Proceedings of the 2007 IEEE
International Conference on Integration Technology
March 20 - 24, 2007, Shenzhen, China

Networked PID Control System Modeling and Simulation Using Markov Chain

Lanzhi Teng and Peng Wen
School of Engineering and Surveying
University of Southern Queensland
West Street, Toowoomba QLD 4350, Australia
{teng & wen}@usq.edu.au

***Abstract* - This paper presents a new model for a Networked PID Control System. In this model a Markov chain is included to represent the unreliable link of the shared networks. In this study we identify the minimum measurement information involved in the PID control system and build it up as an embedded Markov chain. This converts a networked PID control system into a jump control system. This jump control system is evaluated in simulation and the results agree with our analysis.**

Index Terms* - *PID Controller, Networked Control System, Packet Drop, Markov Chain

I. INTRODUCTION

Networked control system (NCS) is an important research area. Examples are given in [1-3]. NCS is a feedback control system that uses data network as a medium for the feedback path [4-5], [6-12]. The introduction of computing communication network improves the efficiency, flexibility, reliability, and reduces the cost of installation, reconfiguration, and maintenance. However, the integration of computing communication and control system causes different kinds of time delays and packet loss which come from the communication channel sharing, operating environment, etc. This degrades a system's performance and possibly cause the system instability [4-5], [7-16]. For a networked linear control system, Wen et al identify the minimum measurement information involved in the system as minimum packet drop sequences, and modeled as Markov chain [7-8]. This paper extends the method to a PID control system over NCS with Markov chain. The new model is employed to analyze the stochastic behavior with packet drops and to identify the minimum measurement information involved in the PID control system. This minimum packet drop sequences is embedded in a Markov chain.

This paper is organized as the followings. Section 1 briefs the background of NCS and our work. Section 2 introduces the new model for a networked PID control system using an embedded Markov chain. In section 3, we demonstrate the model through numerical examples using Simulink. Section 4 sums our work and the conclusion.

II. MARKOV CHAIN BASED MODEL FOR PACKET DROP SEQUENCES

The block diagram in Fig. 1 shows the structure of the NCS. In this NCS, the process and the controller are connected to each other through a network. A PID controller is selected for the networked control system.

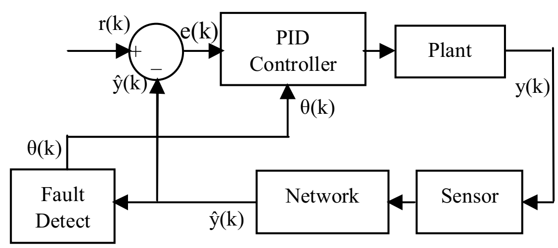


Fig. 1 Networked Control System with PID Controller

A. PID control and minimum drop sequence

With advances in digital technology, the science of automatic control now offers a wide spectrum of choices for control scheme. However, more than 90% of industrial controllers are still implemented based around PID algorithms, particularly at lowest levels, as no other controllers match the simplicity, clear functionality, applicability, and easy of use offered by the PID controller. Its wide application has stimulated and sustained the development of various PID tuning techniques, sophisticated software packages, and hardware modules. With much of academic research in this area maturing and entering the region of "diminishing returns", the trend in present research and development of PID technology appears to be focus on the integration of available methods so as to get the best out of PID control. Networked PID control system is one of these branches that still go on to find the next key technology for PID.

Numerically, a PID controller can be written as

$$u(k)=u(k-1)+k_1e(k)+k_2e(k-1)+k_3e(k-2)$$

where u(k) is the controller output, u(k-1) previous output, e(k) current error, e(k-1) previous error and e(k-2) the error before the previous one. k_1, k_2, and k_3 are the functions of k_P, k_I, and k_D, and the sampling time T.

From the above difference equation, clearly we can see that the two previous input signals and one previous output signals would affect the controller output directly. To achieve a good performance, a PID control algorithm/controller should be able to access all the above information. Therefore, at least two previous measurements should be considered for a networked system with a PID controller. These measurements correspond to two previous consecutive packets. Based on that, we can conclude that the minimum length of the packet sequences is three, which is the current packet plus two previous packets. For a networked PID control system, therefore, we need to look at only three consecutive packets. Each of the three consecutive packets can be either received or dropped. That gives eight states, i.e. {000}, {001}, {010}, {011}, {100}, {101}, {110}, {111}, where 0 represents packet dropped, 1 received, and the most right bit represents the most recent packet. At consecutive times, the state of the system in the term of packet loss may different. At all these times, the system may change from one state to another or stayed in the same state. The future state depends only on the present and the conditional probability distribution. Therefore, Markov chain can be used to represent the minimum packet loss sequence of a PID controller. The process can be modeled by a Markov chain with state space {0, 1, 2, 3, 4, 5, 6, 7}. At any given instant t_k, the Markov chain stochastic variable θ(k) takes one value from the set {0, 1, 2, 3, 4, 5, 6, 7}.

B. System Modeling

For a given control system shown in Fig 1, it can be described as

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k)$$

where $x \in R_n$ denotes the state vector, $y \in R_m$ the measurable output vector, $u \in R_p$ the control input vector.

The output, y(k), and the dropout process, θ(k), drive the model of the feedback channel which generates the feedback signal. The feedback signal is then added to the exogenous input, r(k), to generate the control input, u(k). We define e(k)=r(k)-ŷ(k) as the system error.

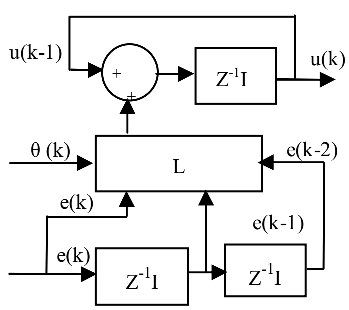


Fig. 2 PID Controller over NCS with Markov Chain

The PID control part can be described in Fig. 2. In the model, the unit delay blocks are used for holding the previous packet signals. Then, when a packet is dropped, the last available feedback signal is used to replace the current packet. For example, if the status is {×10}, y(k-1) is used; if {100}, y(k-2) is used. Of course, if {××1}, packet is received and y(k) is used.

If no packet drops, the dynamics of the overall system with r(k)=0 is

$$\begin{pmatrix} x(k+1) \\ u(k) \end{pmatrix} = \begin{pmatrix} A - k_1BC & -k_2BC & -k_3BC & B \\ -k_1C & -k_2C & -k_3I & I \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-1) \\ x(k-2) \\ u(k-1) \end{pmatrix}$$

If y(k-1) is used, the dynamics of the overall system with r(k)=0 is

$$\begin{pmatrix} x(k+1) \\ u(k) \end{pmatrix} = \begin{pmatrix} A & -k_1BC - k_2BC & -k_3BC & B \\ 0 & -k_1C - k_2C & -k_3C & I \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-1) \\ x(k-2) \\ u(k-1) \end{pmatrix}$$

If y(k-2) is used, the dynamics of the overall system with r(k)=0 is

$$\begin{pmatrix} x(k+1) \\ u(k) \end{pmatrix} = \begin{pmatrix} A & -k_2BC & -k_1BC - k_3BC & B \\ 0 & -k_2C & -k_1C - k_3C & I \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-1) \\ x(k-2) \\ u(k-1) \end{pmatrix}$$

The system has different system matrices, and different transfer functions at each state. The dropout process θ(k) determines system matrix and transfer functions in the above control system. With the PID controller, the system can be tuned or optimized one by one at each mode such that the closed-loop control system would be stable and have best performance.

III. Simulation and Examples

In this section, we study an example to test the theoretical results that are presented in the above section.

Example: Consider the following system

$$A = \begin{pmatrix} 0.2839 & -0.0276 \\ 0.1970 & 0.6821 \end{pmatrix}$$
$$B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$C = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

with initial conditions x(0)=[0 0].

The plant transfer function is represented by

$$H(z) = D + C(zI - A)^{-1}B = \frac{z^{-1} - 0.0869z^{-2}}{1 - 0.9660z^{-1} + 0.1991z^{-2}}$$

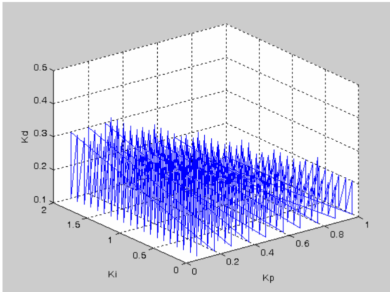
The discrete-time PID controller is derivated by

$$C(z) = \begin{cases} \dfrac{k_1 + k_2z^{-1} + k_3z^{-2}}{1 - z^{-1}}, & \theta(k) = 0,1,3,5,7 \\ \dfrac{(k_1 + k_2)z^{-1} + k_3z^{-2}}{1 - z^{-1}}, & \theta(k) = 2,6 \\ \dfrac{k_2z^{-1} + (k_1 + k_3)z^{-2}}{1 - z^{-1}}, & \theta(k) = 4 \end{cases}$$

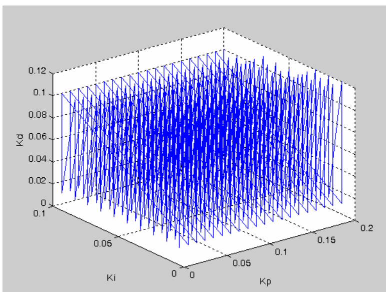
The system transfer function is

$$G(z) = \frac{C(z)H(z)}{1 + C(z)H(z)} = J(z, K_P, K_I, K_D)$$

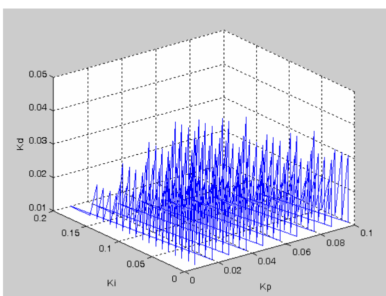
Determine the values of k_P, k_I, and k_D for which the closed-loop characteristic polynomial has all its roots in the unit circle. The stability regions in the space of k_P, k_I, and k_D are calculated using Matlab and are shown in Fig. 3 (a), (b), and (c) for no packet drops, {×10} cases, and {100} cases, respectively. Any PID gains selected from these regions will result in a stable system.



(a) No Packet Drop States



(b) {×10} States



(c) {100} States

Fig. 3 Stable Regions

To make sure the system stable at each mode, we simulated the step responses of the system using Simulink for the following PID controller parameters which are chosen from the stable region data sets

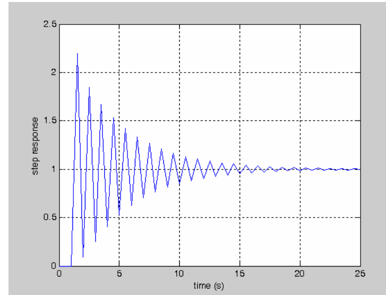
k_{P1}=1, k_{I1}=2, and k_{D1}=0.1 for no packet drop states;

k_{P2}=0.2, k_{I2}=0.1 and k_{D2}=0.1 for packet drop states {×10};

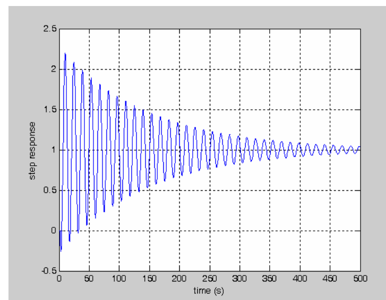
k_{P3}=0.1, k_{I3}=0.11, and k_{D3}=0.01 for packet drop states {100}.

Fig. 4 shows the system step responses for the system model with the sampling time T=0.5s.

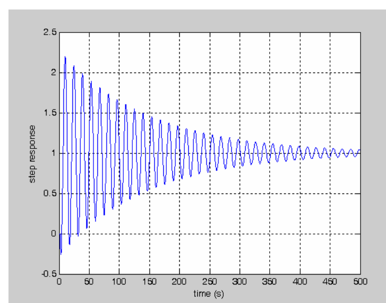
With the above PID controller parameters, we simulate the overall system performance with Simulink. The step response is shown in Fig. 5. The overall system is stable when jump from one mode to another after PID controller tuning.



(a) No Packet Drop States



(b) {×10} States



(c) {100} States

Fig. 4 Step Input Response

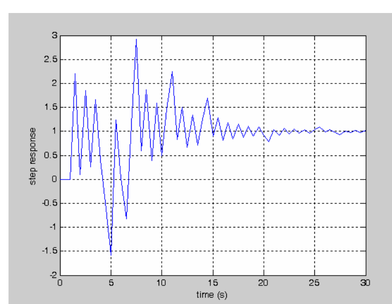


Fig. 5 Overall Step Response of the System

IV. Conclusion and Future Work

In this study, we have identified the minimum drop sequence for a networked PID control system, proposed to model these sequences as a Markov chain, and converted a networked PID control system into a jump control system. Based on this jump control system model, we evaluate the system performance for each of the drop sequences. An example is provided to show how to evaluate the system performance. The results agree with our model prediction. Next, we will investigate the stability and conditions, and the optimization method.

References

[1] G. N. Nair and R.J. Evans, "Communication Limited Stabilization of Linear System", Proceedings of the Conference on Decision and Control, 2000, pp. 1005-1010.

[2] J.K. Yook, D.M. Tilbury, and N.R. Soparkar, "Trading Computation for Bandwidth: Reducing Communication in Distributed Control Systems Using State Estimators", IEEE Transactions on Control Systems Technology, July 2002, Vol 10, No.4, pp. 503-518.

[3] G. Walsh, H. ye, and L. Bushnell, "Stability Analysis of Networked Control Systems", Proceedings of American Control Conference. June 1999.

[4] Xiangheng Liu & Andrea Goldsmith, "Wireless medium access control in networked control system", IEEE American Control Conference, 2004.

[5] B. Azimi Sadjadi, "Stability of networked control system in the presence of packet losses", Proceeding of IEEE Conference on Decision and Control, Hawaii, USA, Dec. 9-12, 2003.

[6] P. Seiler & R. Sengupta, "An H∞ approach to networked control", IEEE Transaction on Automatic control, Vol. 50, No. 3, March 2005.

[7] P. Wen and J. Cao, "Networked Control System Modeling based on Packet Drop Sequences", submitted to IEEE Trans. On Control System Technology, 2006.

[8] Peng Wen & Yan Li, "Networked Control System stability Margin Analysis based on Packet Drop", IEEE Trans. On Control System Technology, Submitted in Dec. 2005.

[9] S. Tatikonda, A. Sahai & S. Mitter, "Stochastic linear control over communication channel", IEEE Trans. On Automatic Control, Vol. 49, No. 9, Sep. 2004, pp. 1549-1561.

[10] Peng Wen &Yan Li, "Deadline fault tolerance in a networked real-time system", The 4th International Conference on Active Media Technology (AMT06), Brisbane, Australia, June 7-9 2006 (Accepted, Jan. 2006).

[11] N. Elia, "When Bode meets Shannon: Control-oriented feedback communication scheme", IEEE Tans. On Automatic Control, Vol. 49, No. 9, Sep. 2004, pp. 1477-1488.

[12] V. Gupta, D. Spanos, B. Hassibi & R. M. Murray, "Optimal LQG control across packet-dropping links", Submitted to IEEE Trans. On Automatic Control, 2005.

[13] Paulo Pedriras et al., "FTP-Ethernet: A flexible real-time communication protocol that support dynamic QoS management on Ethernet-based systems", IEEE Trans. On Industrial Informatics, Col. 1, No. 3, Aus. 2005, pp. 162-172.

[14] P. Seiler & R. Sengupta, "Analysis of communication losses in vehicle control problems", Proceeding of American Control Conference, June 2001, pp.1491-1496.

[15] C. Hadjicostis and R. Touri, "Feedback control utilizing packet dropping network links", IEEEE conference on decision and control, Dec. 2002.

[16] B. Sinopoli, et al., "Kalman filter with intermittent observation", IEEE Transaction on Automatic Control, Vol. 49, No. 9, Sep. 2004, pp 1453-1464.