willing, active participants individually and collaboratively construct knowledge in natural settings from personal interpretations and prior knowledge based on past experiences.

Social constructivism implies that a concept is acquired not only from observations and experiences (O'Leary, 2010), but from how each person interprets those experiences. In other words, individuals form concepts *a posteriori* as opposed to *a priori* (Putnam, 1978). Concepts are said to be attained *a priori* if they can be applied independently of, and prior to, actual experience. Whereas concepts acquired *a posteriori* are so if the knowledge can be applied only as a function of human thought and experiences based on observations through the senses (O'Leary, 2010; Oliver, 2008). Although the divide between these competing epistemologies and ontologies continues over how humans construe and construct knowledge, it is this researcher’s belief that humans cannot separate themselves from their experiences; thus, our past and present physical and observable experiences and perceptions underpin our continuing knowledge development (Corbin & Strauss, 2008a).

*Thought and Language*, written by Vygotsky (1934) is viewed as a seminal work in the field of Education for the understanding of the social foundations of learning, thinking, and dialogue (van der Linden & Renshaw, 2004). In translating Vygotsky’s writings, Cole, Steiner, Scriber, and Souberman (1978) and Hanfmann and Vakar (1962) interpret concepts as abstractions which are obtained when several isolated features, identifications, discriminations and generalisations are synthesised and become a form of thinking and understanding. Thus, in an educational context, conceptual formations are
created through recollection of experiences preserved in memories, practical activity, social interactions and environmentally produced sensations formed during classroom instruction. According to Vygotsky’s social constructivist theory, students will learn if (a) they are placed into a collaborative setting in the classroom, (b) their prior knowledge and understandings are engaged, (c) a strategically chosen practical activity is utilised in the classroom and (d) inquiry and reasoning are provoked. As a result, a process is set in motion by which knowledge is formed. These social constructivist knowledge formation processes are the core of the pedagogy underpinning the problem-based learning approach (Savery & Duffy, 1995) and PBL considered one of the best approaches to social constructivism in an educational environment (Gibbings & Brodie, 2008).

In brief, the researcher’s past experiences and investigations with social constructivism and PBL, both in primary school and university contexts, resulted in a continual change in teaching approaches, progressively towards a more social constructivist paradigm. Therefore, it was natural for the researcher’s social constructivist epistemology to underpin this study’s research design, which required the pre-service teachers to collaboratively solve meaningful, real-world problems practising teachers encounter in their classroom on a daily basis.

### 3.3 The PBL Intervention used in the Pilot Study

The PBL method used in the pilot study was based on Barrows’ (1986) closed loop variation of PBL (Barrows, 1986, 1998; Savery, 2015). In a PBL learning environment the focus is on collaborative, self-directed learning and thinking by students. The closed loop variation of PBL contains an additional, iterative process. As the term closed loop suggests, once students complete their self-directed learning they are then asked to return to the original problem to actively reflect on their research and problem-solving skills (Barrows, 1986; Walker & Leary, 2009). Thus, using the closed loop variation of PBL with the treatment group in the pilot study ensured that

- the pre-service teachers were placed into a collaborative setting in the classroom;
- a strategically designed real-world problem to be solved that related to their future teaching career was utilised;
- their problem solving skills were addressed and facilitated;
they were prompted to inquire and reason for themselves while searching for an established pedagogy which they believed would satisfy the real-world problem; self-assessment and peer assessment were utilised; and the pre-service teachers planned and demonstrated their solutions (Barrows, 1998).

After completing the learning cycle of the closed loop PBL method and accompanying self-directed learning, the pre-service teachers were asked to evaluate their research, processes and solution(s) to the problem. Lastly, they were asked to return to the original problem to reflect on how they might have improved their research and reasoning processes on the basis of what they learned during their self-directed learning, thereby closing the loop (Barrows, 1986).

The closed loop PBL method was chosen to underpin the intervention used in this study for several reasons.

- Within the Productive Pedagogies model, PBL is among the 20 observable classroom teaching and learning practices students experience which were investigated and endorsed by the team of researchers who conducted the Queensland School Reform Longitudinal Study (QSRLS) (Lingard et al., 2003).
- Among the other 19 elements of the Productive Pedagogy model, seven are underpinned by PBL (higher-order thinking, substantive conversation, connectedness to the world, student control, social support, engagement and self-regulation).
- Based on the key findings of the QSRLS, teachers with high ratings on the Productive Pedagogy measure express a strong sense of efficacy in improving student outcomes.
- Research findings provide strong support that classroom practices such as those measured by the Productive Pedagogies model lead to improved academic outcomes for students (Lingard et al., 2003; Lingard et al., 2001).
- PBL has also been shown to produce statistical gains for cognitive outcomes in teacher education studies when the problems target the intended learning outcomes and application of knowledge (Gijbels et al., 2005; Walker & Leary, 2009).
However, this was an untested pedagogical approach in this mathematics education subject; so, meeting the intended learning outcomes of the subject was an important consideration in the decision to change the pedagogical approach and trial PBL. It was considered an ethically unacceptable outcome to disadvantage students for the benefit of the research study. Ultimately, the PBL teaching intervention chosen for this study needed to achieve the intended learning outcomes of the subject while using PBL-type problems which characteristically have a level of ill-structuredness representative of decision-making problems (Jonassen & Hung, 2008).

Ill-structured problems are complex problems that cannot be solved by a simple algorithm. Such problems do not necessarily have a single correct answer but require learners to consider alternatives and provide a reasoned argument to support the solution they generate. (Hmelo-Silver & Barrows, 2006, p. 24)

In PBL, the purpose of ill-structured problems is to help learners “develop their ability to adaptively apply their knowledge to deal with complicated problem situations that are normally seen in real world settings” (Hung, 2011, p. 531). The ill-structured problems are the type encountered in workplace practice which have many plausible solutions, many of which are not evident from the outset (Jonassen, 1997). Ill-structured problems often contain uncertainty about the strategies or principles needed to solve them (Hung, 2011). Adding to this multi-faceted definition, ill-structured problems often require the learners to use their beliefs and make professional judgements about the problem (Jonassen, 2000).

Based on the above descriptions of the term ill-structured, the PBL problems designed for this study are defined as real-world and open-ended, but with specific solution criteria which students needed to accommodate in their solutions. The problems required students to gather data and make decisions (Jonassen, 2000); the type of decisions that face teachers in their daily work. The problems are therefore described as real-world because they are considered to be of the type the students would encounter in their future careers as teachers. The term open-ended is used to indicate that the problems do not have a single correct answer and may have several acceptable solutions. Structuring the problems with criteria was a strategic decision designed to prevent the pre-service teachers from deviating from the learning outcomes of the subject (Hung, 2009, 2011). For example, a specific criteria embedded in the problems was a requirement for the pre-service teachers to access specific year-level strands and sub-strands from the Australian Curriculum and
make use of them in the design of their solutions. The rationale behind designing the PBL problems in this manner was to ensure the criteria guided the pre-service teachers to research the subject’s intended and appropriate content. Thus, the PBL problems used in this study are termed real-world, open-ended decision-making problems. One example of the problems used in the pilot study is presented in the following section.

The design of such real-world, open-ended decision-making problems is considered a key element in the development of a PBL program (Jonassen, 2011; Sockalingam & Schmidt, 2011). Other fundamental elements of a PBL program are the students themselves, when formed into groups, as self-directed learners, and the tutor who assumes the role of facilitator (Hmelo-Silver & Barrows, 2015; Hung, 2011). Each of these three elements requires careful consideration prior to the commencement of any effective PBL pedagogical approach and each will be discussed separately, as they were conceptualised for the pilot study.

3.3.1 Key Elements of a PBL Intervention

Successful implementation of the PBL pedagogical approach relies on students’ learning being driven by the students themselves as they investigate and search for appropriate solutions to real-world, open-ended decision-making problems. Solving PBL problems provides the stimulus for learning as the students appreciate that the problems relate to the type of challenges they will encounter in their future career. Similarly, the problems designed for this study needed to achieve both the learning objectives of the mathematics education subject and be positioned in proximity to the learners’ future teaching careers (Hung, 2006). Another consideration was that “different kinds of problem solving in different contexts and domains call on different skills” (Jonassen, 2000, p. 64) and involve different levels of difficulty (Jonassen & Hung, 2008). As determined by the literature review, the problems for this study were designed primarily as decision-making problems which were conceptualised using Jonassen’s (2000) typology of problems’ benchmarks.

According to Jonassen (2000), this type of multi-faceted problem necessitates an initial consideration of how to design the problems as real-world while also requiring the pre-service teachers to work towards achieving the lesson’s objective of remediating children’s difficulties in mathematics. Equally important was the alignment of the problems with the learning objective of the mathematics education subject so the pre-
service teachers’ development of the targeted mathematics PCK and the ability to enact that PCK was facilitated. In short, if the problems were not written with a clear, specific goal the pre-service teachers may not have engaged with the correct research or reasoning processes, thus deviating from the intended learning outcomes (Hung, 2006). Figure 8 illustrates one of the four problems presented to the treatment and control groups for measurement (Appendix A contains the entire collection of problems).

The aim of this activity is for you to demonstrate your ability to design a lesson which has young students exploring the concept and skills for measuring mass.

<table>
<thead>
<tr>
<th>Scenario: You are attending your 15-day practicum and your mentor teacher is asking you to design a lesson on the topic of mass for her Year 4 class. She informs you the students’ prior knowledge in this area is quite limited. She then provides you with the following guidelines:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Since the students’ conceptual understanding is limited, your mentor asks that you revisit the related content from the Year 2 and Year 3 ACARA content descriptors.</td>
</tr>
<tr>
<td>The design of your lesson should, therefore, provide the students with the opportunity to:</td>
</tr>
<tr>
<td>• revisit their Year 2 and Year 3 prior knowledge;</td>
</tr>
<tr>
<td>• be appropriately introduced through real-world, concrete activities to the concepts related to the measurement of mass appropriate for their year level;</td>
</tr>
<tr>
<td>• apply, in a social constructivist learning context, the Year 4 measurement skills; and</td>
</tr>
<tr>
<td>• demonstrate and explain their understanding to their peers using their own language.</td>
</tr>
</tbody>
</table>

*Figure 8: Example of one of the Real-world, Open-ended Measurement Problems Posed to both the Treatment and Control Groups in Week 4*

The second key element to consider when developing a PBL pedagogical approach is the amount of cognitive scaffolding provided to the groups and individuals. Some of the most successful implementations of PBL have first supported students’ self-directed learning and collaborative skills while they are in the process of adapting to their new PBL context (Jonassen, 2011). It should not be assumed that learners are naturally skilled in self-directed learning, working in groups, or at solving complex problems.
Most learners do not naturally possess these cognitive capabilities; rather, they develop these cognitive skills with sufficient training. Therefore, it is crucial to calibrate the levels of researching and reasoning processes required for solving the problem with the learners’ levels of cognitive readiness as well as their self-directed learning skills, or comfort level with PBL. (Hung, 2006, p. 64)

It was therefore decided to utilise the first three weeks of the semester to scaffold the pilot pre-service teacher treatment group’s collaborative skills required for working productively in groups and their ability to engage with PBL-style problems. The scaffolding provided generally involved using increasing levels of problem complexity and degrees of self-directedness across the tutorials and weeks.

The third key element of a PBL program is the role of the instructor as the facilitator. A PBL facilitator does not deliver lecture content to the students using a traditional teacher-led instructional approach. Rather, the facilitator has a responsibility to avoid simply providing his or her own knowledge of the topic. The objective is to move the students toward self-discovery of the desired outcome, allowing them to own the knowledge rather than being taught the information and/or the solution. In this way, as the real-world problem is presented, for which the solution is not immediately apparent, the students become actively engaged and assume responsibility for their own learning while creating a solution for the problem. One trait of being actively engaged is the presence of facilitated, student-led discussions during and after collaborative group work. In these situations, the instructor is required to use complex facilitation skills.

If the students are not guided by the teacher to consider all the steps in the hypothetic deductive reasoning processes, always to question whether they have learning needs as they work, and to choose and use a variety of resources in their learning, then objectives are compromised. (Barrows, 1986, p. 485)

For the pilot, the researcher, with prior experience in implementing a PBL approach, facilitated the learning of the PBL treatment group (n=15). Another lecturer, skilled in implementing a traditional teacher-led approach, agreed to teach the subject content to the control group (n=15) of pre-service teachers who were enrolled in the subject on a different campus. This sampling decision reduced the risk of interaction between groups.

In summary, three key elements of a PBL program were considered and attended to in the design and implementation of the pilot study. Based on the work pioneered by Barrows (1986), and advanced by Jonassen (2000), Hung (2006) and Jonassen & Hung (2008), the
structure of the problems were considered central to the treatment program’s design. Second, the make-up of the student groups and their readiness to work in a PBL context was addressed. Hence, experiences with using collaborative skills for working with increasing levels of complex problems were provided before the students were required to solve the real-world, open-ended decision-making problems. Third, the researcher, experienced in the PBL method and another lecturer experienced in the teacher-led instructional approach, facilitated student learning with the two student cohorts throughout the pilot study. The next section describes how aims 1 and 2 of the pilot study were addressed and how both teaching interventions were implemented.

3.4 The Pilot Study: Aims 1 and 2

Aims 1 and 2 of the pilot were to assess the study’s research design and to implement and refine the closed loop PBL pedagogical process used with the treatment group. The pilot study was undertaken in the same mathematics curriculum and pedagogy subject and at the same regional campuses as the main study, but a year earlier. Two on-campus cohorts, from two different satellite campuses of the same university, were used forming a treatment group (n=15) and a control group (n=15) respectively. Thus, the participants were not randomly chosen. The sampling technique whereby participants are not randomly chosen and which makes use of existing groups of students is called convenience sampling. Convenience sampling involves including participants who are available and easily recruited (Johnson & Christensen, 2014). Both groups were third year pre-service teachers enrolled in the same semester-long mathematics curriculum and pedagogy subject, and the Australian Curriculum strands and sub-strands of Algebra, Measurement, Geometry and Probability and Statistics provided the content for the subject.

To properly assess the research design, the researcher took on the role of facilitator for the PBL treatment group in the pilot. This decision contributed to the researcher’s deeper understanding of the closed loop PBL method, and how to deliver the teaching method for maximum effect. For example, when the treatment group of pre-service teachers asked questions, it became evident that deciding how much information to provide versus how much redirection should occur affected their level of engagement. It was observed that providing too much information decreased the amount and number of times the pre-
service teachers self-directed themselves to search for pertinent information related to the learning objectives of the topic. In response, subsequent student questions were either redirected to a resource, or queried by the facilitator as to whether they were on the right track, or even asking the right question. This questioning strategy was designed to redirect the students to think critically about their query, and after reflection, self-direct their learning, thus requiring them to take responsibility for obtaining the appropriate information for themselves. In terms of ethical considerations, as an insider researcher, the power differential was minimised by (a) not assigning marks to their completed lesson plan designs or lesson presentations and (b) negotiating with the participants to be co-investigators (Greene, 2014).

As the person in charge of the subject, the researcher was able to recruit a lecturer to teach the control group; one who characteristically uses a traditional didactic approach. The lecturer subsequently agreed to teach the control group in a traditional teacher-led environment at the other campus. The pilot was conducted in accordance with all required ethics protocols. Ethics approval for the study was sought and obtained from the university’s Human Research Ethics Committee. The Participant Information Sheet gave an assurance to the pre-service teachers that the data collected would be limited to activities they normally undertake throughout the semester. Additionally, it was stated in the Participant Information Sheet that all data collected would be de-identified, and the students were free to withdraw from the project at any stage. Participation was voluntary and all participants who agreed provided their informed written consent.

For both cohorts, the semester was 15 weeks in duration with 10 weeks being designated for on-campus attendance. During each of the 10 on-campus classes, both cohorts were presented with the same content topics and real-world, open-ended tutorial problems related to the topics. In week 1 of the semester the treatment group and control group of pre-service teachers were asked to respond to the pre-test instruments comprising the MTEBI and MPCKI (described in detail in a subsequent section). The learning objective for weeks 2 and 3 was for the pre-service teachers to know how, and be able, to move school-aged students from an understanding of simple geometrical and number patterns to an understanding of algebra. The strategy to utilise weeks 2 and 3 in this manner accommodated a revisit of place value (underpinned by patterning) for both groups of pre-service teachers, while simultaneously preparing the treatment group of pre-service
teachers to begin working and learning in a PBL environment. This preparation for the treatment group required a different pedagogical approach compared to the instructional approach used with the control group of pre-service teachers.

While the semester’s content was identical for both groups, how the content was presented was different for the two cohorts. The control group was taught the content using direct instruction in a 1-hour lecture on Tuesdays (or they had the option of watching the recorded lecture), followed by a 2-hour tutorial on Fridays when they engaged with the real-world, open-ended decision-making tutorial problems using a traditional teacher-led instructional approach. Alternatively, the PBL treatment group was expected to investigate the week’s content and the same real-world, open-ended tutorial problems during a 3-hour workshop each Wednesday. Students in this group worked independently and in small working groups to investigate the problem that was posed and to create their solution. They did not receive an on-campus lecture; rather, they were able to view the recorded lecture if they chose as one of the resources made available for their workshop.

In terms of describing the specific approach to closed loop PBL that was used with the treatment group of pre-service teachers, the 3-hour workshops from week 4 and week 5 will provide the context. While the focus of the next section is on describing the PBL teaching intervention, for comparison and research replication purposes, descriptions will also be provided of the traditional teacher-led instruction used with the control group.

In week 4 both cohorts were divided into four groups. Each of the four PBL groups were asked to provide solutions to one of the four real-world, open-ended problems which were based on a particular measurement concept and skill from ACARA (Australian Curriculum Assessment and Reporting Authority, 2016). The PBL groups were given the full three hours to analyse their problem, conduct any necessary research, examine resources and design their solutions in the form of a lesson plan. In week 5, the PBL groups presented their solutions in the form of a lesson delivered to their peers. In the control group, only two of the four real-world, open-ended measurement problems were used with the four groups in week 4. Essentially, two groups worked on one of the problems during the 2-hour teacher-led tutorial while the other two groups solved the other problem. In week 5, the process was repeated for the control groups with the other two measurement problems. Table 3 provides the description of how the content and
pedagogical strategies were delivered to the control group during the 1-hour lecture and then how they engaged with the tutorial tasks during the 2-hour tutorial.

Table 3
Outline of the Teacher-directed Approach used with the Control Group during the Pilot Study

<table>
<thead>
<tr>
<th>Control Group</th>
<th>Traditional Teacher-led Process</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1-hour teacher-directed lecture.</strong></td>
<td>Each Tuesday the control group of pre-service teachers attended a lecture in a traditional lecture theatre. The lecturer displayed a PowerPoint presentation using the theatre’s LCD projector screen. With the theatre room chairs organised in rows, the lecturer delivered, slide-by-slide, using a teacher-led instructional approach, the week’s curriculum content and pedagogical strategies related to the content. In this hour long learning environment, except in the case where any student questions were answered by the lecturer, the students were primarily passive listeners.</td>
</tr>
<tr>
<td><strong>1x2-hour teacher-directed tutorial.</strong></td>
<td>The Friday following each Tuesday lecture, a 2-hour tutorial was conducted in a classroom with the tutorial problems framed around the week’s curriculum and pedagogy topic. The tutorial session began with the lecturer informing the students that, after being divided into groups, and during the allocated tutorial time, they would create a lesson plan in response to one of two real-world, open-ended problems (see Figure 8 for an example measurement problem). Their response should demonstrate their ability to address difficulties children experience with specific mathematics concepts and skills. Next, led by the lecturer and complemented with PowerPoint slides, the two real-world, open-ended tutorial problems were presented and unpacked. This was accompanied by a review of the pedagogical strategies the students would need to solve the problems. Relevant concrete materials were brought to class by the lecturer who explained how each could be used in the solutions of each problem and provided any further clarity when asked. Students then placed themselves into four groups of three or four and were tasked with developing a lesson plan which addressed the specific real-world, open-ended tutorial problem. Visiting each group in turn, the lecturer answered questions and modified students’ reasoning processes. The lecturer confirmed or suggested possible activities and solutions in terms of the pedagogical strategies that were ‘expected’ to be used. The students then reorganised their solutions and wrote-up the solutions to the problems on a provided lesson-plan template. Circulating to each group for the second time, the lecturer established that each group’s solutions met the subject’s learning outcomes. Groups which struggled were assisted to develop an ‘acceptable’ solution. In a teacher-led whole class discussion, each group in turn provided their solution to how they would enact their PCK to remediate the children’s difficulties. Feedback and/or alternative solutions were provided by the lecturer. Any questions asked by other students were generally answered by the lecturer.</td>
</tr>
</tbody>
</table>

Table 4 provides a description of how the treatment group was facilitated in their self-discovery of the content and pedagogical strategies during their 3-hour workshop, in order to compare and contrast the approaches of the two different lecturers. Also described is the student-led process used during the following week’s 3-hour workshop where the treatment groups delivered their solutions in the form of a simulated lesson which addressed the mathematics content covered that fortnight.
Table 4
Outline of the PBL Approach used with the Treatment Group in the Pilot Study

<table>
<thead>
<tr>
<th>Treatment Group using the Closed Loop PBL Process</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3-hour, student-directed workshop.</strong> Each Wednesday the PBL group of pre-service teachers presented themselves to a classroom for their 3-hour workshop, with no lecture component. The room chairs were prearranged into four groups of three or four students each, with each group placed in teams based on their preferred teaching level.</td>
</tr>
</tbody>
</table>

At the start of each workshop, the lecturer informed the students that each group would create a lesson plan in response to one of four real-world, open-ended problems to demonstrate their ability to address difficulties children experience with particular mathematics concepts and skills. Using the closed loop PBL process, the groups spent the remainder of the workshop engaged in a process of discovery/research to determine the information they needed in order to remediate the children’s difficulties and allow them to provide a solution to the problem posed. It was the group’s decision how to best utilise the time remaining in their 3-hour workshop. If they chose to engage outside the workshop, the following seven days outside of class were also at their disposal. They were also free to complement their lesson using a PowerPoint presentation if they chose. Relevant concrete materials (the same materials as provided to the control groups) were available to the groups with no explanation of their possible use in relation to the problems. Other resources made available during the workshop were (a) iPads with internet access, (b) a variety of textbooks aimed at teaching primary school mathematics, (c) access to the recorded lectures, (d) the PBL facilitator as a coach/mentor and (e) blank lesson plan templates (hard copy and electronic versions) on which they could populate their lesson plan. Alternatively, they were free to design their own template.

While the groups engaged with their problem, the lecturer, using a PBL facilitation process, supported the students’ thinking by responding to their questions with probing questions of his own… questions which the students should be asking themselves to guide their thinking. Thus, Socratic Dialogue (van der Linden & Renshaw, 2004), where the lecturer was neither the author nor transmitter of knowledge, but rather an assistant to the learners’ search for solutions to the problems, was engaged. This dialogue included questioning the students’ search for evidence as well as the justification for their choice of lesson activities which they believed would address the difficulties children experience with the particular mathematics topics. At the onset, the students were frustrated because their questions were being answered with more questions by the facilitator, even if it was in order to assist them to search for evidence and apply reasoned arguments. In this way the students were enabled to centre their thinking on the learning objectives of the subject, and, they were guided towards identifying what they knew and what they needed to find out. This iterative approach was undertaken so the students would become more confident in identifying the specific information they needed to discover to effectively solve the problem that was posed.

The following week, each group’s written solutions, presented on a completed lesson plan template, was submitted. Additionally, during the 3-hour workshop each group took turns delivering the lesson to their peers in a simulated classroom using their choice of materials and teaching strategy, and provided a rationale for the pedagogical approach they chose to underpin their lesson. After delivering their lesson, informal feedback was provided to the group members by their peers and the lecturer, and the group delivering the lesson was also allocated time for self-assessment. To complete the closed loop PBL process, the pre-service teachers individually responded to a set of reflection questions (Appendix B). These questions requested they revisit the original problem and reflect on the effectiveness of their process in solving the problem, both individually and as a group. They responded to questions such as: If you were to revisit the original problem, what improvements would you make to your reasoning process?
An illustration of the semester’s weekly schedule used in the pilot study and designed specifically for the treatment group is provided in Figure 9. As a result of semester break, no classes were held in weeks 6 and 7. Additionally, no classes were held in weeks 12, 13 and 14 while the pre-service teachers attended their three-week practicum.

Figure 9: Outline of Semester’s Weekly Schedule for Treatment Group

The difference for the control group was that in weeks 2, 4, 8 & 10, two groups each worked on one of the four problems and the other two groups worked on another one of four problems. Then in weeks 3, 5, 9 & 10 the same process was used with the other two of four problems. Additionally, the control groups did not deliver their designed lesson plan solution to their peers in a simulated classroom setting.
In summary, during the class held in the first week of the semester, both cohorts responded to the pre-test MTEBI and MPCKI. For the remainder of the semester’s teaching weeks, the lectures and tutorials were delivered to the control group in a traditional teacher-led manner. They were provided with a 1-hour teacher-led lecture every Tuesday, followed by a 2-hour teacher-led tutorial every Friday. During week 2 and week 3 the pre-service teachers from the treatment group were provided scaffolding, using increasing levels of problem complexity and degrees of self-directedness, to assist them to develop their collaborative skills for working productively in groups while solving the same tutorial tasks as the control group. The treatment group solved the tasks using a collaborative, student-centred, self-directed approach with only Socratic-style dialogue facilitation from the instructor. Then in weeks 4, 8 and 10 each of the four PBL treatment groups was given a real-world, open-ended problem related to the week’s topic and was required to work through a solution in the form of a lesson plan. In weeks 5, 9 and 11, each PBL group presented their lessons to their peers and received feedback from both their peers and the facilitator. In the last week of the semester, both cohorts of pre-service teachers were asked to respond to the post-test MPCKI and MTEBI.

The primary requirement of the PBL approach is to present real-world, open-ended problems for students to solve (Jonassen, 2011). The problems designed in the pilot were also scrutinised in terms of how well the pre-service teachers’ solutions would align with the intended learning outcomes of the mathematics education subject, which was to assist them to enact their PCK. Based on the lesson plan solutions presented by the treatment group of pre-service teachers, it was determined by the researcher that the problems given to them each week were appropriately designed to meet the subject’s intended learning outcomes. Furthermore, providing the PBL problems to the treatment group in one week, then requiring them to deliver their group’s lesson plan solution the following week, was observed as a practical arrangement considering the amount of time each iteration of the closed loop PBL process requires. However, in terms of aims 1 and 2 of the pilot, certain modifications were made to the main study which will be discussed in section 3.6. Aim 3 of the pilot study, the development and validity testing of the MTEBI and MPCKI, is described in the following section.
3.5 The Pilot Study: Aim 3

Aim 3 of the pilot study was addressed using a larger cohort of pre-service teachers (n=238) from the same subject but who were studying online. This larger sample allowed two scaled measurement instruments to be trialled and validated. In regards to teaching mathematics, the instruments were chosen or developed to produce evidence of the impact of closed loop PBL on pre-service teachers’ self-efficacy, teaching outcome expectancy and PCK, and each instrument will be described separately. The first to be described is the Mathematics Pedagogical Content Knowledge Instrument (MPCKI). The MPCKI was designed specifically for this investigation to measure the pre-service teachers’ mathematics PCK in specific mathematics topics related to the subject the pre-service teachers were studying. The subsequent section provides a description of the pre-existing Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) designed by Huinker and Enochs (1995) to measure pre-service teachers’ personal mathematics teaching efficacy and mathematics teaching outcome expectancies, and used in this study.

3.5.1 Development of the Mathematics Pedagogical Content Knowledge Instrument (MPCKI)

A literature search identified three potentially useful instruments for measuring pre-service teachers’ mathematics PCK. Firstly, Cheang et al. (2007), from the National Institute of Education in Singapore, initiated a project titled Knowledge for Teaching Primary Mathematics (MPCK Project). For the project they developed a 16-item, short answer questionnaire to measure some aspects of mathematics PCK. Eight of the 16 items covered topics associated with whole numbers, fractions or decimals, which were not the content covered in the mathematics subject used in this investigation, and in which the pre-service teacher participants were enrolled. The other eight short answer items were a mix of measurement and geometry questions that aligned with the specific subject’s content. Of those eight items, several were noted by the researcher as potential items for inclusion into the instrument under development for this study - the MPCKI.

In Australia, Callingham and Beswick (2011) presented a report regarding an instrument developed for their national Building the Culture of Evidence-based Practice in Teacher Preparation for Mathematics Teaching (CEMENT) project. The CEMENT instrument consists of three scales: beliefs, MCK, and mathematics PCK. Of the three scales, two were of particular relevance to this study. The CEMENT team’s mathematics PCK scale,
which used a multiple-choice format, included item topics associated with measurement, geometry and probability, as well as whole numbers, fractions and decimals. Therefore, not all items on the instrument were deemed suitable for the purpose of this study. However, a select number of measurement, geometry and probability multiple-choice questions were considered as potential items for inclusion in the MPCKI.

In terms of measuring beliefs, the CEMENT team’s beliefs scale required the pre-service teachers to respond to 10 belief statements by indicating the extent of their agreement based on a five-point Likert scale. Unfortunately, the CEMENT project’s analysis of the belief scale did not separate self-efficacy from outcome expectancy.

The third useful instrument for measuring future teachers’ mathematics PCK came from the TEDS-M Study. In 2011, a set of items from the TEDS-M test were released. The test bank is comprised of 34 questions, 10 of which are mathematics PCK questions, which required either a constructed or multiple-choice response. Of the 10 mathematics PCK questions, five were perceived by the researcher as potential survey items for this study’s MPCKI due to their alignment with the content and learning objectives of the mathematics subject used in the main study.

These three studies (MPCK Project, CEMENT, TEDS-M) formed the basis for the development of the MPCKI with item contributions and permission from Callingham and Beswick (2011) and Cheang et al. (2007), and contributions and acknowledgement of the use of several released items of the TEDS-M (Australian Council for Educational Research for the TEDS-M International Study Centre, 2011). The items used from the three contributing instruments were chosen because of their real-world attributes and alignment with the content of the mathematics subject in which the student participants were enrolled.
Table 5 provides the list of the 12 scenarios of classroom teaching problems used in the MPCKI and identifies the contributing instruments for each, with one algebra scenario designed by the researcher. The MPCKI was designed with 54 multiple-choice items identified as measuring pre-service teachers’ mathematics PCK, as defined by this study and based on previously well-researched definitions of the construct. The 54 questions in the MPCKI are situated in 12 scenarios of classroom teaching (three algebra, three measurement, three geometry, and three statistics & probability). Each teaching scenario contains four to five of the 54 multiple-choice questions. Among the mathematical topics of the teaching scenarios are: parallelograms and rhombi, perimeter and area, length measurement, probability and graph interpretation, degrees of angles, patterning and solving equations.

Table 5
*Items in the MPCKI, Identifying the Original Instruments They were drawn from*

<table>
<thead>
<tr>
<th>MPCKI Item</th>
<th>MPCK Project (Cheang et al., 2007)</th>
<th>CEMEMT (Callingham &amp; Beswick, 2011)</th>
<th>TEDS-M (Tatto et al., 2008)</th>
<th>Designed by researcher</th>
</tr>
</thead>
<tbody>
<tr>
<td>1   Algebra</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2   Geometry</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3   Measurement</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4   Probability &amp; Statistics</td>
<td>√</td>
<td></td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>5   Geometry</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6   Geometry</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7   Probability &amp; Statistics</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8   Algebra</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9   Algebra</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10  Measurement</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11  Measurement</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12  Probability &amp; Statistics</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
As an example, a measurement question from the TEDS-M (Tatto et al., 2008) was modified into a mathematics teaching scenario on the topic of measurement for use on the MPCKI. The scenario was restructured on the MPCKI so that the pre-service teachers needed to respond to four multiple-choice questions in relation to the scenario (Figure 10). A similar structure was used for all 12 scenarios encompassing the 54 items on the MPCKI. The complete set of MPCKI questions can be found in Appendix C.

<table>
<thead>
<tr>
<th>When teaching children length measurement for the first time, Mrs. Brown prefers to begin by having the children measure the width of their book using paper clips, then again using pencils. Below, there are four possible reasons why Mrs. Brown would use this strategy to teach length measurement. For each, indicate whether you believe it is a Correct reason, Partially correct reason, or Incorrect reason.</th>
<th>Correct reason</th>
<th>Partially correct reason</th>
<th>Incorrect reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using familiar/different units enables understanding of what measurement is and that any object/unit with length can be used to measure.</td>
<td>[]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Using non-standard units of length to measure gives differing numbers of units for the same length and shows that we need standard units.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The teacher knows that the students will enjoy their work if they can use hands-on materials.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Using objects of different lengths helps children learn how to decide which unit/object is the most appropriate to measure a given length.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>

*Figure 10: Example Measurement Items in the MPCKI*

Following the selection and modifications of the MPCKI’s 12 mathematics teaching scenarios, the 54 individual items were tested for face validity, inter-rater agreement, and construct validity (Bornstein, 2004; Creswell, 2009; Drost, 2011; Graham, Milanowski, & Miller, 2012). Descriptions of the process and findings resulting from the face validity
interpretations, inter-rater agreement measure and the construct validity test are described in the following section.

### 3.5.1.1 Judgements Regarding Face Validity of the MPCKI

Face validity is a subjective estimate on the degree to which one might view how well a measure operationalises a construct (Bornstein, 2004; Drost, 2011). To test the items for face validity, the MPCKI was examined by a sample of five experts in mathematics PCK, including the researcher. All five were mathematics education academics from three Australian universities who were chosen on the basis of their experience in the field of mathematics PCK for pre-service teachers. The objective was to have each expert provide their respective judgments regarding whether they felt the MPCKI appeared to be a good measure of mathematics PCK (face validity). After an examination of the instrument, each of the experts agreed the 54 items within the MPCKI measured mathematics PCK.

### 3.5.1.2 Measuring Inter-rater Agreement

In general, inter-rater agreement is the percentage of how frequently two or more evaluators agree on the same rating to an identical situation (Gisev, Bell, & Chen, 2013; Graham et al., 2012). This was necessary so that cross checks and a consensus of the best fitting answers of the MPCKI could be established. The process for measuring inter-rater agreement began with each mathematics PCK expert completing the MPCKI (International Test Commission, 2014). Obtaining the appropriate consensus value has important implications for the validity of the study’s results (Stemler, 2004) since the conclusions which are formed from the analysis will only be as good as the quality of the measures (Bond & Fox, 2007). Therefore, prior to determining inter-rater agreement, a method for computing what level of agreement would be sufficient for this type of study needed to be established. There are three common methods for computing inter-rater agreement: intra-class correlation, the percentage agreement measure, and the Cohen’s kappa measure (Graham et al., 2012; Stemler, 2004).

The intra-class index is useful when there are five or more rating categories. Since the items in the MPCKI are not composed of five or more categories, the intra-class correlation index was deemed unsuitable for the inter-rater agreement test.
The percentage agreement measure is calculated by adding up the number of items agreed upon by the raters and dividing by the total number of items rated. Then multiplying that fraction by 100, yielding a percentage of agreement value. This method is considered easy to calculate, easy to conceptualise and easy to explain. One important disadvantage is that it does not account for agreement due to chance as do correlation measures. Hence, percentages of agreement calculations may report rates much higher than warranted (Hayes & Hatch, 1999). In short, the percentage agreement measure is not a statistical analysis which eliminates correlation coefficients between raters who agree by chance (International Test Commission, 2014). Thus, the index was considered an unacceptable measure for determining the inter-rater agreement amongst the five experts’ answers to the 54 questions of the MPCKI.

Cohen’s kappa measures the inter-rater agreement between two raters using a two-level rating scale of nominal/categorical variable, and where agreement due to chance is factored out (Graham et al., 2012; Pallant, 2011). Due to the restriction of measuring between only two raters, the Cohen’s kappa test was unsuitable for this study. Fortunately, Cohen’s kappa has been extended so that the number of raters can be more than two. Fleiss’ kappa can be used when nominal categories are assessed by multiple raters and, like Cohen’s kappa, corrects for agreement being reached by chance (Gisev et al., 2013). Fleiss’ kappa, however, does not assume that the raters have all assessed all items (Gisev et al., 2013). This was not a concern for this study as each of the five experts agreed to assess all 54 items. As a result, it was determined that the agreement percentage among the mathematics PCK experts’ ratings would be best determined using Fleiss’ kappa. Fleiss’ kappa values were calculated to determine inter-rater agreement with multiple raters using an online calculator (https://mlnl.net/jg/software/ira/). Figure 11 shows the value produced by the inter-rater agreement analysis (Geertzen, 2012).

<table>
<thead>
<tr>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 raters and 54 cases</td>
</tr>
<tr>
<td>1 variable with 270 decisions in total</td>
</tr>
<tr>
<td>no missing data</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1. answer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fleiss</strong></td>
</tr>
<tr>
<td>A obs = 0.848</td>
</tr>
<tr>
<td>A exp = 0.334</td>
</tr>
<tr>
<td>Kappa = 0.772</td>
</tr>
</tbody>
</table>

*Figure 11: Fleiss’ Kappa Value for the Inter-rater Agreement of the 54 MPCKI Items*
For most purposes, [Kappa] values greater than 0.75 or so may be taken to represent excellent agreement beyond chance, values below 0.40 or so may be taken to represent poor agreement beyond chance, and values between 0.40 and 0.75 may be taken to represent fair to good agreement beyond chance. (Fleiss, Levin, & Paik, 2003, p. 604)

### 3.5.1.3 Testing for Construct Validity of the MPCKI

Further empirical analyses of the 54 items using the Rasch model (Bond & Fox, 2007) were also used to endorse the legitimacy of the items’ response key. Rasch analyses provide indications of stability, replicability and fit of the test items. The fit statistic delivers evidence of inadequate fit of items based on the argument that “persons whose ability is close to the item’s difficulty should give a more sensitive insight into that item’s performance” (Bond & Fox, 2007, p. 43). As a result, any item responses which diverge excessively (incur the most unexpected responses), based on the difficulty level, from the ability level of the person sample can be flagged for closer scrutiny. Causes for why items diverge unacceptably from the model’s expectancy include being ambiguously written, scoring misinterpretation, or the item does not fit the ‘latent trait’. A latent trait is the fictitious straight line used to represent the theoretically perfect representation of the construct (Bond & Fox, 2007). These analyses of the Rasch model were obtained while testing the MPCKI for construct validity.

One approach to establish construct validity is to examine the fit of the instruments’ items to the underlying construct using Rasch modelling techniques (Bond & Fox, 2007). Fit indices returned from Rasch analyses provide indicators of unidimensionality and evidence of items within a test which do not fit the representation of the construct. Furthermore, logit values presented on a scale show the order of the items’ difficulty along the latent trait, as well as the amount by which the items vary in difficulty (Bond & Fox, 2007).

In order to establish a pre-test post-test design which also allowed both the MPCKI and MTEBI to be completed and analysed within the time allotted, these logit values and the associated statistics provided useful information. Using the results of the Rasch analyses, the 54 items of the MPCKI were organised into two tests for the main study, a 36 item pre-test MPCKI (Appendix E) and a 36 item post-test MPCKI (Appendix F). Each
MPCKI test was organised into equivalent levels of difficulty with 17 questions being used across both tests and each test shown to measure mathematics PCK. Additionally, the approach used to develop the two variations of the MPCKI reduced the potential testing and instrumentation threats. A testing threat suggests that an improvement on a post-test is a result of having taken a matched pre-test. Conversely, instrumentation threat can occur in quantitative measurement when the pre-test instrument and post-test instrument are not equivalent in terms of measuring the same construct (Fraenkel, Wallen, & Hyun, 2015; Johnson & Christensen, 2014; Withrow, 2013). Therefore, for the purposes of this study, the Rasch model was utilised to provide (a) evidence of a single mathematics PCK construct, (b) estimations of items which unexpectedly did not fit the model, (c) the order of difficulty of the MPCKI items along the fictitious straight line used to represent the construct and (d) information on the item difficulty of the items, which was used to split the 54 items into two equivalent item tests.

To establish construct validity of the MPCKI in the pilot study, all 54 items of the MPCKI were loaded into Qualtrics electronic survey software and presented to 238 on-line pre-service teachers during the first week of the pilot study. For studies involving the Rasch model, a minimum of 20 items and a sample size of 200 examinees are sufficient (Wright & Stone, 1979; Zubairi & Kassim, 2006). Item scoring was dichotomous (right or wrong) and the Qualtrics software automatically scored the items. Specifically, each item had three response choices. The correct response was coded as 1. The two incorrect responses were coded as 0.

Rasch model analyses of the data set were conducted using WINSTEPS Version 3.81.0 (Linacre, 2014b). The initial analysis was run to test for unidimensionality and to obtain the fit and difficulty levels of all the MPCKI test items against the latent trait. Along with providing this evidence as a data set, the Rasch model can illustrate the results graphically. Figure 12 illustrates the fit statistics in the form of the bubble chart (Bond & Fox, 2007) produced by WINSTEPS.
The fit of each item to the model is determined along the horizontal axis. Fit values are reported as a standardised $t$ scale and acceptable values fall between -2.0 and +2.0 (Bond & Fox, 2007). The size of each bubble is related to the estimate of the measurement error of that item, where larger bubbles indicate greater error (standard error) of that estimate (Bond, 2003).

Figure 12: Bubble Chart Showing how each of the 54 Items Fit the Scale Measured by the MPCKI

The item fit statistics underpinning the bubble chart reveal all but 2 items (33.1 and 30.3) fall on or within $+$ or $-$ 2 standard deviations. Fit statistics also provide indications of item misfit. Based on guidelines suggested by Bond and Fox (2007), for this type of multiple-choice test, items with mean square (MNSQ) values falling between 0.7 – 1.3 are considered to fit the model. MNSQ statistics are reported in two different ways: infit and outfit. The weighted infit statistic provides measures for items which are close to the person’s ability (Gracia, 2005). Ideally, when the infit MNSQ is 1, the observed variance is exactly the same as the predicted variance (Yu, 2010). The outfit statistic is not weighted, and is therefore more sensitive to the influence of outlying scores (Bond & Fox, 2007). Therefore, the outfit statistic “reflects large differences between observed and expected values for items that are far from the person’s ability” (Gracia, 2005, p. 7).
The summary fit statistics for this scale are shown in Table 6. The MNSQ infit and outfit statistics for all 54 items of the MPCKI fell in the 0.9 - 1.1 and 0.9 - 1.3 range, respectively; that is, these items appear to be behaving as expected and each item is a productive measure of the construct in the sense they are contributing information to the scale.

Table 6
*Item (N=54) Fit Statistics and Separation and Reliability Indices*

<table>
<thead>
<tr>
<th></th>
<th>INFIT</th>
<th>OUTFIT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MNSQ</td>
<td>ZSTD</td>
</tr>
<tr>
<td>MEAN</td>
<td>1.00</td>
<td>.0</td>
</tr>
<tr>
<td>S.D.</td>
<td>.04</td>
<td>1.0</td>
</tr>
<tr>
<td>SEPARATION</td>
<td>6.21</td>
<td>ITEM</td>
</tr>
</tbody>
</table>

The mean square infit and outfit statistic for the 54 items collectively were 1.00 and 1.00; therefore, the expected mean square value of 1.00 was achieved. The ZSTD for the infit and outfit are the standardisation of the fit scores, reported in various interval-scale forms such as *t* or *z*. Basically, the mean squares are transformed so they are distributed like *t*, with an expected mean of 0 (Bond & Fox, 2007), which again was achieved. The item reliability value produced by WINSTEPS was high at 0.97. Hence, all the items are working together consistently toward one measure. Based on these results, the MPCKI is determined as having good construct validity, and suggests the order of item estimates can be relied upon to be replicated with other suitable examinees (Bond & Fox, 2007).

In terms of item difficulty, the bubble chart (Figure 12), represents the order of item difficulty vertically with the upper most items representing the more difficult questions for the 238 pre-service teachers who took the MPCKI. Conversely, the items nearest to the bottom of the chart represent the less difficult items for the 238 pre-service teachers. The scaled intervals along the vertical axis illustrate the difficulty level of the items in relation to the mean in terms of logit values. “Based on the logic of order, the Rasch analysis software programs perform a logarithmic transformation on the item and person data to convert the ordinal data to yield interval data” (Bond & Fox, 2007, p. 29). Specific to this analysis, the actual performance of the 238 pre-service teachers and 54 MPCKI items determined the interval sizes which allowed for the difficulty of the items to be placed along an interval scale. Furthermore, measured estimates of the 54 items provide
clarification of how far apart these items are, relative to each other, based on the 238 pre-service teachers who took the test.

Not only can the Rasch model provide evidence of a single dimension along with the difficulty order of the items along that dimension in a bubble chart, but WINSTEPS can also produce a variable map which illustrates the measures by which the items varied in difficulty. In this way the performance of the pre-service teachers tested can be demonstrated on the same latent trait as the 54 MPCKI items, “and with similar meaningful ability distances revealed between the students” (Bond & Fox, 2007, p. xiii). The person-item mapping in Figure 13 illustrates the difficulty level of the items in relation to the mean (+M, set at 0) in terms of logit values, alongside the pre-service teachers’ ability levels with their mean denoted at M.

The MPCKI items are to the right of the vertical line, and pre-service teachers are to the left denoted by a #, where each # represents three pre-service teachers. According to the item statistics returned from the Rasch analyses, the 54 items were ordered by average ability (measure) that ranged from -1.91 for item 34.1 to 2.87 for item 36.1. That is, in terms of level of difficulty the 54 items were distributed across most of the -2 to +3 logit range suggesting good separation. Item separation is used as an indication of the item hierarchy and how well the items are separated by the participants taking the test (Linacre, 2012; Wright & Stone, 1999). Good item separation (> 3) and good item reliability (> 0.90) indicate the sample is large enough to draw conclusions about the hierarchy of item difficulties, and that the person sample size is large enough (Linacre, 2012, 2014a). The item separation and item reliability values returned from the Rasch analysis are provided in Table 6. The item separation (6.21) and item reliability (0.97) values signify that the order of item estimates defines a distinct hierarchy along the dimension, and can be relied upon to be replicated with other appropriate samples of pre-service teachers (Bond & Fox, 2007).
<table>
<thead>
<tr>
<th>MEASURE</th>
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<th>Map</th>
<th>ITEM</th>
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<td>more</td>
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<td>36.1</td>
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<td>39.4</td>
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<td>3</td>
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<td></td>
<td>36.2</td>
<td>41.5</td>
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</tr>
<tr>
<td>-2</td>
<td>+T</td>
<td>less</td>
<td>frequent</td>
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*Figure 13: Person-Item Map for the MPCKI Pilot Data (N=238)*

In relation to the participant sample (of pre-service teachers), given the range of varying difficulties of the items, the person mapping in Figure 13 shows lack of good separation suggesting the sample of pre-service teachers is a relatively homogenous group in terms of mathematics PCK knowledge. An examination of the statistics (Table 7) underpinning the person mapping provided by Rasch analyses shows the mean square infit and outfit...
values of 1.00 and 1.00, respectively, suggesting the pre-service teachers are behaving as expected by the Rasch model. However, Rasch reliability values are dependent upon the ways in which the items separate the people on the scale (Callingham, 2015).

<table>
<thead>
<tr>
<th>Table 7</th>
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<tbody>
<tr>
<td><strong>Participant (N=231) Separation and Reliability Indices</strong></td>
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<tr>
<td></td>
</tr>
<tr>
<td>MEAN</td>
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<tr>
<td>S.D.</td>
</tr>
<tr>
<td>SEPARATION = .42; ITEM RELIABILITY = .15</td>
</tr>
<tr>
<td>LACKING RESPONSES</td>
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<tr>
<td>VALID RESPONSES</td>
</tr>
<tr>
<td>CRONBACH α PERSON Raw Score “TEST” RELIABILITY = .65 (approximate)</td>
</tr>
</tbody>
</table>

The Rasch analyses returned a separation value of 0.42 which refers to the number of statistically distinct groups the pre-service teachers can be separated into. Such a separation value suggests this group of pre-service teachers is extremely homogenous in the way they answered the 54 items (ability measures between 0.62 for the most able person to -1.15 for the least able pre-service teacher in terms of answering the 54 items). This corroborates what the person-item map illustrates - even with a good separation of items, there is a clustering of the pre-service teachers who are responding to the items in much the same way. In essence, the items are well separated across six ability levels, but the items are not discriminating well between the high and low achievers. However, a person reliability Cronbach alpha value of 0.65, from a Rasch perspective, suggests there is correlation among the ways that these pre-service teachers have answered the 54 items. At the pre-test this type of person separation, and a lower person ability mean (M) compared to the item difficulty mean (+M), is what you might look for in a pre-test post-test situation. For example, in terms of this study, it is hypothesised that after learning has taken place and been measured, a greater separation between the control group and the PBL treatment groups’ level of mathematics PCK should occur. Furthermore, the researcher’s hypothesis proposes that both groups’ mean (M) scores would move higher towards the mean difficulty of the items, suggesting learning has taken place for both groups; but with the PBL treatment group’s mean moving significantly higher than the control group’s mean.
To review, Rasch model analyses of the data set were conducted to provide empirical evidence of the validity of the MPCKI. Firstly, the summary item statistics for the 54 measured items indicate the items are working together and collectively measuring a single construct, which has been described as mathematics PCK in this study.

Secondly, output from the person-item map (Figure 13), revealed the empirical hierarchy of the difficulty levels of the items as adequate across an appropriate range from easy to difficult. Additionally, each item difficulty was estimated on a logit scale. These indices identified that an appropriate spread of items were present along the model’s latent trait (Bond & Fox, 2007) measuring pre-service teachers’ mathematics PCK.

Thirdly, the related statistics revealed the amount by which the items vary in difficulty. These item statistics had practical applications; allowing the items to be separated, based on their levels of difficulty to create the pre-test MPCKI and post-test MPCKI for the main study (Appendix E and Appendix F). This is because items which band together on the same logit levels suggest these items are measuring approximately the same particular intellectual skill. This feature of Rasch allowed for the controlling of the MPCKI items presented to the pre-service teachers in each iteration (pre-test and post-test), so that both assessments were validated as measuring the same constructs at the same or similar difficulty level (Hill & Ball, 2004).

The last reason for utilising Rasch analyses was to identify items which were a misfit for the proposed scale. To identify misfitting items, the Rasch analyses provide estimates for each item and each pre-service teacher independently. For example, item 36.1 was flagged for closer inspection. As Figure 13 illustrates, and the item statistics from the Rasch analyses report, it was the most difficult item for this sample of pre-service teachers. The item statistics also identified item 36.1 as having the most negative correlation (-0.11). This statistic suggests the choice responses for the item may have been ambiguous, whereby the pre-service teachers whose ability level was low answered the question correctly by chance. Conversely, choice responses of pre-service teachers who incorrectly answered questions, despite the model’s expectation that the item was within the student’s zone of success, often appear in the item statistics as difficult (International Test Commission, 2014). However, after examining the item infit and outfit values, 1.03 and 1.31 respectfully, it was determined those values fall within the guidelines of 0.7 and
1.3 (Bond & Fox, 2007). Removing item 36.1 from the MPCKI and rerunning the Rasch test merely caused another item to show a similar negative correlation. This checking mechanism suggested item 36.1 did not detract from the unidimensionality of the scale. Therefore, based on the item statistics and MNSQ values, the 54 items appear to be working together in a cohesive manner. As a result, it was decided to keep item 36.1 as one of the 54 items of the MPCKI.

The previous sections have described the face validity, inter-rater agreement and construct validity testing of the MPCKI. The next section describes the pre-existing MTEBI, developed by Huinker and Enochs (1995), which was considered for this study to be a useful pre-existing instrument to measure pre-service teachers’ personal mathematics teaching efficacy and mathematics teaching outcome expectancies.

### 3.5.2 Measuring Self-efficacy for Teaching and Outcome Expectancy

It was considered important to investigate the impact closed loop PBL may have on pre-service teachers’ self-efficacy for teaching and teaching outcome expectancy based on the literature, with respect to the previously researched impact of PBL and the aims of the pilot. In order to measure self-efficacy for teaching and teaching outcome expectancy, a pre-existing instrument was examined. Based on its applicability to the pilot study, and the literature reviewed, Huinker and Madison’s (1997) MTEBI was examined to determine its suitability for the pilot study.

Designed for use with pre-service teachers, the MTEBI was composed of two subscales; a personal mathematics teaching efficacy subscale containing 13 items, and a mathematics teaching outcome expectancy subscale containing 8 items (Huinker & Enochs, 1995). The 21 items (Appendix D) had five response categories: Strongly Disagree, Disagree, Uncertain, Agree and Strongly Agree. Huinker and Madison (1997) initially validated the MTEBI with 324 pre-service teachers. A reliability analysis of the two subscales produced alpha coefficients of 0.88 for the self-efficacy for teaching mathematics subscale and 0.77 for the mathematics teaching outcome expectancy subscale. Alpha reliability coefficients at levels between 0.75 and 0.90 are acceptable to good (Gliem & Gliem, 2003). In 2000, the MTEBI was subjected to a confirmatory factor and model fit analysis which confirmed that the two subscales are independent and
showing good model fit, adding to the validity and reliability of the instrument (Enochs et al., 2000).

The 21 items of the MTEBI were loaded into the Qualtrics electronic survey software and trialled with the same 238 on-line pre-service teachers as the MPCKI. Having the pre-service teachers from the pilot study answer the MTEBI questions afforded this researcher the opportunity to confirm reliability of each of the subscales. The pre-service teachers’ responses were loaded into the Statistical Package for the Social Science (SPSS) Version 19 (IBM Corp.). Consistent with using the same 21 items and wording as Huinker and Madison, eight of the thirteen self-efficacy questions were negatively worded. As a result, these eight items were recoded prior to analysis. The reliability scale test statistics (Cronbach’s alpha coefficient) for the 13 self-efficacy items and 8 outcome expectancy items are provided in Table 8.

<table>
<thead>
<tr>
<th>Reliability Statistics Self-efficacy for Teaching</th>
<th>Cronbach’s Alpha</th>
<th>N of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.864</td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reliability Statistics Teaching Outcome Expectancy</th>
<th>Cronbach’s Alpha</th>
<th>N of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.792</td>
<td>8</td>
</tr>
</tbody>
</table>

According to the guidelines provided by Pallant (2011), both subscales within the MTEBI show good internal consistency when used with this group of pre-service teachers (Cronbach’s Alpha values near or above 0.8 are considered optimal).

The 21 items of the MTEBI were also subjected to principal components analysis using SPSS. Prior to performing the principal component analysis, the suitability of data for dimension reduction and factor analysis was assessed. The sample size was more than acceptable at over 200 with at least five cases for each of the variables. Inspection of the correlation matrix revealed the presence of many coefficients of 0.3 and above. Although some individual pairwise correlations were below 0.1, no item showed consistently low correlations between all pairwise comparisons. Further, the Kaiser-Meyer-Olkin value
was 0.89, exceeding the recommended value of 0.6 for adequate sampling (Kaiser, 1970, 1974), and Bartlett’s Test of Sphericity (Bartlett, 1954) reached statistical significance, supporting the suitability of the correlation matrix for factor analysis. Principal components analysis revealed the presence of three components with eigenvalues exceeding 1, explaining 28.9%, 14.8% and 5.9% of the variance, respectively. An inspection of the scree plot revealed a clear break at the third component. Using Cattell’s (1966) scree test, it was decided to retain the two components for further investigation. The two-component solution explained a total of 43.7% of the variance with the self-efficacy subscale items contributing 28.85% and the outcome expectancy subscale items contributing 14.82%. To aid in the interpretation of these two subscales, Oblimin rotation was performed. The rotated solution revealed both components showing a number of strong loadings, and all variables from each subscale loading substantially on only one component. The interpretation of the two components was consistent with Huinker and Madison’s (1997) and Enochs et al. (2000) findings of the MTEBI forming two subscales. The results of this analysis support the use of Huinker and Madison’s (1997) MTEBI in this study for measuring the pre-service teachers’ self-efficacy and outcome expectancy to address the two secondary research questions.

In summary, the pilot study informed the main study in three ways. First, the pilot study provided the means to determine whether the research design was feasible. Second, it allowed for the exploration of the application and implementation of the closed loop PBL method as the instructional intervention. Third, it provided for the validity and reliability testing of the MTEBI and MPCKI. Additionally, the following outcomes which resulted from the pilot study allowed for several design issues to be identified and modified where necessary prior to the main study.

### 3.6 Outcomes of the Pilot Study: Modifications made for the Main Study

The pilot study was undertaken in the same mathematics curriculum and pedagogy subject as the main study, but a year earlier. The pre-service teachers (N=30) who participated in the semester-long study formed a treatment group (n=15) and a control group (n=15). The control group was taught the subject content using a traditional teacher-led instructional approach. Alternatively, the treatment group was facilitated using the closed loop PBL method. As such, each group engaged with the same content
and weekly tutorial problems; however, the pedagogical approach used with each group was different. Two instruments were used to collect data in the pilot study. The multiple-choice items of the MPCKI developed for this study were used to measure pre-service teachers’ mathematics PCK and/or the ability to enact their PCK. The existing MTEBI was employed to measure pre-service teachers’ self-efficacy for teaching mathematics and mathematics teaching outcome expectancy. Based on the outcomes of the pilot, changes were made to the teaching intervention and research design in preparation for the main study.

Firstly, it was observed that the groups of pre-service teachers who favoured teaching early childhood students struggled with some of the higher level mathematics content. When those particular groups began working on the more challenging place value questions in week one, the group members struggled with the content. Consequently, they could not, or were reluctant to, provide input to the group. As a result, this strategy for building solidarity among the group members was considered ineffective. Furthermore, in subsequent weeks when these groups were given a teaching scenario which targeted middle school students’ difficulties, most members struggled with their own lack of MCK; and, the intended learning outcome of developing their mathematics PCK was limited. As a result, it was determined that for the main study, forming heterogeneous groups based on a mix of preferred teaching years would better suit the goals of PBL and the research design. In this way, group members with proficient content knowledge of higher grades mathematics would complement and support those group members whose MCK was limited.

Secondly, the findings from the analysis of the pre-service teachers’ responses to the MPCKI multiple-choice survey revealed no significant difference in overall mean scores at the conclusion of the intervention period. However, in terms of developed PCK, this outcome was considered a positive result. This conclusion by the researcher was reached as a result of the broader literature and an examination of the responses to both groups’ end-of-semester mathematics PCK exam questions. A meta-synthesis of eight meta-analyses of PBL studies suggests a multiple-choice survey designed to measure the attainment of content knowledge favours traditional instruction over a PBL pedagogical approach (Strobel & van Barneveld, 2009). Thus, it was considered by the researcher that perhaps the PBL intervention used with the treatment group was effective enough to cause
the non-significant findings between the two groups in terms of developed mathematics PCK. Subsequently, the researcher reflected on whether the MPCKI was an appropriate instrument for measuring pre-service teachers’ ability to enact their PCK. In other words, can or should the MPCKI be used to measure both constructs? The conceptual framework of the TEDS-M study suggests that the multiple-choice format of the MPCKI may have been inadequate to measure both mathematics PCK and the ability to enact the PCK (Tatto et al., 2011). Three of the four released questions from the TEDS-M test which measured enacting PCK were in constructed-response format and one was a multiple-choice item. The multiple-choice items were worth one point and the constructed-response questions were worth two points. Reasons why the test scored multiple-choice items differently than constructed-response items, for measuring particular aspects of PCK, are provided:

In theory, multiple-choice items can be used to measure any of the knowledge domains. However, because of the situated nature of teacher knowledge, this format does not allow respondents to provide detailed information of situations and demonstration of the knowledge that is required to teach mathematics. In contrast, open constructed-response items allow respondents to develop a response to a question and to demonstrate the depth of their thinking on mathematics knowledge and mathematics teaching knowledge. (Tatto et al., 2008, p. 42)

In short, the structure of the multiple choice items do not provide detailed insights into the reasons behind the pedagogical choices prompted by the items, and do not discriminate among different ‘levels’ of PCK (Chick, 2012). The researcher’s conclusion was further supported based on the inspection and observations of the treatment group’s more sophisticated responses to their PCK exam questions compared to previous groups taught. As a result of the MPCKI results from pilot study, and the potential challenge of assessing the complex nature of teachers’ PCK using multiple-choice items (Callingham & Beswick, 2011; Chick et al., 2006; Roche & Clarke, 2011; Tattoo et al., 2008), the decision was made to use the pre-service teachers’ end-of-semester exam mathematics PCK questions to measure their ability to enact their PCK in the main study. The decision to use separate instruments, the MPCKI to measure the pre-service teachers’ mathematics PCK using the multiple-choice structure, and semester exam PCK questions which require constructed responses to measure their ability to enact their mathematics PCK, is supported by research undertaken on PBL (Albanese & Dast, 2014; Dochy et al., 2003; Strobel & van Barneveld, 2009, 2015). This decision also elicited a consideration to split the main research question into two questions (one for investigating the impact of PBL on pre-service teachers’ PCK levels and one for determining PBL’s impact on their ability to enact their PCK).
Subsequently, it was concluded by the researcher that both constructs could be investigated and reported by altering the question. Initially the research question was written:

What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ mathematics PCK and/or their ability to enact their PCK?

Subsequently, the research question was changed to:

What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ mathematics PCK and their ability to enact their PCK?

Lastly, it was observed by the researcher that the treatment group of pre-service teachers were actively engaged in curriculum and pedagogical conversations during their group work, more-so than in previous groups taught. The conversations were consistently on-task and at the cognitive levels of application, synthesis and creation. This was evident based on the higher-order questions the group members asked and the group’s conversations which ensued as a result of responses and feedback they received to their questions. Also observed was the high level of learned PCK demonstrated by the pre-service teachers during their simulated lessons. Furthermore, this was the first subject that the pre-service teachers had studied in their degree which used a PBL pedagogical approach and as such, it was determined from the pilot that it was crucial to hear directly from the students about the value they placed on the PBL learning approach. As a result, it was decided to use interviews to obtain the pre-service teachers’ impressions of the impact of the closed loop PBL intervention on their learning and attitudes with respect to their mathematics PCK, self-efficacy for teaching and teaching outcome expectancy.

3.7 Chapter Summary

The pilot chapter allowed for the exploration of the application and implementation of the closed loop PBL method as the pedagogical intervention in the main study (Aims 1 and 2). Secondly, it allowed for the development and validity testing of the MPCKI and confirmatory factor analysis of the MTEBI (Aim 3), and then further inquiry into the MPCKI’s appropriateness to measure both PCK and the ability to enact PCK. Subsequently, the findings from the pilot data provided direction and justification for the
use of additional data collection techniques to answer the main research question and address the research problem. Hence, the MPCKI’s multiple-choice format was used in the main study to measure pre-service teachers’ mathematics PCK and eight end-of-semester PCK exam questions formed the constructed-response instrument used to measure pre-service teachers’ ability to enact their PCK. The researcher concluded that the MTEBI would be used again in the main study since it had been subjected to a confirmatory factor and model fit analysis which verified the validity and reliability of the pre-existing instrument. The previously validated MTEBI was regarded as useful in the sense it would provide the appropriate amount of evidence of the impact of the closed loop PBL instructional approach on pre-service teachers’ self-efficacy for teaching mathematics and mathematics teaching outcome expectancy. The pilot study also highlighted the need to include the student voice, through interviews that were designed to elicit their views on the efficacy of the PBL approach that they had experienced. It was expected that the inclusion of interview data will also enhance the trustworthiness of the study by providing another data source for the purpose of triangulation (Guba & Lincoln, 1989), and it will assist to explain the impact of PBL on pre-service teachers. As such, the findings of the pilot justified undertaking a full main study using a mixed methods approach. The next chapter describes the main study’s methodology used to investigate the research questions.
4 The Main Study

4.1 Overview

This chapter presents the methodological design considerations of the main study, as they were informed by the pilot. The participants, sites and additional and refined instruments used to collect the quantitative and qualitative data are described. Protocols for ensuring the quality of the research are also explained. The chapter concludes with an evaluation of the strategies employed to ensure the research was conducted ethically.

4.2 Research Design

The main study drew on the literature when considering how to combine qualitative and quantitative research methods by using the strengths of each in a way that best explores the research problem and examines the relationships between the study’s independent and dependent variables (Creswell, 2014; Johnson, Onwuegbuzie, & Turner, 2007). The MPCKI and MTEBI instruments were the data sources for the quasi-experimental non-equivalent comparison-group design of the main study (Creswell, 2014; Johnson & Christensen, 2014). The condition which makes this quantitative component of the study a non-equivalent comparison-group design is the use of a treatment group (PBL) and an untreated comparison (control) group, both of which are administered the same pre-test and a post-test. Due to the control and treatment groups being comprised from existing class groups, a random sample was not possible. When participants are not randomly assigned, an appropriate design to use is quasi-experimental (Johnson & Christensen, 2014). The MTEBI and MPCKI instruments were administered prior to the intervention and following the intervention to both the control and treatment groups. Hence, a pre-test, post-test design, to measure change over time of the treatment group and control group’s self-efficacy for teaching mathematics, mathematics teaching outcome expectancy and mathematics PCK, was employed to examine the impact of the PBL intervention and the traditional teacher-led instructional method, respectively.

In terms of measuring the pre-service teachers’ ability to enact or demonstrate their mathematics PCK, eight end-of-semester PCK exam questions (Appendix G) were used with both cohorts as the main study’s other quantitative component. Since the data were collected from the semester exams, following the intervention, a post-test only design with non-equivalent groups (Johnson & Christensen, 2014) was employed. This design
compared the post-test performance of the treatment group of pre-service teachers with that of the control group of pre-service teachers to investigate any differences in their ability to demonstrate their PCK.

The eight mathematics PCK questions from the exam were chosen based on their structure which required constructed-responses, allowing the students to demonstrate their ability to enact their mathematics PCK, as learned and experienced during the semester. Three of the eight exam questions, which are interrelated, are provided below:

1. Provide a revisit, through an orientating phase activity, which addresses the first and second steps of the four step process for teaching measurement, as outlined in this subject, which would introduce the concept of area of a circle to a Year 6 class. Make sure you mention the specific language and materials you would use.

2. Still revisiting the topic, continue outlining the activity which addresses the third step of the four step process for teaching measurement that would introduce to a Year 6 class the concept of area of a circle. Make sure you mention the specific language and materials you would use.

3. Now in the enhancing phase, outline the activity which addresses the fourth step of the four step process for teaching measurement that would allow a Year 6 student to scaffold their understanding of area of a rectangle to the area of a circle. Mention the specific language and materials you would use.

The responses to each question were scored based the pre-service teachers’ ability to: 1) describe and/or illustrate different ways to model a concept, 2) describe approaches for teaching a particular mathematical concept, 3) discuss the resources and student language used to support their teaching, and 4) make connections between concepts and topics, allowing students to generalise the knowledge (i.e. area of a rectangle to area of a circle). It was expected the analysis of the responses would provide evidence of the impact of the two teaching interventions on pre-service teachers’ ability to enact their mathematics PCK.

The eight exam questions were not subjected to reliability and validity testing using a software package prior to the main study. Rather, consistency with how the exam questions have been answered by over 1,000 past students, with the same demographic
characteristics, has remained stable, indicating a high degree of reliability (Corbin & Strauss, 2008a). In terms of face validity, an examination of the test questions from five experts in the field of mathematics education judged the questions as measuring the ability to enact mathematics PCK in pre-service teachers.

Additionally, semi-structured interviews were conducted with the treatment group of pre-service teachers’ at the end of the intervention to obtain their perspectives of how their mathematics PCK, their ability to enact their PCK, their self-efficacy for teaching and teaching outcome expectancy were affected by the PBL approach, in comparison to their previous experiences with traditional teacher-led instruction. This qualitative data was added to the main study to “confirm changes or lack of changes in the pre-test post-test analysis of the quantitative data and provide insight into the reasons for any changes” (Huinker & Madison, 1997, p. 111).

It was expected that the mixed methods design used in the main study would provide an expanded and deeper understanding of the impact of the PBL intervention on pre-service teachers’ mathematics PCK (Johnson & Christensen, 2014; Onwuegbuzie & Leech, 2005; Somekh & Lewin, 2009). Figure 14 provides a conceptual representation of the main study’s mixed methods research design.
4.3 Participants and Setting

The recruited participants (N=37) were pre-service teachers from a four-year Bachelor of Education degree, in their third year of the program, at a regional Queensland university in Australia. All participants were enrolled in a mathematics education subject at one of
two different campuses of the university. Thus, using convenience sampling, these two campus class groups naturally formed a control group and a treatment group for the purposes of the study. Convenience sampling is a quantitative sampling technique which involves including participants who are available and easily recruited (Johnson & Christensen, 2014). Also, by separating the treatment group and control group on different campuses, the risk of interaction between groups was largely avoided.

The participants of the control group (n=20) were taught by an instructor who agreed to teach the control group using a traditional teacher-directed pedagogy on one campus. The treatment group participants (n=17), who were situated on a different campus to the control group, were instructed independently of the researcher for the main study. By not taking on one of the teaching roles, the researcher was hoping to demonstrate whether the PBL intervention could be successfully implemented by other lecturers. The recruited lecturer, who typically uses a constructivist approach to teaching agreed to use the PBL pedagogical approach, as conceived by the researcher and described in the previous chapter, to deliver the subject. Prior to the start of the study the researcher provided the lecturer with professional development and relevant literature on the closed loop PBL framework, the practice of facilitating a PBL classroom, the importance of preparing groups to work collaboratively in a PBL environment and the art of using Socratic dialogue. The approach of the main study was designed in the same manner as the pilot study except for the modifications implemented as a result of the outcomes from the pilot, as described in the previous chapter.

4.4 Data Collection Method
Quantitative and qualitative instruments were used to collect data at different stages throughout the four month study to answer the three research questions:

1. What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ mathematics PCK and their ability to enact their PCK?

2. What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ self-efficacy for teaching mathematics?
3. What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ mathematics teaching outcome expectancy? Table 9 identifies the specific instruments, sorted by ‘Time’ (when they were used in the study at each data collection point), to collect data on the identified dependent variables (a) mathematics PCK, (b) enacting PCK, (c) self-efficacy for teaching mathematics and (d) mathematics teaching outcome expectancy, and which research question each instrument addressed.

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Instrument</th>
<th>Time</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pre-MPCKI</td>
<td>First week of semester</td>
<td>C and T</td>
</tr>
<tr>
<td>2 and 3</td>
<td>MTEBI</td>
<td>First week of semester</td>
<td>C and T</td>
</tr>
<tr>
<td>1</td>
<td>Post-MPCKI</td>
<td>Last week of semester</td>
<td>C and T</td>
</tr>
<tr>
<td>2 and 3</td>
<td>MTEBI</td>
<td>Last week of semester</td>
<td>C and T</td>
</tr>
<tr>
<td>1, 2 and 3</td>
<td>Semi-Structured Interviews</td>
<td>Last week of semester</td>
<td>T</td>
</tr>
<tr>
<td>1</td>
<td>Semester Exam Responses</td>
<td>Post-semester</td>
<td>C and T</td>
</tr>
</tbody>
</table>

Note: C=Control group; T=Treatment group

Data collection point 1 took place in the first week of the semester. Using hard copies, both the control and treatment group completed the MTEBI (Huinker & Madison, 1997) and the 36-item MPCKI pre-test (Appendix E). The pre-service teachers completed both surveys in approximately 40 minutes. This same process was followed in the last week of the semester, at data collection point 2, using the same MTEBI and the 36 item MPCKI post-test (Appendix F).

At data collection point 3, qualitative data were collected post-intervention by audio-taping semi-structured interviews with the treatment group of pre-service teachers. The interviews (each approximately 15 minutes in duration) were conducted with 16 volunteer pre-service teachers from the treatment group during the last week of the semester. Fourteen were interviewed in person and two interviews were conducted by phone. Each interview was conducted one-to-one (student to researcher) in a confidential location. The interview process involved asking six pre-constructed questions (Table 10) and following
these with additional questions based on the pre-service teacher’s response to the pre-constructed questions.

Table 10
*Interview Questions Posed to the Treatment Group of Pre-service Teachers in the Main Study*

<table>
<thead>
<tr>
<th>Pre-constructed Interview Questions used in Main Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Was the PBL method different than the teaching approach used in your other subjects? How was it different?</td>
</tr>
<tr>
<td>2. Do you prefer the problem-based learning/teaching method which required you to work together to research and solve the task, to other learning/teaching methods? Why or why not?</td>
</tr>
<tr>
<td>3. How did PBL affect your understanding of teaching mathematics?</td>
</tr>
<tr>
<td>4. Do you feel the PBL teaching method has been effective in helping you develop your ability to teach mathematics effectively compared to having teacher-led lectures and teacher-led tutorials?</td>
</tr>
<tr>
<td>5. Would you use the PBL method when you become a teacher? If so, why?</td>
</tr>
<tr>
<td>6. Is there a way you would have rather been taught in this subject?</td>
</tr>
</tbody>
</table>

The pre-constructed interview questions were designed to obtain the pre-service teachers’ views of how closed loop PBL impacted the development and enactment of their mathematics PCK, their self-efficacy for teaching and how effective they felt closed loop PBL was in terms of being an effective pedagogy. The purpose for conducting the interviews was to incorporate the student voice with respect to the impact of PBL in comparison to their experiences with traditional instruction.

Data collection point 4 took place the week following the end of the semester, when the pre-service teachers sat their semester exam for the subject. The exam consisted of mathematics content questions, pedagogical questions and PCK questions. Data used from the exam were student responses to the eight mathematics PCK questions (Appendix G) which were similar to, and required similar responses as the real-world, open-ended problems posed during the semester.
4.5 Data Analysis

Prior to describing the method of analysis for each qualitative and quantitative data set in detail, Table 11 provides a summary of the alignment between each research question, each data set and the method of analysis used.

Table 11
Demonstration of Alignment between RQs, Data Collection Instruments and Data Analysis Method

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Data Source</th>
<th>How Data were Analysed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2 and 3</td>
<td>Semi-structured Interviews</td>
<td>Manual interpretative analysis aided by the QUAGOL guide. NVivo was used to store and analyse transcribed interviews, separate student responses by questions (as nodes) and create themes.</td>
</tr>
<tr>
<td>1</td>
<td>Pre/post MPCKI</td>
<td>ANOVA (comparison of means) and a Paired-samples t-test</td>
</tr>
<tr>
<td>1</td>
<td>Semester Exam Responses</td>
<td>Independent Samples t-test</td>
</tr>
<tr>
<td>2</td>
<td>Pre/post MTEBI</td>
<td>ANOVA (comparison of group means) and a Paired-samples t-test</td>
</tr>
<tr>
<td>3</td>
<td>Pre/post MTEBI</td>
<td>ANOVA (comparison of group means) and a Paired-samples t-test</td>
</tr>
</tbody>
</table>

4.5.1 Qualitative Data Analysis

The context and rich descriptions captured during the semi-structured interviews were transcribed verbatim and stored, using the NVivo (QSR, 2012) qualitative data analysis software package, version 10. In order to optimise the analysis and interpretation of the pre-service teachers’ interviews, the Qualitative Analysis Guide of Leuven (QUAGOL) was utilised (Dierckx de Casterle, Gastmans, Bryon, & Denier, 2012). The QUAGOL, inspired by the constant comparative method (Corbin & Strauss, 2008b), is designed as a guide which facilitates a comprehensive process of analysis of qualitative interview data. Outlined in Table 12, the guide is characterised by a process involving 10 stages which are divided between two phases. The first phase encompasses a systematic preparation of the coding process. The second phase utilises a qualitative software program to complete the systematic coding process.
At stage 1, and as a response to avoiding an over-reliance on a software package, or of moving too quickly or exclusively to coding the data, this study utilised NVivo within the guidelines of the QUAGOL framework. Moving through stage 1, the process included four steps:

1. Listening to each audio-taped interview.
2. Transcribing each interview verbatim into NVivo.
3. Proofing each transcribed interview by reading each one for accuracy while listening to the audio-tapes.
4. Storing each transcribed interview into NVivo.

Table 12
QUAGOL Method for Analysing, Interpreting and Summarising Qualitative Data (Diercks de Casterle et al., 2012)

<table>
<thead>
<tr>
<th>Stage</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Thorough (re)reading of the interviews (a holistic understanding of the respondent’s experience).</td>
</tr>
<tr>
<td>2.</td>
<td>Narrative interview report (a brief abstract of the key storylines of the interview).</td>
</tr>
<tr>
<td>3.</td>
<td>From narrative interview report to conceptual interview scheme (concrete experiences replaced by concepts).</td>
</tr>
<tr>
<td>4.</td>
<td>Fitting-test of the conceptual interview scheme (testing the appropriateness of schematic card in dialogue).</td>
</tr>
<tr>
<td>5.</td>
<td>Constant comparison process (forward-backwards movement between-within case and across-case analysis).</td>
</tr>
</tbody>
</table>

Phase 2: Actual Coding Process

<table>
<thead>
<tr>
<th>Stage</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>Draw up a list of concepts (a common list of concepts as preliminary codes).</td>
</tr>
<tr>
<td>7.</td>
<td>Coding process – back to the ‘ground’ (linking all the relevant prose to the appropriate codes).</td>
</tr>
<tr>
<td>8.</td>
<td>Analysis of concepts (descriptions of concepts, their meaning, dimension and characteristics).</td>
</tr>
<tr>
<td>9.</td>
<td>Extraction of the essential structure (conceptual framework or storyline).</td>
</tr>
<tr>
<td>10.</td>
<td>Description of the results (description of the essential findings).</td>
</tr>
</tbody>
</table>

During stage 2 the researcher again read each interview in order to write a narrative report on each interview, which included key quotations and summarised comments. The report
conveyed the essence of the interviewee’s story in relation to each of the study’s three research questions.

Stage 3 required the researcher to develop a conceptual interview scheme for each interview which would guide the coding process in the subsequent stages. At this stage the researcher located the concrete experiences within each interview. Along with the accompanying researcher’s narrative report, relevant concepts which appear across the experiences were identified. Essentially, working within the guidelines of the QUAGOL framework, themes emerged.

In stage 4 the appropriateness of the conceptual interview schemes created in stage 3 was determined. This stage marked the framework’s first iterative movement which required using a forward and backward, and between within-case and across-case movement, to identify themes common across the students’ stories (Dierckx de Casterle et al., 2012). The iterative process entailed rereading each interview while listening to the audio recordings, while also keeping in mind the related conceptual interview scheme. This stage served as a checking mechanism to determine whether the content populated into the conceptual interview schemes accurately reflected identified themes in relation to the three research questions. Conversely, the process served to check that the themes developed in the conceptual interview scheme aligned to the interview data. Basically, it was during this stage that the conceptual interview schemes were further developed or modified.

Stage 5 is characterised by another iterative forward-backward checking, but also included a within-case and across-case analysis. The purpose of this stage was to test previously identified essential common themes or identify new themes using a constant comparison process (Corbin & Strauss, 2008b). New or previously identified essential themes were checked for their presence across all 16 interviews which had not yet been evident to the researcher at that time. It is at this stage that existing schemes were adapted and refined, based on any new insights.

Stage 6 began the start of phase two, where the actual coding process begins by drawing up a list of preliminary common themes. Four themes were extracted from the data and identified as (a) effect of PBL on learning, (b) ability to teach more effectively, (c) reasons
for using PBL and (d) dissatisfaction with traditional instruction. The identified themes were added to NVivo as nodes. The nodes provided a way of organising the interview responses around the four common preliminary themes (Bazeley & Jackson, 2013). The list of themes was considered preliminary because the nodes were not yet populated with raw interview data. Thus, they were not yet empirically supported (Dierckx de Casterle et al., 2012).

In stage 7, with the aid of NVivo, the researcher manually coded each interview transcript. The process involved rereading each interview against the list of themes at hand, and characterising each significant passage of each interview with one of the four themes. As each passage was identified with one of the four themes, that text was dragged and dropped into the corresponding node (theme) created in the previous stage. At the end of this process the four themes were examined for their ability to capture all significant ideas, beliefs and suggestions as interpreted by the researcher. In essence, themes were confirmed, modified, added to or rejected as determined by the interview data and the researcher’s synthesis of the information.

The analysis conducted during stage 8 was guided by stage 7, when it was determined whether each passage fitted within each theme; or, whether other passages should be split into subthemes. Alternatively, a synthesis of the analysis might suggest combining various themes into one. At the completion of stage 8 the researcher determined that subthemes did exist which represent self-efficacy and outcome expectancy. These subthemes were identified as (a) new confidence and (b) control over student achievement. The interview data which conceptualised these themes and subthemes, based on the researcher’s synthesis of the information, will be presented in the next chapter.

At stage 9 the researcher synthesised the themes and subthemes into a coherent storyline. The process for obtaining a comprehensive and complete storyline involved revisiting the conceptual interview schemes, as a collection, and verifying the collection against each interview using the research questions as the basis.

In stage 10 the researcher confirmed that the students’ narratives created a storyline which addressed the research questions, yet retained the students’ voices. These voices will be
presented using specific quotations from the students’ interviews in the results chapter. Strategically, the storylines were designed to convey the students’ rich descriptions when answering each of the three research questions, alongside the quantitative results.

In terms of credibility, the QUAGOL framework allowed the researcher to avoid such issues as (a) an over-reliance on qualitative software packages, (b) use of a line-by-line coding scheme which sacrifices much of the contextual richness of the interviewee’s story, (c) coding from a pre-conceived notion and (d) loss of the integrity and uniqueness of each interviewee’s responses (Dierckx de Casterle et al., 2012).

### 4.5.2 Quantitative Data Analysis

For the three dependent variables (a) mathematics PCK, (b) self-efficacy for teaching mathematics and (c) mathematics teaching outcome expectancy, the pre-service teachers from both cohorts had a pre-test score and a post-test score from their completed MTEBI and MPCKI. Additional analysis was employed to examine the dependent variable ‘enacting mathematics PCK’ using the results of the pre-service teachers’ mathematics PCK end-of-semester exam questions.

Reliability and validity of the full set of 54 items of the MPCKI was established in the pilot study. Reliability and validity tests of the MPCKI pre- and post-tests were conducted by examining the item infit and outfit statistics and the Cronbach alpha (α) coefficients provided by Rasch model analyses using WINSTEPS Version 3.81.0. The mean square item infit and outfit statistics for the pre-test were 1.0 and 1.03, respectively. The mean square item infit and outfit statistics for the post-test were 1.0 and 1.0, respectively. With an expected mean of 0, the standardisation of the fit scores, reported as ZSTD infit and outfit values, were both 0 indicating good construct validity for both the MPCKI pre-test and post-test. The item reliability value produced by WINSTEPS was high (α > 0.95) for both the pre-test and post-test MPCKI indicating that both versions had good internal consistency.

To analyse both groups’ responses from the MPCKI at two time periods, overall PCK mean scores were calculated for each cohort at each time period. The overall mean scores for each MPCKI scale (pre- and post-test) were determined by adding up the raw scores of the 36 items for each pre-service teacher and calculating the average. The mean score
obtained from the MPCKI ranged from 0-1 as a result of each of the 36 responses being coded as a 0 or 1 [0-36/36].

The collected and analysed responses to the eight end-of-semester PCK questions were an independent data set measuring PCK in a different format to the multiple-choice questions of the pre-test and post-test MPCKI. The exam PCK questions (Appendix G) were similar to the real-world, open-ended type of problem responses required during the semester (Appendix A). These PCK exam questions required the pre-service teachers to design components of lesson plans pertaining to teaching measurement, geometry and probability and statistics to school-aged children. The student responses to the eight PCK exam questions, worth a total of 18 marks, were scored using a partial credit approach. The first PCK exam question was a 3-part, constructed-response question, with each part worth two marks for a total of six marks. The responses to the PCK exam question were scored either 0, .5, 1, 1.5, or 2 and coded using those marks. Scores awarded between 0 and 2 were determined as being answered to different degrees of correctness based on a marking rubric. A mark of 2 was applied to an entirely correct response. Conversely, a mark of 0 indicated a completely incorrect response. The second exam question was also a three-part, constructed-response question, but each part was worth three marks. The responses to the PCK exam question were marked using the same partial credit approach but on a zero to three scale (0, .5, 1, 1.5, 2, 2.5, or 3) and coded using those marks. The third question was a two-part question with each part worth two marks. This exam question was also marked using a partial credit approach (0, .5, 1, 1.5, or 2) and coded using those marks. The exam questions from both groups were scored by one person, the lecturer who taught the same subject but to the online students.

The overall mean scores from each MPCKI test for both cohorts were then calculated and analysed for interpretation. Additionally, the total scores for each cohort from the semester exam PCK questions were collected and compared using t-tests. The two instruments were used to investigate the study’s first research question:

What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ mathematics PCK and their ability to enact their PCK?
The MTEBI is composed of two subscales: a mathematics teaching efficacy subscale which contained 13 items, and a mathematics teaching outcome expectancy subscale which contained eight items (Huinker & Enochs, 1995), as determined by a factor analysis. The calculated means from the individual Likert item scores were analysed and interpreted as a summed rating scale, a Likert scale (Clason & Dormody, 1994; Johnson & Christensen, 2014; Leung, 2011). To analyse both groups’ responses from the Likert scale responses, overall mean scores were calculated for both cohorts in each of the two subscales. The raw scores of the 13 items of the self-efficacy subscale for each pre-service teacher from the control group were added and then the mean calculated. Thus, the mean score obtained from the self-efficacy subscale of the MTEBI ranged from 1-5 [13-65/13]. Likewise, the mean score obtained from the teaching outcome expectancy subscale of the MTEBI ranged from 1-5 [8-40/8]. The means from each subscale for both cohorts were then compared to investigate the study’s two secondary research questions:

What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ self-efficacy for teaching mathematics?

What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ mathematics teaching outcome expectancy?

The mean scores for each of the three dependent variables for both cohorts were compared using the Statistical Package for the Social Sciences (SPSS) version 22.0 (IBM, 2013). SPSS offers the user many different statistical models. In order to draw accurate conclusions, the correct statistical test must be chosen which is appropriate to the research question(s), the nature of the data, and the number of variables and groups in the study (Pallant, 2011). Additionally, assumptions should be met to ensure the results of the tests are accurate (Field, 2013). The remainder of this section describes the processes the researcher took to address these assumptions and determine which statistical tests were appropriate to accurately draw conclusions for each research question.

Initially, Pearson correlations were calculated to determine the strength of the relationship among the three dependent variables. The findings for each of the pairwise comparisons,
as highlighted in Table 13, show the relative strength of association between the three dependent variables. Due to the very low correlations between the dependent variables, this study confidently used a mixed between-within subjects analyses of variance (ANOVA) to compare the mean scores (between-subjects) at two points in time and within-groups across time (Field, 2013; Pallant, 2011). This method is also called a repeated measures ANOVA in SPSS, or split-plot ANOVA (Pallant, 2011). This study used two measures (pre-intervention and post-intervention).

Table 13
Pearson Correlations among the Dependent Variables

<table>
<thead>
<tr>
<th></th>
<th>SE</th>
<th>OE</th>
<th>PCK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Self-efficacy (SE)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pearson Correlation</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>37</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Outcome Expectancy (OE)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pearson Correlation</td>
<td>.025</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.883</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>37</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td><strong>PCK</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pearson Correlation</td>
<td>.076</td>
<td>-.152</td>
<td>1</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.654</td>
<td>.370</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>37</td>
<td>37</td>
<td>37</td>
</tr>
</tbody>
</table>

Prior to analysing the results from the ANOVAs, it was important to check that certain assumptions were not being violated. The general assumptions which apply to parametric tests when comparing group means include (a) dependent variables are measured on a continuous or interval scale rather than discrete or categorical scales, (b) measurements are not influenced by other measurements (independence of observations), (c) populations samples are normally distributed and (d) samples are obtained from populations of equal variance (homogeneity of variance) (Field, 2013).

First, the data collected from the MPCKI and MTEBI (as summated rating scales) were treated as continuous data, and classified as scale variables in SPSS. Second, in terms of independence of observations, the participants from each of the two cohorts were formed from two separate campuses. As a result, there were limited interactions between participants across cohorts. Third, to test whether the scores were normally distributed, a Shapiro-Wilk test was conducted in SPSS for each of the data sets (Razali & Wah, 2011). Tests of Normality statistics returned non-significant values indicating the samples did
not deviate significantly from normality. The fourth assumption to be met was that of homogeneity of variance/covariance. To test these assumptions, SPSS provides the Levene’s Test of Equality of Error Variances statistic. The Levene’s test returned a non-significant value indicating no significant deviation from equality of variances for each of the times (pre and post) between the two groups (treatment and control) (Field, 2013; Pallant, 2011). Although Field (2013) lists these four basic assumptions that must be met for these types of tests to be accurate, Pallant (2011) lists an additional assumption to be met. The additional assumption is that the scores are obtained using a random sample from the population; yet concedes, “this is often not the case in real-life research” (Pallant, 2011, p. 205). Johnson and Christensen (2014), also suggest that random sampling is not always practical. Given that convenience sampling was used in this study, and the sample sizes were moderate, caution was taken during interpretation of the results. With the assumptions for the parametric tests addressed, the consideration and justification for using a mixed between-within subjects ANOVA were established.

An independent samples \( t \)-test was considered for comparing the mean scores of the pre-service teachers’ semester exam PCK questions. Prior to conducting the independent samples \( t \)-test, the same five assumptions for parametric tests were addressed. The data collected from the marked, end-of-semester PCK questions were treated as continuous data, and classified as a scale in SPSS. Since the data collected from the participants are the same as those collected for the ANOVA, the independence of observations and random sampling assumptions had already been established. Tests of Normality statistics returned a non-significance value for this data set, indicating the samples for the PCK variable did not deviate significantly from normality. In terms of homogeneity of variance, the Levene’s Test of Equality of Error Variances statistics returned a non-significant value for the data set, indicating the variances of the two cohorts are equal for the PCK variable. Subsequent to meeting the assumptions, an independent samples \( t \)-test was conducted which compared the mean scores on pre-service teachers’ ability to enact mathematics PCK between the control and treatment groups’ coded end-of-semester exam PCK questions.

In summary, this study used quantitative and qualitative methods for collecting and analysing data in order to capitalise on the unique strengths of each method (McMurray, Pace, & Scott, 2004). However, ensuring validity in mixed methods research is complex.
This is because the researcher must parallel the qualitative criteria of credibility, transferability, interpretive validity and dependability with the quantitative rigor associated with validity and reliability (Denzin & Lincoln, 2013; Guba & Lincoln, 1989). The following section describes how the integrity of the research was maintained.

4.6 Quality of the Research

The aim of this study was to investigate the impact of the social constructivist, PBL teaching method on pre-service teachers’ PCK and ability to enact their PCK, self-efficacy for teaching and teaching outcome expectancy, compared to a traditional teacher-led instructional approach. In terms of each of the three research questions, these variables have been clearly identified within the study and been linked to the literature.

Threats to internal validity such as history and maturation were reduced by using a control group and a treatment group (Johnson & Christensen, 2014). By using a two-group design in this research, any differences between the two groups cannot be attributed to these internal threats as long as they affect both groups equally. Secondly, the internal validity threat of instrumentation was addressed. A Rasch analysis found both versions of the MPCKI to be measuring mathematics PCK, as defined by this study, at comparable difficulty levels. Fourthly, internal validity in terms of a testing threat was reduced by developing two variations of the MPCKI for the main study, both of which were determined by Rasch analyses to be measuring the same construct. Lastly, face validity was established by five experts in the field of mathematics PCK who examined the 54 items of the MPCKI and the eight semester exam PCK questions. In each instrument, the items were viewed by the experts as valid measures of mathematics PCK and the ability to enact PCK, respectively.

The research quality of the main study was enhanced further by employing a mixed methods approach. Firstly, the researcher added a qualitative component to the study. Semi-structured interviews were conducted to obtain additional evidence for answering the research questions. Secondly, each interview was conducted one-on-one and in a confidential location and the interview data were then collected, transcribed, transformed, and analysed using the 10-step process described by the QUAGOL framework. The QUAGOL guide provided objectivity to the qualitative analysis process by employing a
systematic guide for comprehensively and accurately identifying themes, as well as the interviewees’ experiences and inferred meanings.

Using a mixed methods approach in the main study allowed for different data sets to be collected, which were then analysed separately, and the results compared during interpretation; thus, allowing the quantitative and qualitative data to inform each other (Creswell, 2014; O'Leary, 2010). Furthermore, each method counteracted the other’s weaknesses. “For example, the inclusion of quantitative data can help compensate for the fact that qualitative data typically cannot be generalised. Similarly, the inclusion of qualitative data can help explain relationships discovered by quantitative data” (Onwuegbuzie & Leech, 2005, p. 383).

4.7 Ethical Considerations

The study had university Human Research Ethics approval (H13REA002) (Appendix H), prior to data collection, and was conducted in accordance with all required ethics protocols. Participation in the study was voluntary and participants were provided with a Participant Information Sheet and Informed Consent Form (Appendix I). Participant confidentiality was maintained and all data were de-identified. All students who agreed to participate in the study gave their informed written consent.

In the main study, which was a point of difference from the pilot study, it was considered important to remove the researcher as the instructor of the intervention group. This removed the problems generally associated with insider-research. As an insider-researcher, the researcher runs the risk of perceived power and bias (Unluer, 2012), in his/her dual role, and of “projecting one’s own views onto participants” (Greene, 2014, p. 4). As a result, this researcher removed himself from participation in the implementation of the PBL intervention and from marking any assessments in the subject. His interaction with the pre-service teachers was to conduct the semi-structured interviews at the conclusion of the intervention period. It was important for the researcher to collect these data so that he had first-hand knowledge of the students’ responses and reactions to the questions and to ensure the interviews were undertaken as similarly as possible with each participant. The researcher was also the author of the weekly recorded lectures that were available on the university’s leaning management system. These
recordings were accessible by all online and on-campus pre-service teachers provided they were enrolled in the subject.

4.8 Chapter Summary

The focus of this study was to compare the effectiveness of two different pedagogical approaches in a tertiary third year mathematics education subject. The chapter presented the conceptual framework which underpinned the methodology and context for the study. It reviewed and justified the use of the mixed methods, quasi-experimental comparison-group design to examine the impact of the closed loop PBL teaching method compared to a traditional teacher-led instructional approach on pre-service teachers in regards to the three research questions. The issues regarding reliability and validity of the instruments used to collect the data were presented and each data collection method described. The quality assurance techniques and protocols for collecting and analysing the qualitative and quantitative data were discussed. Ethical considerations were explored. This lays the groundwork for the next chapter which provides the results from the data analysis for each of the three research questions.
5 Results
The previous chapter described the research methodology of the main study. This chapter will begin with the demographic data of the two participant groups. Subsequently, the results of the data analyses from the main study are presented. For clarity and ease of reference, the results obtained for each of the three research questions will be presented separately.

5.1 Demographic Data
This study compared the effects of Barrows’ (1986) closed loop PBL pedagogical approach to traditional teacher-led instruction on pre-service teachers’ mathematics PCK, their ability to enact their PCK, their mathematics teaching self-efficacy and outcome expectancy. The participants involved in this study were 37 pre-service teachers in their third year of a four-year initial teacher education program at a regional university in Queensland, Australia. Table 14 summarises the demographic information for each group.

Table 14
Demographic Data for the Treatment and Control Group Cohorts of Pre-service Teachers

<table>
<thead>
<tr>
<th>Cohorts</th>
<th>Treatment group Campus 1</th>
<th>Control group Campus 2</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Students</td>
<td>17</td>
<td>20</td>
</tr>
<tr>
<td>Gender:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Females</td>
<td>88%</td>
<td>92%</td>
</tr>
<tr>
<td>Males</td>
<td>12%</td>
<td>8%</td>
</tr>
<tr>
<td>Age</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>20-45 yrs.</td>
<td>20-47 yrs.</td>
</tr>
<tr>
<td>Mean</td>
<td>28 yrs.</td>
<td>25.5 yrs.</td>
</tr>
<tr>
<td>Median</td>
<td>24 yrs.</td>
<td>21 yrs.</td>
</tr>
<tr>
<td>Prior Teaching Experience</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt; 10 days</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>10 – 15 days</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>16 – 25 days</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>&gt;26 days</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Teacher Aide</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
The participants of the control group and treatment group were from each of two different campuses of the university. The participants of the control group (n=20) studied at one campus, and were taught by an instructor who used a traditional teacher-led approach. The treatment group participants (n=17) studied at a different campus, and were facilitated by an instructor in a workshop environment using the closed loop PBL teaching approach. The level of mathematics PCK was tested for each group using the pre-test MPCKI. The results showed no significant difference between the groups at the pre-test. Likewise, the levels of self-efficacy for teaching and teaching outcome expectancy were tested for each cohort with the MTEBI at the pre-test. The findings revealed no significant difference between the two groups at the pre-test for either variable.

Similar to the pilot study, over the 10 on-campus classes, both groups were presented with the same content topics and real-world, open-ended tutorial problems related to the topics. The remainder of this chapter will present the quantitative and qualitative results of the study for each of the research questions in turn.

5.2 Research Findings for Research Question 1 (RQ1):

What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ mathematics PCK and their ability to enact their PCK?

In order to answer this research question, three instruments were used. The two quantitative measures were the MPCKI and the coded, end-of-semester, mathematics PCK exam questions. The MPCKI pre-test result was compared to the MPCKI post-test result for both groups, and eight items from the end-of-semester exam assessed the pre-service teachers’ ability to enact their PCK in both groups. The qualitative data were obtained from semi-structured interviews which solicited from the PBL treatment group their views of how PBL affected the development of their mathematics PCK; and, how effective they felt PBL was in terms of developing their ability to enact their PCK. It was anticipated that using this mixed methods approach would provide appropriate data to answer the research question by allowing the quantitative and qualitative data to inform each other (O'Leary, 2010).
5.2.1 Results of the MPCKI – Measuring the Level of PCK

A mixed between-within repeated measures ANOVA was conducted to assess the impact of the closed loop PBL teaching method (treatment group) compared to traditional teacher-led instruction (control group) on pre-service teachers' mathematics PCK mean scores at two points in time (pre-intervention and post-intervention). No significant group by time interaction was obtained, $F(1, 35) = .031, p = .86$ (Table 15). However, a significant main effect for time was found, $F(1, 35) = 14.33, p < .01$, partial eta squared = .29 with an increase in PCK scores from Time 1 to Time 2.

Table 15
Mathematics PCK Scores for the Treatment Group and Control Group at two Points in Time

<table>
<thead>
<tr>
<th>Point in Time</th>
<th>Pre-intervention</th>
<th>Post-intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>M</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>17</td>
<td>.431</td>
</tr>
<tr>
<td>Control Group</td>
<td>20</td>
<td>.413</td>
</tr>
<tr>
<td>Difference</td>
<td>.018</td>
<td></td>
</tr>
</tbody>
</table>

A paired-samples $t$-test was conducted to evaluate the impact of closed loop PBL on the treatment group of pre-service teachers’ mathematics PCK scores (Table 16). There was a statistically significant increase in PCK mean scores pre-intervention ($M = .431, SD = .083$) to post-intervention ($M = .500, SD = .088$), $t(16) = 3.035, p < .05$ (two-tailed). The mean increase in PCK scores was .069 with a 95% confidence interval ranging from .021 to .116. The eta squared statistic (.33) indicated a large effect size.

Table 16
Paired-samples $t$-test Mathematics PCK Results (Treatment Group of Pre-service Teachers)

<table>
<thead>
<tr>
<th>Point in Time</th>
<th>Mean</th>
<th>SD</th>
<th>$t$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-intervention</td>
<td>.431</td>
<td>.083</td>
<td></td>
</tr>
<tr>
<td>Post-intervention</td>
<td>.500</td>
<td>.088</td>
<td>2.453</td>
</tr>
</tbody>
</table>

$N = 17$  $df = 16$  Two-tailed  $p < .05$

Similarly, a paired-samples $t$-test was conducted to evaluate the impact of a traditional teacher-led instructional approach on the control group of pre-service teachers’ PCK scores (see Table 17). There was a statistically significant increase in PCK scores pre-
intervention ($M = .413, SD = .071$) to post-intervention ($M = .475, SD = .093$), $t(19) = 2.453, p < .05$ (two-tailed). The mean increase in PCK scores was .062 with a 95% confidence interval ranging from .009 to .116. The eta squared statistic (.27) indicated a large effect size.

The results indicate that both teaching methods were able to assist pre-service teachers to enhance their mathematics PCK from Time 1 (pre-intervention) to Time 2 (post-intervention). Figure 15 illustrates the profile plots comparing the two teaching methods pre-intervention to post-intervention for the treatment group and control group.

![Mean PCK Scores Over Time 1 and Time 2](image)

*Figure 15: MPCKI Results of the Mean PCK Scores from the Treatment and Control Group over Time 1 and Time 2*
5.2.2 Results of the Semester Exam – Measuring Enacting PCK

An independent samples *t*-test was conducted to compare the end-of-semester exam scores, indicating the two group’s ability to enact their PCK. There was a statistically significant difference in mean scores between the treatment group (M = 12.79, SD = 0.67) and the control group (M = 8.58, SD = 0.64); *t* (35) = 4.55, *p* < .001 (see Table 18 and Table 19).

Table 18
*Group Statistics for the Independent Samples t*-test for Mathematics PCK

<table>
<thead>
<tr>
<th>Subgroup</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCK scores</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment</td>
<td>17</td>
<td>12.79</td>
<td>2.7786</td>
<td>.6739</td>
</tr>
<tr>
<td>Control</td>
<td>20</td>
<td>8.575</td>
<td>2.8344</td>
<td>.6338</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td>4.219</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 19
*Independent Samples t*-test Statistics for Mathematics PCK

<table>
<thead>
<tr>
<th></th>
<th>Levene's Test for Equality of Variances</th>
<th>t-test for Equality of Means</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>Sig.</td>
<td><em>t</em></td>
</tr>
<tr>
<td>PCK scores</td>
<td>.072</td>
<td>.790</td>
<td>4.553</td>
</tr>
</tbody>
</table>

Figure 16 illustrates the box plot representing the group statistics. Although there was some overlap between the distributions, the mean increase in enacting mathematics PCK scores was 4.22 with a 95% confidence interval ranging from 2.34 to 6.10. The eta squared statistic (.37) indicated a large effect size. These results indicate the pre-service teachers facilitated using the closed loop PBL teaching method had significantly greater ability to enact their mathematics PCK compared with the control group of pre-service
teachers taught using traditional teacher-led instruction, as measured by the common PCK exam questions.

![Box Plot of the Mean Exam PCK Scores for the Treatment Group and Control Group](image)

**Figure 16:** Box Plot of the Mean Exam PCK Scores for the Treatment Group and Control Group

### 5.2.3 Results from the Interview Questions

The intention of this section is to report the qualitative results obtained from the treatment group’s interviews with respect to answering RQ1. Sourced from their lived experiences with the PBL intervention, each pre-service teacher’s interview responses were transcribed and stored in NVivo. The responses which pertained to RQ1 were categorised during the QUAGOL process within the identified themes: (a) effect of PBL on learning, (b) ability to teach more effectively and (c) dissatisfaction with traditional instruction. From those themes representative statements were extracted. These representative statements will be grouped and presented based on how the researcher interpreted the messages in the pre-service teachers’ stories in relation to the themes, and subsequently, RQ1. Each idea will be briefly discussed and illustrative quotations provided.
Many students commented on how PBL affected their understanding of teaching mathematics (levels of PCK). These comments were categorised during the QUAGOL process under three themes (a) effect of PBL on learning, (b) ability to teach more effectively and (c) reasons for using PBL. Representative statements of the pre-service teachers’ lived experiences are provided:

*It [PBL] made me be more engaged in the learning because I was excited to go and teach and make the lesson. We got to actually be a teacher and take the class and teach a lesson. So it was more real life.*

*It [PBL] gave me headaches (laugh). I had to really, really think. Because it enabled you to work together to solve a problem. So I had to be alert all the time. It gave me new ideas on how to teach, so new perspectives. So I have a bigger repertoire.*

*It [PBL] was more student-led. As a group we went and explored the different ideas and the resources to find out what we wanted to do to work out the actual method of how we were going to teach it. So we were trying to incorporate what we had learned into how we were going to teach it.*

*You got an insight or an aspect of seeing the way other people would teach. So you’ve got your own thoughts what you would do and then you see how they would teach it.*

When asked if they felt PBL had been more effective than traditional instruction for developing their ability to teach mathematics effectively, 15 of 16 students responded in the affirmative. Specifically, 11 answered “Yes”. Three answered “Definitely”, and one responded with “I think it was”. The remaining student did not answer directly; rather, the student responded in a manner which represented an explanation. The transcribed responses were categorised during the QUAGOL process under the themes (a) effect of PBL on learning, (b) ability to teach more effectively and (c) dissatisfaction with traditional instruction. Reasons which emerged regarding why they thought PBL was more effective than traditional instruction for developing their ability to teach mathematics effectively include:

- PBL provides teaching experience.
- PBL requires higher cognitive demand.
- PBL provides immediate feedback on learning.

*It [PBL] solidifies the approach that I was going to use to teach maths.*
I actually have just finished my prac and the first subject that I had to teach was Year 7 algebra. Learning in a problem-based learning environment... I think it really improved the way I was able to teach that lesson.

Having to actually get up and do it [teach] and using strategies... So you’re seeing what works and what doesn’t work. And you’re getting feedback as well on what they [peers and facilitator] think worked and what didn’t work.

It’s helped my mind learn to structure sequences for lesson planning in specific relation to mathematics.

The view provided by students regarding a dissatisfaction with traditional teacher-led instruction, in terms of developing their mathematics PCK is further demonstrated by the following representative responses:

- It [traditional instruction] doesn’t help my learning. It [traditional instruction] doesn’t make me think about what I should be learning to get the answers.
- It [traditional instruction] was all about recall and trying to remember things.
- With lecturing, I listen but it doesn’t make sense to me. I’ll forget it as soon as I walk out.

5.3 Research Findings for Research Question 2 (RQ2):

What impact will using closed loop PBL, compared to a traditional teacher-led instructional method, in a mathematics education subject have on pre-service teachers’ self-efficacy for teaching mathematics?

Two instruments, one quantitative and one qualitative, were used to answer this question. The pre-service teachers’ responses from the self-efficacy subscale of the MTEBI provided the quantitative comparison between the two groups. The rich descriptions collected from the treatment group’s semi-structured interviews allowed their views, regarding how the closed loop PBL teaching method impacted their self-efficacy for teaching, to be heard in greater depth than could be measured by the MTEBI alone.

5.3.1 Results of the MTEBI Self-efficacy Subscale

A mixed between-within subjects ANOVA was conducted to assess the impact of the closed loop PBL teaching method compared to traditional teacher-led instruction on the pre-service teachers’ self-efficacy mean scores at two points in time (pre-semester and post-semester). There was no significant interaction for the group by time analysis, $F (1,$
The main effect of time was significant, however, $F(1, 35) = 22.43, p = .000$, with both groups showing an increase in self-efficacy across the study period (Table 20). The main effect comparing the two types of teaching methods was not significant, $F(1, 35) = 1.16, p = .29$.

Table 20
Self-efficacy for Teaching Scores for the Treatment Group and Control Group at two Points in Time

<table>
<thead>
<tr>
<th>Point in Time</th>
<th>Pre-intervention</th>
<th>Post-intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>M</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>17</td>
<td>3.76</td>
</tr>
<tr>
<td>Control Group</td>
<td>20</td>
<td>3.63</td>
</tr>
<tr>
<td>Difference</td>
<td>0.13</td>
<td>0.24</td>
</tr>
</tbody>
</table>

A paired-samples $t$-test was conducted to evaluate the impact of closed loop PBL on the treatment group of pre-service teachers’ self-efficacy for teaching mathematics (Table 21). There was a statistically significant increase in self-efficacy mean scores pre-intervention ($M = 3.76, SD = .633$) to post-intervention ($M = 4.17, SD = .396$), $t(16) = 3.281, p < .01$ (two-tailed). The mean increase in scores was .41 with a 95% confidence interval ranging from .678 to .146. The eta squared statistic (.40) indicated a large effect size.

Table 21
Paired-samples $t$-test Self-efficacy Results (Treatment Group of Pre-service Teachers)

<table>
<thead>
<tr>
<th>Point in Time</th>
<th>Mean</th>
<th>SD</th>
<th>$t$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-intervention</td>
<td>3.76</td>
<td>.633</td>
<td></td>
</tr>
<tr>
<td>Post-intervention</td>
<td>4.17</td>
<td>.396</td>
<td>3.281</td>
</tr>
</tbody>
</table>

A paired-samples $t$-test was also conducted to evaluate a traditional teacher-led instructional approach on the control group of pre-service teachers’ self-efficacy for teaching mathematics (Table 22).
There was a statistically significant increase in self-efficacy mean scores pre-intervention \((M = 3.63, SD = .649)\) to post-intervention \((M = 3.93, SD = .568)\), \(t(19) = 3.389, p < .01\) (two-tailed). The mean increase in scores was \(0.30\) with a 95% confidence interval ranging from \(.479\) to \(.113\). The eta squared statistic \((.41)\) indicated a large effect size.

Table 22
Paired-samples t-test Self-efficacy Results (Control Group of Pre-service Teachers)

<table>
<thead>
<tr>
<th>Point in Time</th>
<th>Mean</th>
<th>SD</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-intervention</td>
<td>3.63</td>
<td>.649</td>
<td></td>
</tr>
<tr>
<td>Post-intervention</td>
<td>3.93</td>
<td>.568</td>
<td>3.389</td>
</tr>
<tr>
<td>(N = 20)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(df = 19)</td>
<td></td>
<td></td>
<td>Two-tailed</td>
</tr>
<tr>
<td>(p &lt; .01)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 17 illustrates the profile plots comparing the two teaching methods pre-intervention to post-intervention for the treatment group and control group.

*Figure 17: MPCKI Results of the Mean Self-efficacy for Teaching Scores from the Treatment and Control Group over Time 1 and Time 2*
5.3.2 Results from the Interview Questions

The pre-service teachers generally felt that as a result of learning with the PBL method and delivering lessons to their peers in a simulated classroom, they were more prepared to teach during their three-week practicum and as a future teacher. Student responses which exemplify this view were classified during the QUAGOL process under the themes (a) ability to teach more effectively, (b) reasons for using PBL, (c) dissatisfaction with traditional instruction and (d) the subtheme, new confidence. From those themes and subthemes, representative statements emerged from the pre-service teachers’ responses and provided additional clarifying data for RQ2:

I can probably go out into a classroom and teach the lessons that I taught and know what to do better, and I’ve got the confidence.

Now I know how to teach those sort of things. I learned a lot and I am more confident in teaching.

Being able to do it [teach in front of a class], it actually gives you the confidence, and you know then that you can put it into practice when you actually become a teacher.

It made me confident in the fact that we had to get in front of a class and do it in front of the other students.

Views regarding comparisons between traditional teacher-led instruction and PBL in terms of developing self-efficacy for teaching were also expressed by students, such as:

Instead of just like sitting there listening to how you could teach it…. If you just got given the answer I don’t think I would have learned as much.

The other subjects were teacher-led. They didn’t encourage discussion and collaborative learning. I need to know how to put things into practice. They never actually got us to put into practice - which is what I needed.

5.4 Research Findings for Research Question 3 (RQ3):

What impact will using closed loop PBL, compared to a traditional teacher-led instructional method, in a mathematics education subject, have on pre-service teachers’ mathematics teaching outcome expectancy?

The data for RQ3 were gathered from two instruments, one quantitative and one qualitative; (1) the pre-service teachers’ responses to the teaching outcome expectancy
subscale of the MTEBI, (2) along with the responses provided by the treatment group in
the semi-structured interviews.

5.4.1 Results of the MTEBI Outcome Expectancy Subscale
A mixed between-within subjects ANOVA was conducted to assess the impact of the
closed loop PBL teaching method compared to traditional teacher-led instruction on pre-
service teachers’ outcome expectancy mean scores at two points in time (pre-semester
and post-semester). There was no significant interaction effect between teaching methods
and time, $F (1, 35) = .04, p = .85$. The main effect for time was not significant, $F (1, 35) = 1.45, p > .05$ (Table 23). The main effect comparing the two types of teaching methods
was also not significant, $F (1, 35) = .32, p = .57$.

<table>
<thead>
<tr>
<th>Point in Time</th>
<th>Pre-intervention</th>
<th>Post-intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>M</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>17</td>
<td>3.58</td>
</tr>
<tr>
<td>Control Group</td>
<td>20</td>
<td>3.48</td>
</tr>
<tr>
<td>Difference</td>
<td>0.10</td>
<td>0.07</td>
</tr>
</tbody>
</table>

A paired-samples $t$-test was conducted to evaluate the impact of closed loop PBL on the
treatment group of pre-service teachers’ mathematics teaching outcome expectancy. The
analysis indicated there was no significant increase in outcome expectancy mean scores
pre-intervention to post-intervention. A paired-samples $t$-test was also conducted to
evaluate the traditional teacher-led instructional approach used with the control group of
pre-service teachers’ mathematics teaching outcome expectancy. Again, the analysis
indicated no significant increase in outcome expectancy mean scores pre-intervention to
post-intervention.

Figure 18 illustrates the profile plots comparing the two teaching methods pre-
intervention to post-intervention for the treatment group and control group.
5.4.2 Results from the Interview Responses related to Outcome Expectancy

The treatment group of pre-service teachers were not explicitly asked during their semi-structured interviews what impact closed loop PBL had on their teacher outcome expectancy development, as it was felt they may not fully comprehend the construct. Instead, evidence was extracted from their explanations of the positive impact they felt closed loop PBL had on their learning; and, their responses to the question, would you use PBL when you become a teacher, and if so, why? Student responses which represent those views were classified during the QUAGOL process under the themes (a) ability to teach more effectively, (b) reasons for using PBL and (c) the subtheme, control over student achievement. Representative statements were extracted and revealed details which provide insights into RQ3 as evidenced by:

*I did benefit from PBL... So, I think PBL would be more effective for students.*

*I think it’s a good way for students to learn and be engaged.*
I remember going to other classes and we all sit there.... Where with PBL everyone is engaged. I was listening to people around me and they are all talking and it’s all about the work. That’s definitely something I look at and go, well that would work with my students.

As a student if I’m benefiting from it [PBL] then the students, if done correctly, would benefit from it as well.

When asked if there was a better way they would have rather been taught, 13 of 16 pre-service teachers stated No, two of 16 stated I don’t think so, and one student did not answer directly; rather, the student responded in a manner which represented an explanation:

I was surprised by the lack of content that we were given. But other than that, I really enjoyed the way that we were given problems to solve to learn greater understanding.

Responses for why they believed there was not a better way to be taught the subject include:

I really enjoyed the way that we were given problems to solve to learn greater understanding.

Because it was literally giving us a lesson that we could be confronted with in a real life situation.

Students get to respond to a problem and the kids work well like that.

It works. I want to go out and find the answers for myself, otherwise I won’t remember it.

I think that’s [PBL is] the best way to learn.

5.5 Conclusion

The results for the three research questions, from the data that were collected in this study, are mixed in relation to the impact of PBL on pre-service teachers’ PCK, self-efficacy and outcome expectancy. The next chapter will synthesise the results and provide a possible explanation for the results that were discussed in this chapter, with reference to the relevant literature. Limitations of this research and directions for further research are also described.
6 Discussion and Conclusion

6.1 Overview

This study set out to investigate the effect of the social constructivist, closed loop PBL method (Barrows, 1986), in comparison to traditional instruction, on pre-service teachers’ PCK in a tertiary mathematics education subject. The literature review with respect to self-efficacy suggested that pre-service teachers may work harder and persist longer at maximising their levels of PCK if they have high teaching self-efficacy and believe that possessing sound PCK will have a positive effect on their students’ learning (Bandura, 2006a; Biggs, 1989; Enochs et al., 2000; Garvis, Pendergast, & Keogh, 2012). Bandura theorised that performance accomplishments, vicarious experiences and verbal encouragement or persuasion are effective types of experiences for creating a strong sense of efficacy (Bandura, 1994). Mastery experiences from performance accomplishments, such as those which a PBL intervention provides (Dunlap, 2005), were stated as the most powerful source in creating a strong sense of efficacy (van Dinther et al., 2011). Hence, a relationship was hypothesised to exist between levels of self-efficacy for teaching (Bandura, 1977) and the goals and characteristics of the PBL teaching model (Savery, 2015). The nature of the relationship is based on creating a strong sense of efficacy by using the PBL method which allows the pre-service teachers to experience mastery from delivering solutions to real-world situations they will encounter in their teaching career, being provided with positive feedback regarding their teaching, as well as observing their peers teaching successfully (vicarious experiences). Teachers who judge themselves capable of promoting academic success evoke academic attainments in their students regardless of whether they teach advantaged or disadvantaged students (Bandura, 1994). Consequently, self-efficacy for teaching mathematics and mathematics teaching outcome expectancy were considered worthy dependent variables to be tested. Figure 19 illustrates the theoretical model for the PBL pedagogical intervention used with the treatment group of pre-service teachers.
Figure 19: Theoretical Framework for the PBL Intervention of the Research (Bandura, 1977; Dewey, 1938; Shulman, 1986; Vygotsky, 1975)
The literature review also revealed studies which used different types of questionnaires to capture the complex elements of pre-service teachers’ mathematics PCK. Some researchers preferred using multiple-choice questions while others used constructed-response formats depending on the attribute of PCK being measured (Callingham & Beswick, 2011; Cheang et al., 2007). Because of PCK’s multifaceted nature and complex interactions, the developers of the TEDS-M test used both types of questions (Tatto et al., 2008). This test consists primarily of multiple-choice questions to measure pre-service teachers’ mathematics PCK and constructed-response items to measure their ability to enact their PCK. The reasoning for measuring these PCK constructs differently is that they are different facets of PCK (Chick, 2012; Tatto et al., 2008). Findings that enacting mathematics PCK is difficult to measure using multiple-choice items suggest that “there are different levels of PCK…and unless we ask students/teachers the basis for their decision we do not always get the full picture of their PCK” (Chick, 2012, p. 8). Succinctly, a multiple-choice format does not allow pre-service teachers the ability to elaborate on situations and demonstrate the full range of their knowledge that is required to teach mathematics. On the other hand, constructed-response items allow pre-service teachers to provide different explanations of student problems thus differentiating the respondents based on the detail of their interpretations and ability to provide developmental pathways for students (Hill et al., 2008).

In the context of PBL studies, when constructed-response assessments require a level of elaboration beyond what multiple-choice questions can provide, students who learn using a PBL pedagogical approach appear to perform better than those taught with traditional instruction. Furthermore, when multiple-choice questions were situated in teaching and learning scenarios, traditional instruction was favoured (Albanese & Dast, 2014; Strobel & van Barneveld, 2009, 2015). As a result, this researcher chose to use both question types. The MPCKI tested students’ levels of mathematics PCK using a multiple-choice format, while the end-of-semester exam PCK questions gauged the pre-service teachers’ ability to enact their PCK using constructed-response items.

Additionally, the literature presented in this study found that some studies used qualitative methods such as journals and interviews to obtain additional data on the development of pre-service teachers’ PCK or the impact of PBL on their PCK (Chick, 2007; Chick & Beswick, 2013; Goodnough, 2003; McCray & Chen, 2012), suggesting
that using both quantitative and qualitative data collection methods can advance substantive understanding (Thorne, 2008). It was therefore decided to use multiple instruments within a mixed methods approach in this study to compare the impact of the two instructional approaches, namely, (a) the MPCKI (multiple-choice questions), (b) end-of-semester exam PCK questions (constructed-response items), (c) the MTEBI (self-efficacy and outcome expectancy student surveys) and (d) student interviews.

The MPCKI was created primarily from an amalgam of previously existing instruments and validated by the researcher and was considered, on the basis of the statistical analysis, to be a reliable measure of mathematics PCK in pre-service teachers, as defined by this study. The MPCKI consisted of one original item developed by the researcher along with 11 items that were modified from those used by Callingham and Beswick (2011), Cheang et al. (2007) and the TEDS-M (Australian Council for Educational Research for the TEDS-M International Study Centre, 2011; Tatto et al., 2008).

Measuring pre-service teachers’ ability to enact their mathematics PCK, and independent of the multiple-choice survey items from the MPCKI, were the eight end-of-semester exam PCK questions. The PCK exam questions were designed to be similar to the real-world, open-ended questions used during the weekly tutorials with both the control and treatment groups during the semester. The main difference between the two groups was the pedagogical approach used to present the content (described in section 3.4). This design ensured both groups were exposed to the same subject content, albeit using different pedagogical approaches. The eight end-of-semester exam PCK questions were chosen for this study for two reasons. Firstly, measuring pre-service teachers’ ability to enact their mathematics PCK using constructed-response items is strongly supported in the literature (Ball et al., 2008; Callingham & Beswick, 2011; Cheang et al., 2007; Chick et al., 2006; Tatto et al., 2008). Thus, the eight PCK exam questions chosen for this study were designed as constructed-response items to enable the pre-service teachers to respond in a way that comprehensively demonstrated their ability to (a) model a concept, (b) teach a particular mathematical concept, (c) describe resources and student language used to support their teaching and (d) make connections between concepts and topics, allowing students to generalise the knowledge. Secondly, the eight PCK exam questions were chosen based on the consistency with which the exam questions have been answered by over 1,000 past students, with the same demographic characteristics,
indicating a high degree of reliability (Corbin & Strauss, 2008a). Further, in terms of face validity, an examination of the test questions by five experts in the field of mathematics education judged the questions as measuring the ability to enact mathematics PCK in pre-service teachers.

The MTEBI (Huinker & Enochs, 1995) is well established for accurately measuring pre-service teachers’ mathematics teaching efficacy and mathematics teaching outcome expectancy (Briley, 2012; Enochs et al., 2000; Huinker & Madison, 1997; Moseley & Utley, 2006), and proved to be a useful instrument in the pilot study.

The treatment group student interviews used in the study closely examined the impact of the PBL intervention on students’ perceptions of the way they were required to engage with the content. As a result, the pre-service teachers from the PBL group individually shared their views and ideas during post-intervention semi-structured interviews about how PBL had affected their self-beliefs and mathematics PCK compared with their experiences with traditional instruction used in their other subjects.

However, the differences between the two groups varied, by data source, for each of the three research questions under investigation. This chapter will discuss the results of the study for each research question sequentially and will endeavour to resolve the apparent contradictions found in the results, making appropriate links to the literature. The chapter will close with a synthesis of the results and a discussion of the limitations of the study, its implications for educators and recommendations for further research.

### 6.2 Interpretation of Findings for RQ1

*What impact will using closed loop PBL, compared to a traditional teacher-led instructional approach, in a mathematics education subject, have on pre-service teachers’ mathematics PCK and their ability to enact their PCK?*

To measure and assess the impact of closed loop PBL on pre-service teachers’ mathematics PCK levels, two of the four instruments from this mixed methods approach were used: (a) the MPCKI and (b) the student interview questions. To measure and assess the impact of closed loop PBL on the pre-service teachers’ ability to enact their PCK, the end-of-semester exam PCK questions and student interview questions were utilised.
The comparative levels of mathematics PCK for both groups of pre-service teachers were tested at the start of the study using the MPCKI pre-test and were found to exhibit no significant difference. The results of the ANOVA which compared the pre-test MPCKI and post-test MPCKI mean scores indicated no significant difference in the mathematics PCK of the two groups at the conclusion of the intervention period. Thus, it appeared as expected that the PBL intervention had not achieved an improvement in the treatment group’s mathematics PCK when compared to the control group’s mathematics PCK. In contrast to the results obtained with the MPCKI, the results from the end-of-semester exam PCK questions indicated that the PBL treatment group were able to enact or demonstrate their mathematics PCK at a more advanced level than the control group at the end of the study. Further, the student interview responses indicated that the closed loop PBL approach was considered favourably by the students who participated in the study in comparison to their experiences in their other subjects which used a traditional teacher-led instructional approach. Possible reasons for these results are considered in the following sections.

6.2.1 Discussion of the results for RQ1

The MPCKI’s multiple-choice items and end-of-semester exam’s constructed-response questions were designed to measure mathematics PCK and the pre-service teachers’ ability to enact their PCK respectively, as defined by this study. Thus, it was theorised that both instruments would demonstrate significant outcomes based on their respective functions for assessing each teaching method. However, the results indicated no significant difference between the groups with respect to the level of their PCK at the end of the study, but the PBL intervention group’s ability to enact or demonstrate their PCK was significantly higher than the control group’s ability.

The MPCKI was used to determine that both groups had a similar level of mathematics PCK before the intervention commenced. During the four month intervention period the two groups were exposed to different pedagogical approaches. At the end of the intervention the MPCKI was used to compare the control group and treatment group levels of mathematics PCK. The ANOVA results indicated no significant difference in the effectiveness of the two types of teaching methods. In fact, the results of the paired-samples $t$-tests of each group’s mathematics PCK scores indicated that both teaching
methods were effective in assisting pre-service teachers to develop their mathematics PCK over the course of the study. Based on the responses to the end-of-semester PCK exam questions however, it was found that the PBL group were more competent at demonstrating their ability to enact their mathematics PCK than were the control group, even though both groups had similar levels of mathematics PCK. Two conclusions regarding these results were ultimately reached.

Firstly, a multiple-choice format is not aligned with the type of learning outcomes PBL facilitates and is therefore inappropriate to measure the effects of PBL on learners (Albanese & Dast, 2014; Dochy et al., 2003; Strobel & van Barneveld, 2009, 2015). The findings from a synthesis of eight meta-analyses and systematic reviews of PBL studies conducted over the past 23 years have specified that when multiple-choice questions are used to assess knowledge, traditional instruction was favoured (Strobel & van Barneveld, 2015). Based on this research, and in relation to assessing mathematics PCK in this study, the results of the MPCKI should therefore have favoured the control group of pre-service teachers taught using a traditional teacher-led approach. This was not the case. Both groups performed equally well on the post-test MPCKI. Thus, in this study, closed loop PBL did not hinder the development of the pre-service teachers’ PCK and it was as effective as traditional instruction, when PCK is measured using a multiple-choice test.

Secondly, PBL characteristically produces favourable outcomes when learning is measured using performance or skill-based assessments, similar to the learning which was facilitated by the PBL intervention used in this study (Strobel & van Barneveld, 2015; Walker & Leary, 2009). Support of this statement is found in a meta-analysis of 43 studies on the positive impact of PBL on the enacting of knowledge across a mix of tertiary education disciplines (Dochy et al., 2003). In terms of measuring the ability to enact mathematics PCK, other researchers support the use of PBL as the teaching method and constructed-response items as the assessment type (Chick, 2012; Hill et al., 2008; Strobel & van Barneveld, 2009; Tattö et al., 2008; Walker & Leary, 2009). In this study, both the treatment and control groups created solutions to real-world, open-ended problems in the form of a lesson plan during class time. However, how each group acquired the information to do so and demonstrated mastery of the subject matter were different. The PBL group were required to self-discover and self-direct their own learning while the PBL tutor assumed the role of facilitator. Additionally, the treatment
group of pre-service teachers were required to enact their PCK in a simulated classroom (performance accomplishment). The control group of pre-service teachers were provided direct instruction in the use of teaching strategies and resources which were used to create their lesson plans, but they did not deliver their lessons in a simulated classroom. Their class time was allocated to discussing, using teacher-led discussion, how they would teach the content to children (e.g., enact their mathematics PCK). Subsequently, the eight end-of-semester exam PCK questions answered by both groups of pre-service teachers were designed to measure their ability to enact their mathematics PCK similar to what they had been required to do throughout the tutorials. The two groups’ responses to the exam questions indicated the PBL group of pre-service teachers demonstrated a higher level of ability to enact their PCK than the control group of pre-service teachers. This study’s finding aligns with meta-analysis findings on the positive impact of PBL on outcomes related to application of PCK in teacher education studies (Walker & Leary, 2009). The studies which used closed loop PBL as their pedagogical intervention “indicated some of the largest findings in favour of PBL ($d = 0.54$)” on outcomes at the concept, principle, and enacting level (Walker & Leary, 2009, p. 23). Therefore, the conclusion reached is that the closed loop PBL method used in this study was more effective than the traditional teacher-led approach for developing pre-service teachers’ ability to enact their mathematics PCK when measured using an instrument requiring constructed-responses. The PBL group were in reality better able to teach the required mathematics content than the control group.

The above findings also explain the favourable student interview responses obtained from the treatment group of pre-service teachers. The students all agreed that learning using the PBL approach made them more engaged than traditional instruction and that it positively affected their learning and their ability to enact what they had learned about teaching mathematics. Further discussion is provided in the following section, again making appropriate links to the literature.

### 6.2.1.1 Reflecting on the Semi-structured Interview Responses

The control group of pre-service teachers were not interviewed. Several of the interview questions required the treatment group of pre-service teachers to compare their views on PBL to their experiences with the traditional teacher-led approach used in their other subjects (see Table 10, page 92). For example, interview question 4 asked the pre-service
teachers if they felt PBL had been more effective than traditional instruction for developing their ability to teach mathematics effectively. The interview question asking students about their perception of the PBL approach in relation to how they prefer to learn revealed a nearly unanimous response. Only one student indicated that he was not in favour of the PBL approach because he “disliked working in groups”. The rest of the students indicated they thought PBL “was really effective” compared to other subjects where they only “talk about teaching but don’t practice it”. Almost unanimously they found PBL “very beneficial” and they indicated that they preferred “to learn that way”. One student stated that she “liked being given the problems and going to find the answers”.

The viewpoints shared by this study’s PBL treatment group of pre-service teachers are supported by the literature (Askell-Williams, Murray-Harvey, & Lawson, 2007; Downing, Ning, & Shin, 2011; Edwards & Hammer, 2007; Goodnough, 2003). One such study conducted in a science education methods subject used informal conversational and semi-structured interviews as data collection instruments (Goodnough, 2003). The researcher reported that most of the 28 participating pre-service teachers favoured the PBL experience. “Twenty-one students gave it a strong endorsement, while….four students who disliked the PBL experience did not like group work” (2003, p. 12). The other three pre-service teachers felt they could have learned the content equally well individually.

Further support of this study’s qualitative findings were found in two other studies (Askell-Williams et al., 2007; Edwards & Hammer, 2007). In each investigation interviews were conducted with pre-service teachers enrolled in a semester-long childhood development subject taught using PBL. In Askell-Williams et al. (2007), the pre-service teachers were asked questions related to how the PBL model impacted their knowledge of strategies for teaching and learning. Sample responses from that study include:

_I have learnt a lot more to do with teaching than I have over the past two years of university_ (Askell-Williams et al., 2007, p. 248).

_It [PBL] is an effective way of learning, and… I have realised how much more I have gained from this approach than I probably would have with a more traditional approach_ (2007, p. 250).
Indicative pre-service teachers’ responses provided by Edwards and Hammer (2007) include:

*Application of theories about learning in a ‘real’ scenario has made it easier to understand the theories better* (Edwards & Hammer, 2007, p. 32).

*By making the scenario real it was putting theory into practice* (2007, p. 32).

Rather than using interviews to collect data about student experiences, Downing et al. (2011) used questionnaires with first-year, undergraduate architecture students to measure perceptions of their PBL learning experience compared to traditional instruction. The researchers employed a two-group (control and treatment), pre-test post-test design. The duration of the study was 15 months which covered three semesters of study. The impact of PBL on the students’ experience was reported as significantly greater than that experienced by the students taught using traditional instruction. Statistical significance was found also in the students’ perception of their generic skills development. The Downing et al. (2011) study was significantly longer in duration to this study, but similar results have been achieved.

The conclusion reached by this researcher is that PBL, even when used in only one semester, can positively impact pre-service teachers’ ability to enact their mathematics PCK, and their positivity towards the PBL pedagogical approach as demonstrated in their interviews, bodes well for their continuing development of mathematics PCK.

### 6.3 Interpretation of the Findings for RQ2 and RQ3

*What impact will using closed loop PBL, compared to a traditional teacher-led instructional method, in a mathematics education subject have on pre-service teachers’ self-efficacy for teaching mathematics?*

*What impact will using closed loop PBL, compared to a traditional teacher-led instructional method, in a mathematics education subject, have on pre-service teachers’ mathematics teaching outcome expectancy?*

To measure and assess the impact of the closed loop PBL method on the pre-service teachers’ teaching beliefs, two of the four instruments from this mixed methods approach were used; the MTEBI (Huinker & Enochs, 1995) and the interview questions. The
MTEBI consists of two subscales; a mathematics teaching efficacy subscale and a mathematics teaching outcome expectancy subscale. The difference in the MTEBI mean scores for each subscale, pre-test to post-test, for the control group and treatment group were compared using an ANOVA. The results indicated no significant difference in the effectiveness of the two teaching approaches across the four month duration of the study for either subscale. However, the results of the paired-samples t-test conducted on each group indicated that both teaching methods were effective in assisting pre-service teachers to develop their self-efficacy for teaching mathematics over the course of the study. On the other hand, the paired-samples t-test conducted on each group to evaluate the two teaching methods’ impact on outcome expectancy revealed no significant change pre-intervention to post-intervention for either group. The analysis of the interview transcripts from the treatment group however revealed a new sense of teacher efficacy. Reasons for the differences between the MTEBI results and interview interpretations are considered in the following sections.

6.3.1 Discussion of the results for RQ2

Self-efficacy for teaching is defined as a belief in one’s ability to teach effectively (Enochs et al., 2000). The pre-service teachers indicated in their interviews that their belief in their ability to teach effectively had been impacted positively due to their PBL experiences when compared to their experiences with the traditional instruction used in their other subjects, especially as a result of the ‘teaching practice’ they received by presenting their lessons to their peers. It should be noted that the treatment group of pre-service teachers alluded to self-efficacy using the term confidence. “Confidence is a nondescript term that refers to strength of belief but does not necessarily specify what the certainty is about” (Bandura, 1997, p. 382). Specifying the type of confidence found, one student commented during her interview how solving one of the real-world, open-ended problems in class helped her improve her teaching of algebra during her practicum experience.

*I think I would have got up there and just paddled all this stuff off to the kids. Whereas having been in a problem-based learning environment…. I think it really improved the way I was able to teach that [algebra] lesson. I got the kids more engaged and got them involved in the process. And I probably wouldn’t have done that before.*
More general indicative responses of the enhanced self-efficacy for teaching mathematics stated by the pre-service teachers during their interviews included that PBL gave them “classroom experience” which allowed them to “feel much more capable” at teaching mathematics. Students described how the PBL approach “built confidence”. They attributed this additional confidence to having to “get in front of a class and do it [teach] in front of other students”.

The ANOVA results from the MTEBI indicated no significant difference between the impacts of the two pedagogical approaches on pre-service teachers’ self-efficacy for teaching mathematics at the end of the study. However, the t-tests indicated that each pedagogical approach significantly increased the pre-service teachers’ self-efficacy for teaching mathematics, and the interview responses support a conclusion that there was a degree of enhanced self-efficacy for teaching mathematics felt by the PBL group post-intervention. Previous studies using the MTEBI were conducted over a longer time frame than the four-month duration of this study (Moody & DuCloux, 2015; Swars et al., 2009). Other studies report that beliefs are very resistant to change to the point where the change can be measured (Hoy & Spero, 2005; Iyer & Wang, 2013). It was therefore hypothesised that a longer study would be necessary in order to obtain significant results with the MTEBI, but the weight of the data from the student interviews reflected positively on the closed loop PBL pedagogical approach as implemented in this study.

### 6.3.2 Discussion of the results for RQ3

Teaching outcome expectancy is the belief that effective teaching will have a positive effect on student achievement (Enochs et al., 2000). During their interviews the treatment group of pre-service teachers alluded to their mastery and vicarious learning experiences in class, which they indicated positively affected their outcome expectancy. The notion that mastery experiences strongly influence outcome expectancy is supported by Huinker and Madison (1997). “If they [the pre-service teachers] could understand these ideas, given effective instruction, then surely their future students, whoever they may be, would also benefit from this type of instruction” (Huinker & Madison, 1997, p. 123).

In this study, students suggested that they will most likely use PBL as a teaching approach because they expect it will enhance their ability to increase their students’ learning. An examination of the interview transcripts from the treatment group of pre-
service teachers revealed that they did feel the PBL method was an effective way to learn; and, as a result, they will most likely use it when they become a teacher to enhance student academic achievement. They stated that they had benefitted from PBL and they believed that “children having those experiences in the classroom would benefit similarly”. The pre-service teachers indicated that PBL is “a great way to learn” and “when you are actively engaged in what you are learning, you take a lot more in”. They expressed the belief that they would use PBL with their future students.

In contrast to the interview results, the paired-samples t-tests conducted to evaluate the impact of the two teaching approaches on outcome expectancy revealed no significant change pre-intervention to post-intervention for either group. Additionally, the ANOVA results from the MTEBI indicated no significant difference between the two teaching interventions for outcome expectancy.

Other studies which examined the impact of education subjects on pre-service teachers’ teaching beliefs reported similar findings (Hoy & Spero, 2005; Swars et al., 2009; Woolfolk & Hoy, 1990). The conclusions reached by these researchers is that pre-service teachers’ self-efficacy for teaching increases with enactive and vicarious experiences during coursework and practicum teaching experiences. However, in terms of outcome expectancy, pre-service teachers’ beliefs increase during coursework but decline or stay the same during their teaching experiences. The findings were attributed to the “unrealistic optimism prospective teachers have prior to student teaching about teacher’s abilities to overcome negative influences” (Swars et al., 2009, p. 50).

In this study, both groups of pre-service teachers completed their three-week practicum just five days prior to completing the MTEBI at the post-test. Therefore, the non-significant findings from both groups’ paired-samples t-tests regarding outcome expectancy may have resulted from their “unrealistic optimism” in relation to their ability to teach mathematics effectively, which did not match the reality experienced during their practical experience. Unfortunately, the pre-service teachers were not explicitly asked what impact PBL had on their teacher outcome expectancy as it was believed that they would not accurately interpret the construct. As a result, the beliefs extracted from the interviews in which the treatment group of pre-service teachers alluded to the impact of PBL on their outcome expectancy are limited. But, comments made by the students as
described above and in the previous chapter, indicate that the PBL pedagogical approach did have an impact on their outcome expectancy, even if this impact was not measured by the MTEBI.

Perhaps again the study was not long enough or broad enough to influence sustained change in the pre-service teachers’ outcome expectancy that could be measured by the MTEBI. In conclusion, the items of the MTEBI were not able to identify distinct self-efficacy and outcome expectancy changes specific to the mathematics topics taught in one subject over one semester. Support of this conclusion is found in a longitudinal study that used the MTEBI to investigate the changes which occurred in pre-service teachers’ mathematics beliefs during their teacher education program (Swaras et al., 2009). The teacher education program included two semesters of mathematics methods subjects designed to provide a foundation for the pre-service teachers to use social constructivist pedagogies to teach mathematics. The longitudinal study took place over two full academic years which comprised four semesters of coursework including three semesters of 2-day-a-week field placements followed by a semester of student teaching. Some of the significant changes in mathematics teaching self-efficacy and outcome expectancy occurred during the first mathematics methods subject while some occurred during the second methods subject. The authors concluded that the findings indicate the value of the second mathematics methods subject for supporting continued change in self-efficacy. In terms of outcome expectancy, a slight decrease during the student teaching semester was reported. The explanations provided by the researchers suggest the decline in the pre-service teachers’ outcome expectancy was a result of “more realistic expectations for successful learning outcomes given the demands of teaching, variations across students, and other uncontrollable factors” (2009, p. 62).

Further support was revealed in a more recent study (Moody & DuCloux, 2015). This three-semester long study used the MTEBI to measure the impact of three mathematics content subjects taught in sequence on pre-service teachers’ self-efficacy for teaching mathematics and mathematics teaching outcome expectancy. Each subject focused on different topics in mathematics as a requirement towards a teaching certificate. The subjects were delivered using a social constructivist, student-centred approach on two different groups of pre-service teachers. The researchers’ hypothesis was that a treatment comprising three subjects taught over three semesters (one subject in each of three
consecutive semesters), underpinned by social constructivism and taught by instructors who took on the role of facilitators, was necessary to significantly increase pre-service teachers’ mathematics teaching efficacy. With no significant differences found at the start of the three-subject mathematics sequence for both self-efficacy for teaching mathematics and mathematics teaching outcome expectancy, an independent-samples t-test revealed a significant difference in mean scores between the two groups for both subscales at the post-test.

Overall, researchers who used PBL to enhance teaching efficacy reported mostly positive quantitative results and positive participant responses based on self-reporting instruments, but the majority of these studies were conducted over a longer time frame than this study. However, Downing et al. (2011) stated that regardless of the type of qualitative instruments used, or the duration of the PBL treatment, higher education students reported having a better learning experience in a subject which uses PBL compared to traditional instruction.

In summary, the data analyses in this study revealed divergences in the findings for each of the three research questions, depending on the data collection method used. For each of the divergent findings interpretations have been posed. The following section provides a synthesis of the researcher’s interpretations and conclusion relative to the findings within and across the three research questions.

### 6.4 Synthesis of the Findings

This synthesis of the researcher’s interpretations and ultimate conclusion is based on the literature reviewed, an analysis of the results from the MPCKI, MTEBI, end-of-semester exam, interview responses and a consideration of the study’s research design. Simply stated, based on the study’s results, a PBL investigation longer than one semester in duration, and if possible across more than one subject taught with PBL, may be necessary in order to obtain significant differences between groups taught with and without PBL, when using the MPCKI or MTEBI.

Firstly, during the pilot study the MPCKI was determined by a Rasch model analysis as being a valid and reliable instrument for measuring the construct identified in this study
as mathematics PCK. Prior to the commencement of the intervention in the main study, at the pre-test, the validated MPCKI was used to determine that both groups had a similar level of mathematics PCK. During the four month intervention period the two groups were exposed to different pedagogical approaches. Post-intervention, when the pre-service teachers’ mathematics PCK was measured using multiple-choice items within the MPCKI, the results should have favoured the control group (based on prior studies). However, the PBL group’s PCK was shown to be equally positively affected. This result is viewed as a positive outcome for this closed loop PBL study, even though the mean differences statistically do not indicate a positive difference. The inference being made is that, contrary to previous research, the PBL method used in this study did not disadvantage the PBL students in relation to learning mathematics PCK. Furthermore, if the study was expanded to include other mathematics education subjects over subsequent semesters, perhaps a significant positive difference with the MPCKI may ensue because a longer study would allow the PBL approach to exert more profound impact on the pre-service teachers’ PCK which the instrument is able to measure.

Similarly, the researcher resolved that the non-significant results obtained from the MTEBI analyses were influenced by (a) teaching efficacy beliefs developing over time and appearing to be resistant to change once established (Hoy & Spero, 2005; Iyer & Wang, 2013; Moody & DuCloux, 2015; Swars et al., 2009), (b) the MTEBI not being able to pick up distinct self-efficacy and outcome expectancy changes specific to the mathematics topics taught in one subject during one semester, (c) the pre-service teachers being exposed to the closed loop PBL method for only one semester and (d) the pre-service teachers being exposed to the closed loop PBL method using only one subject during the semester. Therefore, if the study were expanded to include other mathematics education subjects, a positive result on the MTEBI may ensue because the pre-service teachers will become more confident in more areas of mathematics PCK, not just the four topics covered in the subject from this study. Furthermore, the study should be conducted for a longer period of time so that pre-service teachers engage in more teaching and practicum experiences. It is proposed that the additional time would provide the necessary conditions to more positively impact their beliefs, which may then reveal a significant difference between pedagogical approaches when using the MTEBI to collect data.
Another significant conclusion from this study was provided by the findings in favour of PBL’s effectiveness over traditional instruction to enhance the pre-service teachers’ ability to enact their PCK (e.g., to teach mathematics) from the end-of-semester exam data. The statistically significant difference found in the performance of the two groups on the exam questions was most probably related to the pedagogical approaches each group experienced over the four month intervention period. In essence, the closed loop PBL intervention used in this study allowed the PBL group to demonstrate they were more competent in enacting their mathematics PCK than the control group.

This conclusion is also supported by the interview questions (Table 10) posed to the treatment group of pre-service teachers which asked questions specific to the PBL teaching approach employed in relation to the problems and topics of the subject. The pre-service teachers were asked to elaborate about what they learned, how they learned it, and how they felt in terms of teaching mathematics as a result of learning through the PBL method in comparison to their experiences with traditional instruction. When the thematic analysis was completed on the treatment group of pre-service teachers’ interview responses, PBL was consistently viewed as the preferred learning method which (a) positively impacted their ability to teach the mathematics topics practised during the semester and (b) built their teacher confidence more-so than traditional instruction for teaching those mathematics topics. The pre-service teachers attributed their enhanced ability and beliefs to being required to (a) present non-assessed mathematics lessons to their peers in a simulated classroom (mastery experiences), (b) receive constructive peer and instructor feedback (performance accomplishments with verbal persuasions) and (c) observe their peers as they delivered similar mathematics lessons in a simulated classroom (vicarious experiences).

This study has provided evidence that closed loop PBL is a pedagogical approach which can be used in tertiary mathematics education subjects to effectively develop pre-service teachers’ ability to enact their mathematics PCK, as well as build confidence in future teachers of mathematics. The results of this study also highlights its limitations. In the following section the researcher identifies the limitations which need to be considered by the reader when interpreting the results.
6.5 Limitations of the Study

The results synthesis was strengthened by the use of a mixed methods approach. Reflecting on the conduct of the research though it became apparent there were also limitations in the study which should be noted. Firstly, inherent limitations of the study arose when it was decided to use convenience sampling in the study, using pre-service teachers who enrolled in the researcher’s mathematics education subject at the two satellite campuses. As a result, the number of pre-service teachers who participated in the study was limited to how many enrolled in the subject and subsequently who gave written consent to be involved. Relatively small sample sizes (n=17 and n=20) means the results cannot be attributed to the whole or larger population.

Secondly, closed loop PBL was a completely different teaching approach in comparison to what the pre-service teachers had experienced during their twelve or more years of traditional teacher-led instruction. Consequently, a semester-long closed loop PBL treatment employed in one subject was perhaps not long enough for them to become completely comfortable and adept with it and to demonstrate the value of this pedagogical approach in pre-service teacher mathematics education. There is evidence from this study however that the PBL approach positively impacted the students’ PCK, their ability to demonstrate their mathematics PCK and that their self-efficacy for mathematics teaching was enhanced as a result of participating in the PBL intervention.

In terms of teaching efficacy, beliefs and attitudes develop over time and once established are resistant to change. The consensus is that beliefs related to teaching and learning which took twelve years of schooling to develop are not easily changed by newer teaching and learning practices introduced in a teacher education program, especially for students transitioning straight from secondary school into university study (Hoy & Spero, 2005; Iyer & Wang, 2013; Moody & DuCloux, 2015; Swars et al., 2009). A similar conclusion can be drawn from the results of this study. Applying closed loop PBL for just one semester in just one subject potentially limited the impact of the pedagogical approach on the breadth of dependent variables investigated. The researcher acknowledges that this possible limitation should have been considered more closely in the research design.
Lastly, interviewing the control group should also have been part of the research design. Furthermore, interview questions which explicitly probed pre-service teachers’ views regarding how closed loop PBL impacted their self-efficacy for teaching mathematics and mathematics teaching outcome expectancy would have strengthened the interview. Interviewing the control group may have highlighted more clearly the differences in impact of the two pedagogical approaches. The omission of these control group interviews and explicit questions may have limited the degree of the data source used for the purpose of triangulation in the study. However, it is believed these limitations do not detract from the presented findings in the research, but do provide direction for further research.

### 6.6 Directions for Further Research

The literature on PBL is voluminous. Nevertheless, more specific investigations on PBL are still needed in relation to pre-service teachers’ and mathematics PCK. Associated with this study’s research problem, that not all graduating pre-service teachers possess adequate PCK to teach effectively, PBL’s effectiveness to enhance their ability to enact their mathematics PCK (e.g., to teach effectively) is still inconclusive in the literature because many researchers did not identify which variation, or degrees of structure of PBL or the type and difficulty of the problems they used in their studies (Barrows, 1994; Walker & Leary, 2009). However, this study provides some limited evidence that closed loop PBL positively impacts pre-service teachers’ ability to enact their PCK. Therefore, it is proposed that researchers conducting studies in PBL in the future should, in the first instance, provide the extent of the PBL method employed as well as a clear description of protocols (Albanese & Dast, 2014; Goodnough & Nolan, 2008; Newman, 2003).

For example, Barrows suggests that each of the six teaching variations in his PBL taxonomy have varying degrees of impact on four key educational objectives namely, structuring knowledge for use in clinical contexts, developing an effective clinical reasoning process, developing effective self-directed learning skills, and increased motivation for learning. Of all six variations of his PBL taxonomy, Barrows states the closed loop variation is best positioned to enhance all four objectives (Barrows, 1986). It is suggested that future researchers embarking on such a study should clarify they are
using Barrows’ PBL taxonomy and specify which variation of PBL as well (Walker & Leary, 2009).

Other problems associated with past PBL studies are a result of researchers attempting to resolve the debate about the effectiveness of PBL by examining only test scores or interview responses without scrutinising the PBL protocols (Hung, 2011). Examining the actual implementation of the PBL process and the types of problems used during the implementation “may help illuminate how and why the end results were produced, and in turn, shed light on how to improve PBL practices to yield desired learning outcomes that are aligned with the theoretical promises of PBL” (2011, p. 530). For example, studies have shown using properly designed real-world, open-ended problems has a strong impact on student learning when used with the PBL model (Gijselaers & Schmidt, 1990; Schmidt & Moust, 2000; Sockalingam, Rotgans, & Schmidt, 2011).

Future PBL researchers investigating the complex and multi-faceted nature of PCK need to be clear about what they are testing and use instruments that are appropriate to the components of PCK being measured (Albanese & Dast, 2014; Walker & Leary, 2009).

This study was conducted over the course of one semester (four months) with a relatively small sample size (N=37). It is recommended that any replication of the study should be conducted with a larger sample size. Additionally, the study was conducted in isolation. When asked during their semi-structured interviews, the treatment group of pre-service teachers stated that no other lecturers in the university’s School of Education where the study was conducted employed the PBL method in their subjects. Thus, prior to enrolling in this mathematics education subject, the third-year pre-service teacher participants had no prior university experience with PBL. Therefore, it would be worthwhile to replicate the study over a longer period using a broader PBL treatment which included other mathematics subjects and possibly a larger number of students. The hypothesis is that such a modification in the research design would give the pre-service teachers more exposure to learning with PBL and allow them to generalise their mathematics PCK and teacher efficacy more broadly. It would also allow the findings to be generalised to a larger population.
Lastly, it was initially hypothesised in this comparison study that pre-service teachers with high mathematics teaching outcome expectancy would likely believe that possessing sound mathematics PCK will impact positively on their future students’ mathematics academic achievement; which, in turn may motivate them to work harder and persist longer at maximising their levels of PCK. The quantitative findings from this study did not support the hypothesis. Predominantly, pre-service teachers’ teaching outcome expectancy are resistant to change. Thus, the recommendation is that researchers investigating pre-service teachers’ self-efficacy for teaching, and especially outcome expectancy, consider a longer exposure to learning with PBL which would ultimately increase the pre-service teachers’ teaching efficacy and effect significant change.

While the need for further research is acknowledged, the researcher believes the main aim of the study was achieved, which was to determine the effectiveness of closed loop PBL when compared to a traditional teaching pedagogy in a tertiary mathematics teacher education subject. The results from this mixed methods study suggest that closed loop PBL is a promising pedagogy for developing mathematics PCK in pre-service teachers and a more effective pedagogy for developing pre-service teachers’ ability to enact their PCK compared to a traditional teacher-led instructional approach. Consequently, this researcher’s constructivist ontological and epistemological beliefs with respect to the effectiveness of PBL in mathematics education subjects were strengthened by this mixed methods study. These advancements in knowledge and increased conviction of constructivist views form the basis for the researcher’s reflexive account of the entire research process.

### 6.7 Reflexive Account of the Research

Traditional instruction has historically been the instructional method of choice in teacher education, and was for this researcher at the start of his teaching career. The genesis of this learning journey was when the researcher, as a novice teacher, determined that his traditional teacher-centred instructional approach was an ineffective pedagogy and when used with his Year 6 students coincided with increased behavioural problems. As an early career teacher the attempt to minimise behaviour problems was met with success when it was discovered that providing meaningful, real-world problems to the students to solve
collaboratively increased their level of engagement with the content. Continued success using this type of pedagogy in the school system and into tertiary teaching strengthened the researcher’s conviction that using meaningful, real-world problems as the stimulus for learning was a more effective pedagogical approach than using a traditional teacher-led instructional approach. Eventually, this conviction led to action research studies involving PBL (Martin, 2012; Martin & Jamieson-Proctor, 2010). On reflection, the results of those studies could not conclusively support existing knowledge regarding the effectiveness of PBL. A further review of the literature led to a clearer understanding of the variations and degrees of structure of PBL and the necessity for clearly identifying them in future research. This research path set in motion a series of learning events for the researcher that will guide his future research and approach to teaching.

Conducting the pilot study and reporting its findings provided additional learning opportunities. For example, conducting the pilot study advanced the researcher’s knowledge of the closed loop PBL pedagogical process. As a result, the researcher was confident in preparing the lecturer who was recruited to teach the subject to the treatment group of pre-service teachers using the closed loop PBL method.

The choice to conduct interviews in the main study was based on the findings from the pilot study and the literature which states that using a mixed method design allows for an appropriate lens to interpret the quantitative perspective (Creswell & Plano, 2007; Janesick, 2000). However, if the study were to be replicated consideration would be given to interviewing the control group of pre-service teachers as well. Asking for their views regarding how the traditional teacher-led instructional approach in this subject impacted their ability to teach effectively and their teaching efficacy beliefs would benefit the comparative study.

Making the decision to use the real-world, open ended problems with both cohorts during the weekly tutorials may have played a role in the findings, since properly designed problems have a strong impact on student learning when used with the PBL instructional model (Gijselaers & Schmidt, 1990; Schmidt & Moust, 2000; Sockalingam et al., 2011). A comparison of viewpoints from both groups of pre-service teachers may reveal whether the design of the PBL problems given to both cohorts during their tutorials was
the trigger that set in motion an effective learning process which allowed both groups to equally develop their mathematics PCK as measured by the MPCKI.

As a direct result of conducting the main study and reporting its findings, the researcher believes that the MPCKI appeared to allow both groups to demonstrate their developed mathematics PCK and that the exam questions appeared to allow both groups to demonstrate their ability to enact their mathematics PCK they developed during the semester. Furthermore, the treatment group of pre-service teachers unanimously stated that the PBL method used in this study was more effective than their experiences with the traditional instruction used in their other subjects for developing their ability to teach mathematics effectively, and, that PBL positively impacted their teacher confidence more than traditional instruction.

In closing, a constructivist, student-centred pedagogy is advocated in national curriculum documents and described as a teaching approach which uses relevant, real-world and open-ended tasks which allow students to construct deep understanding at a relational level (ACARA, 2016; Donnelly & Wiltshire, 2014; National Council of Teachers of Mathematics, 2011; Sullivan, 2011). The researcher’s constructivist beliefs reflect this view and he explicitly believes closed loop PBL is a pedagogy capable of addressing the research problem and responding to the Ministerial Advisory Group’s recommendation on how teacher education in Australia could be improved to ensure new teachers possess adequate PCK and can teach effectively at the graduate level (i.e., enact their PCK) (Australian Institute for Teaching and School Leadership [AITSL], 2014; Teacher Education Ministerial Advisory Group, 2014). The Ministerial Advisory Group proposes that if PCK and teaching methods are all important elements of teacher effectiveness and effective teaching begins with effective teacher preparation, then university teacher preparation programs should focus their efforts on ensuring their graduates have strong PCK as well as be able to utilise their PCK to enhance student learning outcomes (Teacher Education Ministerial Advisory Group, 2014). The findings in this study regarding developing mathematics PCK also align with Teacher Education Ministerial Advisory Group’s assertion that highly effective teachers possess strong PCK and the expectation that tertiary providers demonstrate that their programs use evidence-based pedagogical approaches which enable pre-service teachers to make a positive impact on the learning of all students (Teacher Education Ministerial Advisory Group, 2014).
This study is considered an initial attempt to provide evidence in determining the impact of traditional teacher-led instruction compared to closed loop PBL, the most student-centred in Barrows’ taxonomy, on pre-service teachers’ mathematics PCK. The closed loop PBL pedagogy was shown as being more effective than a traditional approach for developing pre-service teachers’ ability to enact their mathematics PCK. Therefore, it is proposed that closed loop PBL is a pedagogy, informed by research, which would allow pre-service teachers during their coursework to routinely integrate theory and classroom practice before graduation. Such a delivery model would allow pre-service teachers to reflect on how their teaching practice could be improved, and in turn, make a difference to student learning.
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Appendix A: Real-world, Open-ended, Decision-making Problems
The aim of this activity is for you to demonstrate your ability to design a lesson which has young students exploring the concept and skills for measuring length.

**Scenario:** You are on your 15-day prac and your mentor teacher tells you that you will be teaching length to her Year 1 class. She informs you that when she teaches her students about length for the first time, she prefers to begin by having the children measure the width of their books using paper clips, then again using pencils. **But she states that you may design the lesson in any way you see fit.**

However, she does provide you with the following guidelines:

You are to reference the appropriate ACARA strand(s) and sub-strand(s) for the activities you plan. The design of your lesson should provide the students with the opportunity to:

- revisit their Foundation Year prior knowledge;
- be appropriately introduced through real-world, concrete activities to the concepts of measurement of length appropriate to their year level;
- apply, in a social constructivist learning context, the related Year 1 measurement skills; and
- demonstrate and/or explain their understanding to their peers using their own language.

The lesson is to consist of an introduction phase, an enhancing (application) phase and a synthesising phase. You may use any concrete and/or virtual resources, materials, textbooks, or IT available to you. You may also write-up the lesson plan using the example template provided or create your own lesson plan template.

You may use your choice of materials and teaching strategy. Be prepared to provide a rationale for the pedagogical approach you chose to underpin your lesson.
The aim of this activity is for you to demonstrate your ability to design a lesson plan which revises and then addresses young students’ concept of time and skills for measuring time.

**Scenario:** You have an upcoming interview with a principal for your first teaching job. Several days before the interview the Principal’s secretary rings you and requests that you prepare a response to one of the principal’s interview questions. The question is as follows: “I want to know how you would teach one of my Year 3 classes how to measure time.” The Principal continues stating that from a diagnostic test it has been determined that most of the students lack the conceptual understanding of ‘duration’ or ‘time passing by’. Therefore, you are to cover those year level concepts and skills before covering the Year 3 content for measuring time. You are to reference the ACARA strand / sub-strand(s) for the activities you plan. Therefore:

- You are to identify the prior knowledge you will address in the lesson and revisit that material. Additionally, you will identify the concepts and skills the students need to acquire in Year 3.

- You are to design the activities in your lesson so the students work collaboratively through real-world, concrete activities and apply, in a social constructivist learning context, the Year 3 skills.

- The students are to demonstrate and/or explain their understanding to their peers using their own language.

The lesson is to consist of an introduction phase, an enhancing (application) phase and a synthesising phase. You may use any concrete and/or virtual resources, materials, textbooks, or IT available to you. You may also write-up the lesson plan using the example template provided or create your own lesson plan template.

You may use your choice of materials and teaching strategy. Be prepared to provide a rationale for the pedagogical approach you chose to underpin your lesson.
The aim of this activity is for you to demonstrate your ability to design a lesson plan which sees young students exploring the stages of development of the concept and skills for measuring mass.

Scenario: You are in your 15-day prac and your mentor teacher is asking you to design a lesson plan on the topic of mass for her Year 4 class. She informs you the students’ prior knowledge in this area is quite limited. She then provides you with the following guidelines:

Since the students’ conceptual understanding is limited, your mentor asks that you revisit the related content from the Year 2 and Year 3 ACARA content descriptor(s).

The design of your lesson should therefore provide the students with the opportunity to:

- revisit their Year 2 and Year 3 prior knowledge;
- be appropriately introduced through real-world, concrete activities to the concepts of measurement of mass appropriate to their year level;
- apply, in a social constructivist learning context, the related Year 4 measurement skills; and
- demonstrate and/or explain their understanding to their peers using their own language.

The lesson is to consist of an introduction phase, an enhancing (application) phase and a synthesising phase. You may use any concrete and/or virtual resources, materials, textbooks, or IT available to you. You may also write-up the lesson plan using the example template provided or create your own lesson plan template.

You may use your choice of materials and teaching strategy. Be prepared to provide a rationale for the pedagogical approach you chose to underpin your lesson.
The aim of this activity is for you to demonstrate your ability to design a lesson plan which sees young students exploring the concept and skills for measuring area of a circle.

**Scenario:** You are on your 15-day prac and your mentor teacher is asking you to design a lesson plan which introduces *the concept for measuring area of a circle* to her Year 6 class. She informs you the students’ active prior knowledge on this topic is calculating the area of rectangles but, they lack conceptual understanding of ‘area’. She then provides you with the following guidelines:

Since the students’ conceptual understanding of area is lacking, your mentor asks that you reference the appropriate ACARA sub-strand(s) and content descriptor(s).

Design your lesson so the students have the opportunity to:

- revisit their Year 4 and 5 prior knowledge of ‘area’ of a circle;
- apply, in a social constructivist learning context, the *concept for measuring area of a circle* through real-world, concrete activities; and
- demonstrate and/or explain their understanding to their peers using their own language.

The lesson is to consist of an introduction phase, an enhancing (application) phase and a synthesising phase. You may use any concrete and/or virtual resources, materials, textbooks, or IT available to you. You may also write-up the lesson plan using the example template provided or create your own lesson plan template.

You may use your choice of materials and teaching strategy. Be prepared to provide a rationale for the pedagogical approach you chose to underpin your lesson.
The aim of this activity is for you to demonstrate your ability to design a lesson plan on 2D and 3D shapes underpinned by the *The van Hiele Levels of Geometric Understanding.*

**Scenario:** You are on your 15-day prac and your mentor teacher informs you that her Year 2 students can identify most shapes, but most have misconceptions about the attributes of these 2-D and 3D shapes. She is asking you to develop a lesson plan which addresses these misconceptions. She provides you with the following guidelines. You are to reference the appropriate ACARA strand(s) and sub-strand(s) for these year levels. The design of your lesson should provide the students with the opportunity to:

- revisit their Foundation Year and Year 1 prior knowledge in regards to ‘shape’;

- be **appropriately introduced** through real-world, concrete activities to the concepts of geometry from the appropriate year level content descriptor of ‘shape’;

- apply, in a social constructivist learning environment, the related Year 2 geometry skills; and

- demonstrate and/or explain their understanding to their peers using their own language.

The lesson is to consist of an introduction phase, an enhancing (application) phase and a synthesising phase. You may use any concrete and/or virtual resources, materials, textbooks, or IT available to you. You may also write-up the lesson plan using the example template provided or create your own lesson plan template.

Be prepared to discuss how your design addresses the van Hiele framework. You may use your choice of materials and teaching strategy. Be prepared to provide a rationale for the pedagogical approach you chose to underpin your lesson.
The aim of this activity is for you to demonstrate your ability to design a lesson plan to teach *Location* to students.

**Scenario:** Your mentor’s Year 3 class has been given the honour of being escorts for the city council members who will be visiting the school. He sees this as a perfect opportunity to teach students how to give and follow directions, and being able to use those skills to interpret positions on the school maps and provide directions to a prescribed location.

You are to reference the appropriate ACARA strand(s) and sub-strand(s). The design of your lesson should provide the students with the opportunity to:

- revisit their Year 1 and Year 2 prior knowledge in regards to ‘Location’;
- be **appropriately introduced** through real-world, concrete activities to the concept of geometry from the content descriptor of ‘Location’ appropriate to their year level;
- apply, in a social constructivist learning context, the Year 3 geometry skills; and
- demonstrate and/or explain their understanding back to their peers using their own language.

The lesson is to consist of an introduction phase, an enhancing (application) phase and a synthesising phase. You may use any concrete and/or virtual resources, materials, textbooks, or IT available to you. You may also write-up the lesson plan using the example template provided or create your own lesson plan template.

You may use your choice of materials and teaching strategy. Be prepared to provide a rationale for the pedagogical approach you chose to underpin your lesson.
The aim of this activity is for you to demonstrate your ability to design a lesson plan on 3D shapes underpinned by the *The van Hiele Levels of Geometric Understanding*. The students are presently working at level 2 and are to be moved to level 3 through your activities.

Scenario: You are in your 15-day prac and your mentor teacher tells you that you will be teaching to her Year 5 class *properties of 3D shapes* and the relationship they have with the 2D shapes from which they are created. She provides you with the following guidelines:

You are to reference the appropriate ACARA strand(s) and sub-strand(s) for these years. The design of your lesson should allow the students with the opportunity to:

- revisit their Year 3 and Year 4 prior knowledge in regards to ‘shape’;
- be **appropriately introduced** through real-world, concrete activities to the concepts of geometry from the content descriptor of ‘shape’ appropriate to their year level;
- apply, in a **social constructivist** learning context, the related Year 5 skills; and
- demonstrate and/or explain their understanding back to their peers using their own language.

The lesson is to consist of an introduction phase, an enhancing (application) phase and a synthesising phase. You may use any concrete and/or virtual resources, materials, textbooks, or IT available to you. You may also write-up the lesson plan using the example template provided or create your own lesson plan template.

Be prepared to discuss how your design addresses the van Hiele framework. You may use your choice of materials and teaching strategy. Be prepared to provide a rationale for the pedagogical approach you chose to underpin your lesson.
The aim of this activity is for you to demonstrate your ability to design a lesson plan to teach **both** Location and Transformation using the **Cartesian coordinate system**.

**Scenario:** Your mentor's Year 7 class has been given the honour of designing a school logo for the school. But before the students tackle such a task, your mentor wants to be sure the students have all the pre-requisite skills from Year 5 and Year 6. Hence, your mentor is giving you this teaching task under the following guidelines.

You are to reference the appropriate ACARA strand(s) and sub-strand(s). The design of your lesson should provide the students with the opportunity to:

- revisit their Year 5 skills regarding translations, reflections, of 2D shapes, and identifying line and rotational symmetry;
- revisit the Year 6 skills of working in the Cartesian coordinate system using all four quadrants;
- apply, in a social constructivist learning context, the related Year 5 and Year 6 skills in an integrated activity; and
- demonstrate and/or explain their understanding back to their peers using their own language.

The lesson is to consist of an introduction phase, an enhancing (application) phase and a synthesising phase. You may use any concrete and/or virtual resources, materials, textbooks, or IT available to you. You may also write-up the lesson plan using the example template provided or create your own lesson plan template.

You may use your choice of materials and teaching strategy. Be prepared to provide a rationale for the pedagogical approach you chose to underpin your lesson.
The aim of this activity is for you to demonstrate your ability to design a lesson plan on probability.

**Scenario:** You are taking your Year 3 students to the school fete where many of the games they will play involve probability. You would like to prepare your students so that they understand the concepts associated with games such as a coin toss, spinners, etc. Because you are concerned that the Year 3 curriculum on probability might be a challenge for some of your students, you choose to revisit their Year 2 *Chance* Content Descriptor as well.

You are to reference the appropriate ACARA strand(s) and sub-strand(s) for these year levels.

The design of your lesson should provide the students with the opportunity to:

- revisit their prior knowledge and language related to the Year 2 'Chance' content descriptor;
- be appropriately introduced through real-world, concrete activities to the concepts of chance, likelihood, outcomes, and randomness appropriate to their year level;
- apply, in a social constructivist learning context, the Year 3 skills; and
- demonstrate and/or explain their understanding back to their peers using their own language.

The lesson is to consist of an introduction phase, an enhancing (application) phase and a synthesising phase. You may use any concrete and/or virtual resources, materials, textbooks, or IT available to you. You may also write-up the lesson plan using the example template provided or create your own lesson plan template.

You may use your choice of materials and teaching strategy. Be prepared to provide a rationale for the pedagogical approach you chose to underpin your lesson.
The aim of this activity is for you to demonstrate your ability to design a lesson plan on statistics.

Scenario: Your year 5 class has just completed the first phase of an experiment and the students have collected the data on paper recording sheets. You are ready to ask them to assemble/organise the data on paper and represent/display that data by sketching bar graphs on A3 paper. This all seems so “yesterday” and the Australian Curriculum now requires students to engage with this concept and skill using ICT. Design a lesson which requires the students to use specific technology and/or software to electronically organise and then display the data.

You are to reference the appropriate ACARA strand(s) and sub-strand(s) for these year levels. The design of your lesson should provide the students with the opportunity to:

- revisit their prior knowledge of the related Year 4 ‘Data representation and interpretation’ content descriptor(s);
- be appropriately introduced through real-world, concrete activities to the concepts of data representation and data display appropriate to their year level;
- apply, in a social constructivist learning context, the Year 5 skills using ICT; and
- demonstrate and/or explain their understanding back to their peers using their own language.

The lesson is to consist of an introduction phase, an enhancing (application) phase and a synthesising phase. You may use any concrete and/or virtual resources, materials, textbooks, or IT available to you. You may also write-up the lesson plan using the example template provided or create your own lesson plan template.

You may use your choice of materials and teaching strategy. Be prepared to provide a rationale for the pedagogical approach you chose to underpin your lesson.
Appendix B: Treatment Group Reflection Forms
Name ________________________________________

How well did you and your team use research and the information sources that were available to solve the original problem? i.e. internet, textbook, lecture material, concrete materials, experts etc.

NOT WELL ☐ OK ☐ VERY WELL ☐

Please explain:

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

If you were provided another opportunity at this lesson plan design, what improvements would you make to your reasoning process?

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________
How would you assess your **own performance** during the self-directed learning, research and implementation process?

NOT GOOD □  OK □  VERY GOOD □

Please explain:

________________________________________________________________________

________________________________________________________________________

How would you assess your **group’s performance** during the self-directed learning, research and implementation process?

NOT GOOD □  OK □  VERY GOOD □

Please explain:

________________________________________________________________________

________________________________________________________________________

How would you assess your **tutor’s performance** during your self-directed learning and implementation process?

NOT GOOD □  OK □  VERY GOOD □

Please explain:

________________________________________________________________________

________________________________________________________________________
Appendix C: Full Set of the Mathematics Pedagogical Content Knowledge Instrument Items
In the number sentence, $17 - 9 = \square + 1$, one of your students places an 8 in the empty box. For each teacher intervention provided in the table below, indicate to the right which intervention you *would not use*, *might use*, or *definitely would use* to help the student understand this relationship.

<table>
<thead>
<tr>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Discuss with the student the purpose of the equal sign and about relationships between the left side and the right side of an equation.</strong></td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td><strong>Remind the student that what you do to one side of the equation you must do to the other side.</strong></td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td><strong>Ask the student to solve a similar, yet less difficult problem such as $3 + 4 = \square + 2$</strong></td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td><strong>Provide the student with a balance scale and blocks to create a representation of the equation.</strong></td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td><strong>Advise the student to consider the commutative property of addition.</strong></td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
In an introductory lesson on parallelograms, you provided the following shape to your students.

Along with the above shape, indicate which additional shape(s) below you would NOT use, might use, or definitely would use to help the students improve their understanding about the properties of parallelograms.

<table>
<thead>
<tr>
<th># 2 and # 4 due to their attributes.</th>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td># 2 due to its similarity.</td>
<td></td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td># 1 and # 3 due to their attributes.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td># 3 and # 4 due to their similarities and differences.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td># 1 due to its attributes.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Your class is exploring measurement concepts. In the table below is a list of statements students made. Decide for each statement if **urgent** teacher intervention is required by indicating if you *would NOT intervene, might intervene, or definitely would intervene.*

<table>
<thead>
<tr>
<th>Statement</th>
<th>Would NOT intervene</th>
<th>Might intervene</th>
<th>Definitely would intervene</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area is the space inside a shape.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The diameter of a circle is the same idea as the perimeter of a square.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Volume is the amount of space a shape takes up.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Area is a measurement of the surface.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>As the perimeter increases, the area always increases.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Four of your students are examining the 3 spinners below. They are comparing the probabilities of the spinners stopping over a shaded region.

For each of the four student explanations below, indicate whether teacher intervention is required by indicating if you **would NOT**, **might intervene**, or **definitely would intervene**.

<table>
<thead>
<tr>
<th>Student Explanation</th>
<th>Would NOT intervene</th>
<th>Might intervene</th>
<th>Definitely would intervene</th>
</tr>
</thead>
<tbody>
<tr>
<td>“The probability is twice as large for Spinners 2 and 3 compared to Spinner 1 because they have two regions to stop on and Spinner 1 has only one region.”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>“Spinners 1 and 2 have the same probability since the shaded regions have the same area. Spinner 3 however, has a lower probability than Spinner 2 because the shaded region is a smaller area.”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>“Spinners 1, 2 and 3 have the same probability because the total of the shaded regions are the same size.”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>“The probabilities for Spinners 2 and 3 are the same because those areas are the same proportion of the whole circle. For Spinner 1 however, the probabilities are different because the shaded area for Spinner 1 has a bigger proportion of the whole circle.”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
When asked to measure the angle below with a protractor, Kylie answers that it is 30°. She asks you if she is correct. For each of the following statements indicate if you would not say it to Kylie, might say it to Kylie, or definitely would say it to Kylie.

<table>
<thead>
<tr>
<th></th>
<th>Would NOT say</th>
<th>Might say</th>
<th>Definitely would say</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can you show me which angle you are trying to measure?</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Well done Kylie, you’re absolutely correct.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Remember that angles are about the amount of turn, and the arrow shows the direction of turn.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>You need to subtract that from 360°.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>This one’s tricky because your protractor will only measure angles up to 180°.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Tommy is in Year 5. He states that A is the only rhombus because it's a diamond. For each teacher intervention provided in the table below, indicate to the right which intervention you *would NOT use, might use*, or *definitely would use* to help Tommy develop his understanding of shapes.

<table>
<thead>
<tr>
<th>Intervention</th>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tell Tommy that only A and D are rhombuses</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Tell Tommy that B and D are also rhombuses.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Ask Tommy to turn all the shapes into the same orientation as A.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Ask Tommy to measure the sides of each shape.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Tell Tommy that he’s correct.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
A box contains 18 red cubes, 10 green cubes, 10 yellow cubes and 2 black cubes. Without looking, Sheryl takes a cube from the box and keeps the result hidden. Students were asked to respond to the following question:

What is the chance that the cube is green?

One student says that the chance is 1 in 4. To help interpret this response, indicate for each follow-up question below whether you **would NOT ask**, **might ask**, or **definitely would ask** as the most appropriate follow-up question.

<table>
<thead>
<tr>
<th>Would NOT ask</th>
<th>Might ask</th>
<th>Definitely would ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>I'd ask them a similar question, but with only 8 red cubes instead of 18 red cubes.</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>I'd ask them a similar question with smaller numbers such as: 10 red cubes, 5 green cubes, 4 yellow cubes and 1 black cube.</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>I'd ask them, “How did you work that out?”</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>It is not necessary to ask a follow-up question because the student has responded correctly.</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>I'd ask them, “What is the difference between the chance of 1 in 4 and the chance of 10 and 40?”</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
A teacher gave the following problem to Sally to solve.

The numbers in the sequence 7, 11, 15, 19, 23, … increase by 4. The numbers in the sequence 1, 10, 19, 28, 37, … increase by 9. The number 19 is in both sequences. If the two sequences are continued, what is the next number that is in BOTH the first and second sequence?

Sally answers “27 and 46”.

Below there are five possible reasons for Sally’s response. For each, indicate whether you believe it is a Correct reason, Partially correct reason, or Incorrect reason for her response.

<table>
<thead>
<tr>
<th>Reason</th>
<th>Correct reason</th>
<th>Partially correct reason</th>
<th>Incorrect reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sally misread/misunderstood the question.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Sally gave the next numbers in each sequence rather than the “same” number in each.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Sally answered the question correctly.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Sally interpreted “BOTH” as meaning give “two” answers.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Sally answered only part of the question correctly.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Amy is analyzing the pattern shown below. She notices that each new step has one extra triangle and $t$ denotes the step number in the sequence.

![Pattern Diagram]

In finding a mathematical description of the pattern, Amy explains her thinking by saying: “I see three sticks are being used for each triangle. Then I see that from the second step on, I am counting one stick twice for each triangle, so I have to remove those.”

Variable $n$ in the equations below represents the total number of toothpicks used in each step. Following Amy’s thinking, indicate for each equation whether the equation is the **correct representation**, **partially correct representation**, or **incorrect representation** to her statement.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Correct representation</th>
<th>Partially correct representation</th>
<th>Incorrect representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 2t + 1$</td>
<td>[]</td>
<td>[]</td>
<td>[]</td>
</tr>
<tr>
<td>$n = 2(t + 1) - 1$</td>
<td>[]</td>
<td>[]</td>
<td>[]</td>
</tr>
<tr>
<td>$n = 3t - (t - 1)$</td>
<td>[]</td>
<td>[]</td>
<td>[]</td>
</tr>
<tr>
<td>$n = 3t + 1 - t$</td>
<td>[]</td>
<td>[]</td>
<td>[]</td>
</tr>
</tbody>
</table>
When teaching children about length measurement for the first time, Mrs. Brown prefers to begin by having the children measure the width of their book using paper clips, then again using pencils.

Below there are five possible reasons why Mrs. Brown would use this strategy to teach length measurement. For each, indicate whether you believe it is a **Correct reason**, **Partially correct reason**, or **Incorrect reason**.

<table>
<thead>
<tr>
<th>Reason</th>
<th>Correct reason</th>
<th>Partially correct reason</th>
<th>Incorrect reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using familiar objects such as pencils encourages estimation skills.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Using familiar/different units enables understanding of what measurement is and that any object/unit can be used to measure.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Using non-standard units of length to measure gives differing numbers of units for the same length and shows that we need standard units.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The teacher knows that the students will enjoy their work more if they can use hands-on materials.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Using objects of different lengths helps children learn how to decide which unit/object is the most appropriate to measure a given length.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Suppose you wish to know if your students really understand the formula for the area of a rectangle.

Below there are five teaching strategies you might use for this purpose. For each strategy below, indicate whether you would NOT use, might use, or definitely would use the strategy to determine if they really do understand the formula for the area of a rectangle.

<table>
<thead>
<tr>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Give them the following problem. If a rectangle is 4 cm long and 3 cm wide, what is its area?</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Simply ask them to tell you what the formula is for the area of a rectangle.</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Give them the following problem: “Sketch two rectangles each having an area of 12 cm$^2$.”</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Take a circle and partition it like a pizza and then cut out the pieces. Arrange those pieces to form a rectangle and ask the students to determine the area of the newly formed rectangle.</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Using a rectangle which is 4 cm long and 5 cm wide, ask the students to determine the area using only a square centimetre tile.</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
The following problem was given to Year 3 students.

The graph shows the number of pens, pencils, rulers and erasers sold by a store in one week. The names of the items are missing from the graph. However, pens were the item most often sold. Fewer erasers than any other item were sold. More pencils than rulers were sold. How many pencils were sold?

A. 40
B. 80
C. 120
D. 140

Some Year 3 students would experience difficulty with a problem of this type. Below are several possible reasons why. For each, indicate whether you believe it is a Correct reason, Partially correct reason, or Incorrect reason.

<table>
<thead>
<tr>
<th>Reason</th>
<th>Correct reason</th>
<th>Partially correct reason</th>
<th>Incorrect reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>There is a considerable amount of information to read, organise, sequence and relate to the graph.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The names are missing from the graph and they wouldn’t have experienced this before.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
The items described in the text are listed in a different order to the bars on the graph creating logistic or sequencing challenges.

Year 3 students would have no difficulties interpreting, and reasoning through, this problem.

The language used is quite challenging. Example, “fewer than any other” and “more pencils than rulers”.
Appendix D: Mathematics Teaching Efficacy Beliefs Instrument
**Mathematics Teaching Efficacy Beliefs Instrument**

Name ___________________________________________  Date __________________

Please indicate the degree to which you agree or disagree with each statement below by ticking the appropriate box to the right of each statement.

<table>
<thead>
<tr>
<th></th>
<th>Strongly disagree</th>
<th>1</th>
<th>2</th>
<th>Uncertain</th>
<th>3</th>
<th>4</th>
<th>Strongly agree</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>I will continually find better ways to teach mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Even if I try very hard, I will not teach mathematics as well as I will most subjects.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>I know how to teach mathematics concepts effectively.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>I will not be very effective in monitoring student’s mathematical learning activities in the classroom.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.</td>
<td>Strongly disagree</td>
<td>1</td>
<td>2</td>
<td>Uncertain</td>
<td>3</td>
<td>4</td>
<td>Strongly agree</td>
</tr>
<tr>
<td>---</td>
<td>----------------------------------------------------------------------------------------------------------------</td>
<td>------------------</td>
<td>---</td>
<td>---</td>
<td>-----------</td>
<td>---</td>
<td>---</td>
<td>----------------</td>
</tr>
<tr>
<td>7</td>
<td>I will not be able to teach mathematics effectively.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>The inadequacy of a student’s mathematical performance can be overcome through good teaching.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>I understand mathematics concepts well enough to be effective in teaching elementary mathematics.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>The teacher is generally responsible for the achievement of students in mathematics.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Students achievement in mathematics is directly related their teacher’s effectiveness in mathematics teaching.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child’s teacher.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Strongly disagree</td>
<td>2</td>
<td>Uncertain</td>
<td>4</td>
<td>Strongly agree</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>------------------</td>
<td>---</td>
<td>-----------</td>
<td>---</td>
<td>----------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>I will find it difficult to use manipulatives to explain to students why mathematics works.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>I will be able to answer student’s mathematical questions.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>I wonder if I have the necessary skills to teach mathematics in the future.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Given a choice, I will not invite the principal to evaluate my mathematics teaching.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>When teaching mathematics, I will usually welcome student questions.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>I do not know what to do to turn students onto mathematics.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix E: Pre-intervention MTEBI & MPCKI
The purpose of this questionnaire is to investigate pre-service teachers’ mathematical pedagogical content knowledge and their self-belief in their ability to teach mathematics effectively. You are asked to participate because it has professional implications for you and your future students.

The following 2 requests for information regarding your mother’s maiden name and the first 2 letters of your first name are being asked to ensure no participant’s names are collected on this survey.

What are the first 4 letters of your mother’s maiden name ___ ___ ___ ___

What are the first 2 letters of your first name ___ ___

Gender (M or F) _____

Indicate which year you were born ___ ___ ___ ___

Please indicate the correct response by ticking the appropriate box to the right of each statement.

<table>
<thead>
<tr>
<th>Indicate the level of prior classroom teaching experience you have.</th>
<th>None – only been in the classroom as a student</th>
<th>volunteering with little teaching</th>
<th>10 - 15 days classroom experience at practicums</th>
<th>16 - 25 days classroom experience at practicums</th>
<th>More than 26 days classroom experience at practicums</th>
<th>teaching experience as a teacher aide</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Indicate for each year level the mathematics subject(s) you have studied and passed. Indicate “none” if you did not study maths in that year.

eg: Year 8 - None
eg: Year 11 - Maths A and B

<table>
<thead>
<tr>
<th>Year 8</th>
<th>Year 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>_____________________________</td>
<td>_____________________________</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year 10</th>
<th>Year 11</th>
</tr>
</thead>
<tbody>
<tr>
<td>_____________________________</td>
<td>_____________________________</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>_____________________________</td>
</tr>
</tbody>
</table>

Have you studied and passed EDX1280 or received credit for it?  
Yes [ ]  No [ ]

<table>
<thead>
<tr>
<th>Besides EDX1280, how many other university level mathematics courses have you studied and passed, if any?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>[ ]</td>
</tr>
</tbody>
</table>
Please indicate the degree to which you agree or disagree with each statement below by ticking the appropriate box to the right of each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Strongly disagree</th>
<th>2</th>
<th>Uncertain</th>
<th>4</th>
<th>Strongly agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>I will continually find better ways to teach mathematics.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Even if I try very hard, I will not teach mathematics as well as I will most subjects.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>I know how to teach mathematics concepts effectively.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>I will NOT be very effective in monitoring mathematics activities.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Statement</td>
<td>Strongly disagree</td>
<td>1</td>
<td>2</td>
<td>Uncertain</td>
<td>3</td>
</tr>
<tr>
<td>--------------------------------------------------------------------------</td>
<td>-------------------</td>
<td>---</td>
<td>---</td>
<td>-----------</td>
<td>---</td>
</tr>
<tr>
<td>I will find it difficult to use manipulatives to explain to students why mathematics works.</td>
<td></td>
<td>[]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I will typically be able to answer students’ questions.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I wonder if I have the necessary skills to teach mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Given a choice, I will not invite the principal to evaluate my mathematics teaching.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>When teaching mathematics, I will usually welcome student questions.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I do not know what to do to turn students onto mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Statement</td>
<td>Strongly disagree</td>
<td>2</td>
<td>Uncertain</td>
<td>4</td>
<td>Strongly agree</td>
</tr>
<tr>
<td>--------------------------------------------------------------------------</td>
<td>-------------------</td>
<td>---</td>
<td>-----------</td>
<td>---</td>
<td>----------------</td>
</tr>
<tr>
<td>If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>I will generally teach mathematics ineffectively.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>The inadequacy of a student’s mathematics background can be overcome through good teaching.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>I understand mathematics concepts well enough to be effective in teaching primary school mathematics.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>The teacher is generally responsible for the achievement of students in mathematics.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>Students’ achievement in mathematics is directly related to their teacher’s effectiveness in mathematics teaching.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child’s teacher.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
</tbody>
</table>
The next 12 questions are asking you about your knowledge of teaching mathematics.

In the number sentence, $17 - 9 = \square + 1$, one of your students places an 8 in the empty box. For each teacher intervention provided in the table below, indicate to the right which intervention you would not use, might use, or definitely would use to help the student understand this relationship.

<table>
<thead>
<tr>
<th>Intervention</th>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remind the student that what you do to one side of the equation you must do to the other side.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Provide the student with a balance scale and blocks to create a representation of the equation.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Advise the student to consider the commutative property of addition.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
In an introductory lesson on parallelograms, you provided the following shape to your students.

Along with the above shape, indicate which additional shape(s) below you would NOT use, might use, or definitely would use to help the students improve their understanding about the properties of parallelograms.

<table>
<thead>
<tr>
<th></th>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 and #3 due to their attributes.</td>
<td>[]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>#3 and #4 due to their attributes.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>#1 due to its attributes.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Your class is exploring measurement concepts. In the table below is a list of statements students made. Decide for each statement if urgent teacher intervention is required by indicating if you *would NOT intervene, might intervene, or definitely would intervene*.

<table>
<thead>
<tr>
<th></th>
<th>Would NOT intervene</th>
<th>Might intervene</th>
<th>Definitely would intervene</th>
</tr>
</thead>
<tbody>
<tr>
<td>The diameter of a circle is the same idea as the perimeter of a square.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Volume is the amount of space a shape takes up.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>As the perimeter increases, the area always increases.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Three of your students are examining the 3 spinners below. They are comparing the probabilities of the spinners stopping over a shaded region.

For each of the three student explanations below, indicate whether teacher intervention is required by indicating if you would NOT intervene, might intervene, or definitely would intervene.

<table>
<thead>
<tr>
<th>Explanation</th>
<th>Would NOT intervene</th>
<th>Might intervene</th>
<th>Definitely would intervene</th>
</tr>
</thead>
<tbody>
<tr>
<td>“The probability is twice as large for Spinners 2 and 3 compared to Spinner 1 because they have two regions to stop on and Spinner 1 has only one region.”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>“Spinners 1 and 2 have the same probability since the shaded regions have the same area. Spinner 3 however, has a lower probability than Spinner 2 because the shaded region is a smaller area.”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>“The probabilities for Spinners 2 and 3 are the same because those areas are the same proportion of the whole circle. For Spinner 1 however, the probabilities are different because the shaded area for Spinner 1 has a bigger proportion of the whole circle.”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
When asked to measure the angle below with a protractor, Kylie answers that it is 30°. She asks you if she is correct. For each of the following statements indicate if you would not say it to Kylie, might say it to Kylie, or definitely would say it to Kylie.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Would NOT say</th>
<th>Might say</th>
<th>Definitely would say</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can you show me which angle you are trying to measure?</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Remember that angles are about the amount of turn, and the arrow shows the direction of turn.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>You need to subtract that from 360°.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Tommy is in Year 5. He states that A is the only rhombus because it's a diamond. For each teacher intervention provided in the table below, indicate to the right which intervention you would NOT use, might use, or definitely would use to help Tommy develop his understanding of shapes.

<table>
<thead>
<tr>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tell Tommy that only A and D are rhombuses</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Tell Tommy that B and D are also rhombuses.</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Ask Tommy to measure the sides of each shape.</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
A box contains 18 red cubes, 10 green cubes, 10 yellow cubes and 2 black cubes. Without looking, Sheryl takes a cube from the box and keeps the result hidden. Students were asked to respond to the following question:

What is the chance that the cube is green?

One student says that the chance is 1 in 4. To help interpret this response, indicate for each follow-up statement below whether you would NOT use, might use, or definitely would use as the most appropriate follow-up strategy.

<table>
<thead>
<tr>
<th>Follow-up Statement</th>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td>I’d ask them the same question, but with 8 red cubes, 10 green cubes, 10 yellow cubes and 2 black cubes.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>No follow-up strategy is necessary because the student has responded correctly.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>I’d ask them, “What is the difference between the chance of 1 in 4 and the chance of 10 in 40?”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
A teacher gave the following problem to Sally to solve.

The numbers in the sequence 7, 11, 15, 19, 23, … increase by 4. The numbers in the sequence 1, 10, 19, 28, 37, … increase by 9. The number 19 is in both sequences. If the two sequences are continued, what is the next number that is in BOTH the first and second sequence?

Sally answers “27 and 46”.

Below there are three possible reasons for Sally’s response. For each, indicate whether you believe it is a **Correct reason**, **Partially correct reason**, or **Incorrect reason** for her response.

<table>
<thead>
<tr>
<th>Reason</th>
<th>Correct reason</th>
<th>Partially correct reason</th>
<th>Incorrect reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sally answered the question correctly.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Sally interpreted “BOTH” as meaning give “two” answers.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Sally answered only part of the question correctly.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Amy is analysing the pattern shown below. She notices that each new step has one extra triangle and \( t \) denotes the step number in the sequence.

![Pattern Diagram](image)

In finding a mathematical description of the pattern, Amy explains her thinking by saying: “I see three sticks are being used for each triangle. Then I see that from the second step on, I am counting one stick twice for each triangle, so I have to remove those.”

Following Amy’s thinking, indicate for each equation whether the equation is the **correct representation**, **partially correct representation**, or **incorrect representation** to her statement. Variable \( n \) in the equations below represents the total number of toothpicks used in each step.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Correct representation</th>
<th>Partially correct representation</th>
<th>Incorrect representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 2t + 1 )</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>( n = 2(t + 1) - 1 )</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>( n = 3t - (t - 1) )</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
When teaching children length measurement for the first time, Mrs. Brown prefers to begin by having the children measure the width of their book using paper clips, then again using pencils.

Below there are three possible reasons why Mrs. Brown would use this strategy to teach length measurement. For each, indicate whether you believe it is a **Correct reason**, **Partially correct reason**, or **Incorrect reason**.

<table>
<thead>
<tr>
<th>Reason</th>
<th>Correct reason</th>
<th>Partially correct reason</th>
<th>Incorrect reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using familiar/different units enables understanding of what measurement is and that any object/unit with length can be used to measure.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Using non-standard units of length to measure gives differing numbers of units for the same length and shows that we need standard units.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The teacher knows that the students will enjoy their work if they can use hands-on materials.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Suppose you wish to know if your students really understand the formula for the area of a rectangle.

Below there are three teaching strategies you might use for this purpose. For each strategy below, indicate whether you \textit{would NOT use}, \textit{might use}, or \textit{definitely would use} the strategy to determine if they really do understand the formula for the area of a rectangle.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Give them the following problem. If a rectangle is 4 cm long and 3 cm wide, what is its area?</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Simply ask them to tell you what the formula is for the area of a rectangle.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Give them the following problem: “Sketch two rectangles each having an area of 12 cm$^2$. ”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
The following problem was given to Year 3 students.

The graph shows the number of pens, pencils, rulers and erasers sold by a store in one week. The names of the items are missing from the graph. However, pens were the item most often sold. Fewer erasers than any other item were sold. More pencils than rulers were sold. How many pencils were sold?

- E. 40
- F. 80
- G. 120
- H. 140

Some Year 3 students would experience difficulty with a problem of this type. Below are several possible reasons why. For each, indicate whether you believe it is a Correct reason, Partially correct reason, or Incorrect reason.

<table>
<thead>
<tr>
<th>Reason</th>
<th>Correct reason</th>
<th>Partially correct reason</th>
<th>Incorrect reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>There is a considerable amount of information to read, organise, sequence and relate to the graph.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The names are missing from the graph and they may not have experienced this before.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The language used is quite challenging. Example, “fewer than any other” and “more pencils than rulers”.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Appendix F: Post-intervention MTEBI & MPCKI
The purpose of this questionnaire is to investigate pre-service teachers’ mathematical pedagogical content knowledge and their self-belief in their ability to teach mathematics effectively. You are asked to participate because it has professional implications for you and your future students.

The following request below for name and demographic information will not become part of the survey. It is being asked in case follow-up information is required.

Last name ___________________________

First name __________________________

Gender (M or F) _____

Indicate which year you were born __ __ __ __
Please indicate the degree to which you agree or disagree with each statement below by ticking the appropriate box to the right of each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Strongly disagree</th>
<th>2</th>
<th>Uncertain</th>
<th>4</th>
<th>Strongly agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.</td>
<td>[]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I will continually find better ways to teach mathematics.</td>
<td>[]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Even if I try very hard, I will not teach mathematics as well as I will most subjects.</td>
<td>[]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.</td>
<td>[]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I know how to teach mathematics concepts effectively.</td>
<td>[]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I will NOT be very effective in monitoring mathematics activities.</td>
<td>[]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strongly disagree</td>
<td>2</td>
<td>Uncertain</td>
<td>4</td>
<td>Strongly agree</td>
</tr>
<tr>
<td>------------------------------------------------------------------</td>
<td>-------------------</td>
<td>---</td>
<td>-----------</td>
<td>---</td>
<td>----------------</td>
</tr>
<tr>
<td>I will find it difficult to use manipulatives to explain to students why mathematics works.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>I will typically be able to answer students’ questions.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>I wonder if I have the necessary skills to teach mathematics.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>Given a choice, I will not invite the principal to evaluate my mathematics teaching.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>When teaching mathematics, I will usually welcome student questions.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>I do not know what to do to turn students onto mathematics.</td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
<td></td>
<td>[ ]</td>
</tr>
<tr>
<td>Strongly disagree</td>
<td>1</td>
<td>2</td>
<td>Uncertain</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>-------------------</td>
<td>---</td>
<td>---</td>
<td>-----------</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>I will generally teach mathematics <strong>ineffectively</strong>.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The inadequacy of a student’s mathematics background can be overcome through good teaching.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>I understand mathematics concepts well enough to be effective in teaching primary school mathematics.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The teacher is generally responsible for the achievement of students in mathematics.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Students’ achievement in mathematics is directly related to their teacher’s effectiveness in mathematics teaching.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child’s teacher.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
The next 12 questions are asking you about your knowledge of teaching mathematics.

In the number sentence, \(17 - 9 = \underline{\quad} + 1\), one of your students places an 8 in the empty box. For each teacher intervention provided in the table below, indicate to the right which intervention you would not use, might use, or definitely would use to help the student understand this relationship.

<table>
<thead>
<tr>
<th>Intervention</th>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discuss with the student the purpose of the equal sign and about relationships between the left side and the right side of an equation.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Ask the student to solve a similar, yet less difficult problem such as (7 - 5 = \underline{\quad} + 1)</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Advise the student to consider the commutative property of addition.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
In an introductory lesson on parallelograms, you provided the following shape to your students.

Along with the above shape, indicate which additional shape(s) below you would NOT use, might use, or definitely would use to help the students improve their understanding about the properties of parallelograms.

<table>
<thead>
<tr>
<th># 2 and # 4 due to their attributes.</th>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td># 1 and # 3 due to their attributes.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td># 3 and # 4 due to their attributes.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Your class is exploring measurement concepts. In the table below is a list of statements students made. Decide for each statement if urgent teacher intervention is required by indicating if you *would NOT intervene, might intervene, or definitely would intervene.*

<table>
<thead>
<tr>
<th>Statement</th>
<th>Would NOT intervene</th>
<th>Might intervene</th>
<th>Definitely would intervene</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area is the space inside a shape.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The diameter of a circle is the same idea as the perimeter of a square.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Area is a measurement of the surface.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Three of your students are examining the 3 spinners below. They are comparing the probabilities of the spinners stopping over a shaded region.

For each of the three student explanations below, indicate whether teacher intervention is required by indicating if you *would NOT intervene, might intervene, or definitely would intervene.*

<table>
<thead>
<tr>
<th>Student Explanation</th>
<th>Would NOT intervene</th>
<th>Might intervene</th>
<th>Definitely would intervene</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Spinners 1 and 2 have the same probability since the shaded regions have the same area. Spinner 3 however, has a lower probability than Spinner 2 because the shaded region is a smaller area.”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>“Spinners 1, 2 and 3 have the same probability because the total of the shaded regions are the same size.”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>“The probabilities for Spinners 2 and 3 are the same because those areas are the same proportion of the whole circle. For Spinner 1 however, the probabilities are different because the shaded area for Spinner 1 has a bigger proportion of the whole circle.”</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
When asked to measure the angle below with a protractor, Kylie answers that it is 30°. She asks you if she is correct. For each of the following statements indicate if you *would not say* it to Kylie, *might say* it to Kylie, or *definitely would say* it to Kylie.

<table>
<thead>
<tr>
<th></th>
<th>Would NOT say</th>
<th>Might say</th>
<th>Definitely would say</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well done Kylie, you’re absolutely correct.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Remember that angles are about the amount of turn, and the arrow shows the direction of turn.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>This one’s tricky because your protractor will only measure angles up to 180°.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Tommy is in Year 5. He states that A is the only rhombus because it's a diamond. For each teacher intervention provided in the table below, indicate to the right which intervention you **would NOT use**, **might use**, or **definitely would use** to help Tommy develop his understanding of shapes.

<table>
<thead>
<tr>
<th>Intervention</th>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ask Tommy to turn all the shapes to the same orientation as A.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Ask Tommy to measure the sides of each shape.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Tell Tommy that he’s correct.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
A box contains 18 red cubes, 10 green cubes, 10 yellow cubes and 2 black cubes. Without looking, Sheryl takes a cube from the box and keeps the result hidden. Students were asked to respond to the following question:

What is the chance that the cube is green?

One student says that the chance is 1 in 4. To help interpret this response, indicate for each follow-up strategy below whether you would NOT use, might use, or definitely would use as the most appropriate follow-up strategy.

<table>
<thead>
<tr>
<th>Follow-up Strategy</th>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td>I'd ask them a similar question with smaller numbers such as: 10 red cubes, 5 green cubes, 4 yellow cubes and 1 black cube.</td>
<td>[]</td>
<td>[]</td>
<td>[]</td>
</tr>
<tr>
<td>No follow-up strategy is necessary because the student has responded correctly.</td>
<td>[]</td>
<td>[]</td>
<td>[]</td>
</tr>
<tr>
<td>I'd ask them, “What is the difference between the chance of 1 in 4 and the chance of 10 in 40?”</td>
<td>[]</td>
<td>[]</td>
<td>[]</td>
</tr>
</tbody>
</table>
A teacher gave the following problem to Sally to solve.

The numbers in the sequence 7, 11, 15, 19, 23, … increase by 4. The numbers in the sequence 1, 10, 19, 28, 37, … increase by 9. The number 19 is in both sequences. If the two sequences are continued, what is the next number that is in BOTH the first and second sequence?

Sally answers “27 and 46”.

Below there are three possible reasons for Sally’s response. For each, indicate whether you believe it is a Correct reason, Partially correct reason, or Incorrect reason for her response.

<table>
<thead>
<tr>
<th>Reason</th>
<th>Correct reason</th>
<th>Partially correct reason</th>
<th>Incorrect reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sally misread/misunderstood the question.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Sally gave the next numbers in each sequence rather than the “same” number in each.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Sally answered only part of the question correctly.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Amy is analysing the pattern shown below. She notices that each new step has one extra triangle and \( t \) denotes the step number in the sequence.

![Pattern Diagram]

In finding a mathematical description of the pattern, Amy explains her thinking by saying: “I see three sticks are being used for each triangle. Then I see that from the second step on, I am counting one stick twice for each triangle, so I have to remove those.”

Following Amy’s thinking, indicate for each equation whether the equation is the correct representation, partially correct representation, or incorrect representation to her statement. Variable \( n \) in the equations below represents the total number of toothpicks used in each step.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Correct representation</th>
<th>Partially correct representation</th>
<th>Incorrect representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 2t + 1 )</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>( n = 3t - (t - 1) )</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>( n = 3t + 1 - t )</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
When teaching children length measurement for the first time, Mrs. Brown prefers to begin by having the children measure the width of their book using paper clips, then again using pencils.

Below there are three possible reasons why Mrs. Brown would use this strategy to teach length measurement. For each, indicate whether you believe it is a Correct reason, Partially correct reason, or Incorrect reason.

<table>
<thead>
<tr>
<th>Reason</th>
<th>Correct reason</th>
<th>Partially correct reason</th>
<th>Incorrect reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using non-standard units of length to measure gives differing numbers of units for the same length and shows that we need standard units.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The teacher knows that the students will enjoy their work if they can use hands-on materials.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Using objects of different lengths helps children learn how to decide which unit/object is the most appropriate to measure a given length.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Suppose you wish to know if your students really understand the formula for the area of a rectangle.

Below there are three teaching strategies you might use for this purpose. For each strategy below, indicate whether you *would NOT use*, *might use*, or *definitely would use* the strategy to determine if they really do understand the formula for the area of a rectangle.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Would NOT use</th>
<th>Might use</th>
<th>Definitely would use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply ask them to tell you what the formula is for the area of a</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>rectangle.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Take a circle and partition it like a pizza and then cut out the pieces.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>Arrange those pieces to form a rectangle and ask the students to</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>determine the area of the newly formed rectangle.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Using a rectangle which is 4 cm long and 5 cm wide, ask the students to</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>determine the area using only a square centimetre tile.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The following problem was given to Year 3 students.

The graph shows the number of pens, pencils, rulers and erasers sold by a store in one week. The names of the items are missing from the graph. However, pens were the item most often sold. Fewer erasers than any other item were sold. More pencils than rulers were sold. How many pencils were sold?

I. 40
J. 80
K. 120
L. 140

Some Year 3 students would experience difficulty with a problem of this type. Below are several possible reasons why. For each, indicate whether you believe it is a Correct reason, Partially correct reason, or Incorrect reason.

<table>
<thead>
<tr>
<th>Reason</th>
<th>Correct reason</th>
<th>Partially correct reason</th>
<th>Incorrect reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>The names are missing from the graph and they may not have experienced this before.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The items described in the text are listed in a different order to the bars on the graph creating logistic or sequencing challenges.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
<tr>
<td>The language used is quite challenging. Example, “fewer than any other” and “more pencils than rulers”.</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
</tr>
</tbody>
</table>
Appendix G: End-of-semester Exam PCK Questions
Eight End-of-semester Mathematics PCK Questions

This is a 3-part question, and, each question is worth 2 marks. So, read all three (3) questions before answering the first question.

1. Provide an appropriate revisit, through an orientating phase activity, which addresses the first and second steps of the four step process for teaching measurement, as outlined in this subject, which would introduce the concept of area of a circle to a Year 6 class. Make sure you mention the specific language and materials you would use.

2. Still revisiting the topic, continue outlining the activity which addresses the third step of the four step process for teaching measurement that would introduce to a Year 6 class the concept of area of a circle. Make sure you mention the specific language and materials you would use.

3. Now in the enhancing phase, outline the activity which addresses the fourth step of the four step process for teaching measurement that would allow a Year 6 student to scaffold their understanding of area of a rectangle to the area of a circle. NOTE: It is not until AFTER Year 6 where students begin using the formula. Make sure you mention the specific language and materials you would use.

This is a 3-part question, and, each question is worth 3 marks. So, read all three (3) questions before answering the first question.

The van Hiele framework describes 5 sequential levels of learning specifically related to geometry. The three (3) levels that relate to primary school are: Recognition, Analysis, and Relational. For each of these three (3) levels, detail one appropriate teaching activity that you would use to facilitate children’s learning in the topics.

1. 1st activity (Recognition / Visualisation Level) for 2D shapes (Early Primary). Detail a teaching activity that you would use to facilitate children’s learning of 2D shapes at the Recognition Level. Make sure you mention the specific language and materials you would use.

2. 2nd activity (Analysis Level) – for 2D shapes (Primary). Detail a teaching activity that you would use to facilitate children’s learning of 2D shapes at the Analysis level. Make sure you mention the specific language and materials you would use.
3. 3rd activity (Relational Level) – for 2D and/or 3D shapes (Upper Primary/Middle). **Detail a teaching activity** that you would use to facilitate children’s learning of 2D and/or 3D shapes at the Relational level. Make sure you mention the specific language and materials you would use.

This is a 2-part question. **Part 1 is worth 2 marks and part 2 is worth 1 mark.**
You are preparing an activity for a Year 3 class which involves tossing 2 coins several times. For the activity:

1. Detail an activity which requires the Year 3 students to collect data and communicate the findings to another class. Make sure you mention the specific language and materials you would use.

2. Identify two mathematical concepts you expect the students will develop from engaging in this activity.
Appendix H: Ethics Approval
The USQ Human Research Ethics Committee (HREC,) at its meeting on 12 March 2013, assessed your application and agreed that your proposal meets the requirements of the National Statement on Ethical Conduct in Human Research (2007). Your project has been endorsed and full ethics approval granted.

<table>
<thead>
<tr>
<th>Project Title</th>
<th>Problem-based learning’s impact on pre-service teachers’ pedagogical content knowledge and their ability to apply their knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approval no.</td>
<td>H13REA002</td>
</tr>
<tr>
<td>Expiry date</td>
<td>30 November 2014</td>
</tr>
<tr>
<td>HREC Decision</td>
<td>Approved as submitted</td>
</tr>
</tbody>
</table>

The standard conditions of this approval are:

(a) conduct the project strictly in accordance with the proposal submitted and granted ethics approval, including any amendments made to the proposal required by the HREC
(b) advise (email: ethics@usq.edu.au) immediately of any complaints or other issues in relation to the project which may warrant review of the ethical approval of the project
(c) make submission for approval of amendments to the approved project before implementing such changes
(d) provide a ‘progress report’ for every year of approval
(e) provide a ‘final report’ when the project is complete
(f) advise in writing if the project has been discontinued.

For (c) to (e) forms are available on the USQ ethics website: http://www.usq.edu.au/research/ethicsbio/human For (d) and (e), diarise the applicable dates now to ensure compliance with reporting requirements.

Please note that failure to comply with the conditions of approval and the National Statement (2007) may result in withdrawal of approval for the project.

You may now commence your project. I wish you all the best for the conduct of the project. If you have any further queries please do not hesitate to contact me on 4631 2690 or ethics@usq.edu.au

Melissa McKain
Office of Research & Higher Degrees
Appendix I: Participant Information Sheet and Consent Form
TO: Students

Full Project Title: Problem-based learning’s impact on pre-service teachers’ pedagogical content knowledge and their ability to apply their knowledge.

Principal Researcher: David Martin
Associate Researcher: Professor Romina Jamieson-Proctor
Associate Researcher: Professor Peter Albion

I teach at the University of Southern Queensland (USQ) Fraser Coast Campus in the Faculty of Education, and have begun a postgraduate degree in the Doctor of Philosophy (PhD) program.

You are invited to participate in this research project because it has professional implications for you and for all your future students.

Please read the following plain language description of the research carefully. Its purpose is to explain to you as openly and clearly as possible all the procedures involved so that you can make a fully informed decision as to whether you are going to participate. Feel free to ask questions about any information in the document.

Once you understand what the project is about, if you agree to participate, please sign the Consent Form. By signing the Consent Form, you indicate that you understand the information and that you give your consent to participate in the research project.

1. **Purpose of Research**

The purpose of this study is to determine the impact a problem-based learning teaching approach versus a traditional teacher-centred instructional approach in a university course will have on pre-service teachers’ mathematics pedagogical content knowledge and their self-belief regarding their ability to teach mathematics. The research will contribute to a PhD postgraduate degree.

2. **Procedures**

Participation in this project will involve

- Completing a pre-semester and post-semester questionnaire.
- Answering your scheduled weekly tutorial questions to the best of your ability.
- Completing the end-of-semester exam to the best of your ability.
- If necessary, due to possible disagreeing data from your responses, participating in a post-semester interview.
3. **Confidentiality**

All data collected from questionnaires, tutorial questions, semester exams, or interviews will be de-identified for storage. Confidentiality will be assured in this process as neither your personal information nor grades will be identified and in any publication. Information will be provided in such a way that you cannot be identified.

The data collected electronically will be stored in password protected computer files. Data collected manually, via questionnaires, tutorial questions, semester exams, or semi-structured interviews will be stored in a locked filing cabinet. Data will be destroyed after the mandatory 5 yr term on completion of the study.

Demographic data collected in this study does not identify any peoples from the following groups: pregnant women and the foetus; children and young people; people in dependent or unequal relationships; people highly dependent on medical care; people with cognitive impairment, intellectual disability, or mental illness; people involved in illegal activities; Aboriginal and Torres Strait Islander peoples; people in other countries; other cultural and ethnic groups.

**Voluntary Participation**

Participation is entirely voluntary. **If you do not wish to take part you are not obliged to.** If you decide to take part and later change your mind, you are free to withdraw from the project at any stage.

Your decision whether to take part or not to take part, or to take part and then withdraw, will not affect your relationship with the University of Southern Queensland.

Before you make your decision, a member of the research team will be available to answer any questions you have about the research project. You can ask for any information you want. Sign the Consent Form only after you have had a chance to ask your questions and have received satisfactory answers. If you decide to withdraw from this project, please notify a member of the research team.

4. **Queries or Concerns**

Should you have any queries regarding the progress or conduct of this research, you can contact the principal researcher:

**David Martin**  
**School of Teacher Education & Early Childhood**  
**University of Southern Queensland; PO Box 910**  
**Hervey Bay  QUEENSLAND  4655**  
**Telephone: 4194 3171; Email: david.martin2@usq.edu.au**

*If you have any ethical concerns with how the research is being conducted or any queries about your rights as a participant please feel free to contact the University of Southern Queensland Ethics Officer on the following details.*

Ethics and Research Integrity Officer  
Office of Research and Higher Degrees  
University of Southern Queensland; West Street, Toowoomba 4350  
Ph: +61 7 4631 2690; Email: ethics@usq.edu.au
University of Southern Queensland

Consent Form

TO: Students

Full Project Title: Problem-based learning’s impact on pre-service teachers’ pedagogical content knowledge and their ability to apply their knowledge.

Principal Researcher: David Martin

Associate Researcher: Professor Romina Jamieson-Proctor

Associate Researcher: Professor Peter Albion

- I have read the Participant Information Sheet and the nature and purpose of the research project has been explained to me. I understand and agree to take part.

- I understand the purpose of the research project and my involvement in it.

- I understand that I may withdraw from the research project at any stage and that this will not affect my status now or in the future.

- I confirm that I am over 18 years of age.

- I understand that while information gained during the study may be published, I will not be identified and my personal results will remain confidential.

Name of participant

Signed

Date

If you have any ethical concerns with how the research is being conducted or any queries about your rights as a participant please feel free to contact the University of Southern Queensland Ethics Officer on the following details.

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