Optimal topology selection of continuum structures with displacement constraints

Q.Q. Liang[a,*], Y.M. Xie[a] and G.P. Steven[b]

[a] School of the Built Environment, Victoria University of Technology, PO Box 14428, Melbourne City MC, VIC 8001, Australia
[b] Department of Aeronautical Engineering, The University of Sydney, NSW 2006, Australia

Abstract

This paper deals with optimal topology selection of continuum structures subject to displacement constraints by using the performance-based design concept. The optimal topology of a continuum structure is generated by gradually eliminating underutilized elements from the discretized design domain. A performance index is developed for monitoring the optimization process and used as a termination criterion in the optimization algorithm so that the global optimum can be selected from the optimization history. Maximizing the performance index in the design space is proposed as the performance-based optimization criterion. The performance index can be utilized to compare the efficiency of structural topologies produced by different continuum topology optimization methods. Several examples are provided to demonstrate the capabilities of the performance-based optimization approach in selecting the best configuration for the minimum-weight design of continuum structures with maximum stiffness.

Keywords: Finite element analysis; Performance based optimization; Performance evaluation; Stiffness; Selection; Topology

* Corresponding author. Tel.: +61 3 9688 5016; Fax: +61 3 9688 4139
E-mail address: qqliang88@hotmail.com (Q.Q. Liang)

1. Introduction

Structural topology optimization is the selection of the best configuration for the design of structures. Topology optimization of skeletal structures has been reviewed by Topping [1] and Kirsch [2]. Much more attention has been devoted to the development of continuum topology optimization methods in recent years. The homogenization-based optimization method pioneered by Bendsøe and Kikuchi [3] treats continuum topology optimization as a redistribution problem of composite material that consists of perforated microstructures. The homogenization-based design concept has extensively been adopted in continuum topology optimization [4-8]. Optimal topologies can be generated without using the homogenization theory as discussed by Rozvany et al. [9]. The density function approach given by Mlejnek et al. [10] and by Yang and Chuang [11] uses the element density as continuous design variables to solve the topology optimization problem of continuum structures. The rule-based approach by Rodriguez-Velazquez and Seireg [12] and the evolutionary optimization method by Xie and Steven [13] are discrete variable optimization approaches, which employ binary decision-making criteria to generate optimal topology. The density function and discrete variable optimization approaches are very attractive to practicing engineers due to their simplicity.

The difficulty involved in continuum topology optimization is to incorporate an approximate termination criterion in optimization algorithms to obtain the global optimum. The prescribed material volume has commonly been used in topology optimization approaches as the termination criterion [3-8,14-16]. In these approaches, using different amount of material as the constraint results in different designs. The behavioral constraints have been employed in topology optimization as the termination criterion, which is the only driving force for determining the final results [17,18]. Moreover, no objective functions and constraints are used to control the optimization process in some optimization methods [13,19,20]. The result satisfying these termination criteria may not be the global optimum in the given design space. Therefore, there is a strong need to develop performance-based termination criteria, which can be used to monitor the optimization process so that the global optimum can be determined.

Performance indices have been used to assist the selection of the best configuration for the design of structural components and systems by various researchers. The framework outlined by Ashby [21] is useful for deriving performance indices for the optimization of materials and cross-section shapes for structural components. Burgess [22,23] has used the method proposed by Ashby to develop performance indices known as form factors for measuring the mass efficiency of structural layouts for trusses and beams. However, it is difficult to extend this approach for the optimization of discritized continuum structures because the objective function cannot simply be represented by the separable functional, geometrical and material parameter functions. A performance index proposed by Liang et al. [24] is capable of assisting the selection of optimal topology for the minimum-weight design of continuum structures with stress constraints.

In this paper, a performance-based optimization approach is proposed to solve the optimal topology selection problem of continuum structures with displacement constraints. In this approach, a performance index developed is used to select the optimal topology from the optimization process. Maximizing the structural performance of the topology is considered as the optimal design criterion. The performance index proposed can be incorporated in continuum topology design methods for identifying the global optimum of structures subject to displacement constraints. Examples are given to demonstrate the effectiveness of the proposed approach for the optimal topology design of continuum structures.

2. Topology design problem formulation

For the maximum stiffness topology design, minimizing the mean compliance of a structure is commonly used as the objective function and the constraint is imposed on a somewhat arbitrary chosen material volume. However, it should be noted that the designer usually does not know what percentage of the material volume should be sufficient for supporting applied loads. By using optimization methods based on such a problem formulation, a trial-and-error process cannot be avoided if the designer really wants to select the minimum-weight optimal design for the given domain. A realistic optimization approach for the stiffness design is to use the structural weight as the objective function and the behavioral quantity such as the mean compliance or displacements as the constraints [18,25]. In this study, the weight of a structure is chosen as the objective function and displacements are treated as the constraints. The topology optimization problem of a continuum structure subject to displacement constraints can be stated as follows

$$\text{minimize} \quad W = \sum_{e=1}^{n} w_e(t) \tag{1}$$

$$\text{subject to} \quad u_j - u_j^* \le 0 \qquad j=1,..,m \tag{2}$$

where W is the total weight of a structure, w_e is the weight of the eth element, t is the thickness of elements, u_j is the jth constrained displacement, u_j^* is the prescribed limit of u_j, m is the total number of constraints and n is the total number of elements in the design. Since the thickness of a plane continuum structure has a significant effect on the structural weight and it needs to be specified by the designer in practice, it is treated as one of the design variables. Only the uniform variation of the element thickness is considered in the present study owing to its practical applications [26].

3. Performance-based optimization criterion

The structural weight can be reduced gradually by eliminating underutilized portions from the structure in the optimization process. To obtain the global optimum for the minimum-weight design, the performance of the resulting topology at each iteration must be measured by suing the performance index (PI), which can be derived by using the scaling design approach. The stiffness matrix of a plane stress continuum structure is a linear function of the thickness of elements. The element thickness can be uniformly scaled to keep the critical constraint at the constraint surface [24,27]. By scaling the design, the scaled design variable is expressed by

$$t^s = \varphi t \tag{3}$$

where t^s is the scaled thickness of elements, φ is a scaling factor that is the same for all elements and t is the actual thickness of elements. The equilibrium equation for the structure in the finite element analysis can be expressed as

$$\frac{1}{\varphi}[K^s]\{u\} = \{P\} \tag{4}$$

where $[K^s]$ is the stiffness matrix of the scaled structure, which is calculated by using the scaled design variable t^s, $\{u\}$ is the actural nodal displacement vector and $\{P\}$ is the nodal load vector. The equilibrium equation for the scaled design can also be written in terms of the scaled design variable by

$$[K^s]\{u^s\} = \{P\} \tag{5}$$

From Eqs. (4) and (5), the scaled nodal displacement vector can be obtained as

$$\{u^s\} = \frac{1}{\varphi}\{u\} \tag{6}$$

Therefore, in order to satisfy the displacement constraints imposed on a structure, the actual design needs to be scaled by $\varphi = u_j / u_j^*$, u_j is the absolute value of the *j*th constrained displacement that is the most critical in the design. By using a approaximate scaling factor, the scaled weight of the initial design domain can be represented by

$$W_0^s = \left(\frac{u_{0j}}{u_j^*}\right) W_0 \tag{7}$$

in which W_0 is the actual weight of the initial design domain and u_{0j} is the absolute value of the most critical constrained displacement in the initial design under the applied loads. Similarly, in order to obtain the best feasible design in an iterative optimization process, the current design is scaled to satisfy the prescribed displacement limit at each iteration. The scaled weight of the current design at the *i*th iteration is expressed by

$$W_i^s = \left(\frac{u_{ij}}{u_j^*} \right) W_i \tag{8}$$

where u_{ij} is the absolute value of the most critical constrained displacement in the current design at the ith iteration under the applied loads and W_i is the actual weight of the current design at the ith iteration.

The efficiency of material layouts in a structure at the ith iteration can be determined by the performance index, which is defined as

$$PI = \frac{W_0^s}{W_i^s} = \frac{u_{0j} W_0}{u_{ij} W_i} \tag{9}$$

It can be seen from Eq. (9) that the performance index is a dimensionless number that measures the efficiency of material layouts in resisting deflection and failure of a plane stress continuum structure with a uniform thickness. It depends on the topology but not on the scale of the structure. The performance index of the initial design is equal to unity. The performance of a structural topology is improved when inefficient materials are removed from the design domain. The displacement limit u_j^* is consequently eliminated from Eq. (9). This indicates that the optimal topology for the minimum-weight design of a continuum structure is the same for any value of the prescribed displacement limit. Minimizing the weight of a continuum structure with displacement constraints for a given design space can be achieved by maximizing the performance index. Therefore, the performance-based optimization criterion is stated as

$$\text{maximize } PI = \frac{u_{0j} W_0}{u_{ij} W_i} \tag{10}$$

It is noted that uniformly changing the element thickness does not affect the topology of a plane stress continuum structure and the performance index, but significantly influences the weight of the structure and the constrained displacements. As a result of this, the thickness of elements is not changed in the finite element analysis at each iteration. The displacement limits are usually set to large values in order to obtain the optimum design, which can then be sized to satisfy the actual displacement limits that are required by codes of practice.

4. Element elimination based on displacement criteria

The element elimination criteria can be derived by performing the sensitivity analysis on the constrained displacements due to element removal. The equilibrium equation for a static structure is expressed by

$$[K]\{u\} = \{P\} \tag{11}$$

If the *e*th element is removed from a structure, Eq. (11) can be rewritten as

$$([K] + [\Delta K])(\{u\} + \{\Delta u\}) = \{P\} \tag{12}$$

where $[\Delta K]$ and $\{\Delta u\}$ are the changes of the stiffness matrix and nodal displacements vector, respectively. The change of the stiffness matrix is

$$[\Delta K] = [K_r] - [K] = -[k_e] \tag{13}$$

in which $[K_r]$ is the stiffness matrix of the resulting design and $[k_e]$ is the stiffness matrix of the *e*th element. The change of the displacement vector can be obtained from Eqs. (11) and (12) by neglecting the higher order terms as

$$\{\Delta u\} = -[K]^{-1}[\Delta K]\{u\} \tag{14}$$

A virtual unit load is applied to the direction of the *j*th-constrained displacement to determine the change of the constrained displacement u_j due to an element removal. By multiplying Eq. (14) with the unit load vector $\{F_j\}^T$ in which only the component corresponding to the *j*th-constrained displacement is equal to unity and all the others are equal zero, the change of the constrained displacement can be obtained as

$$\Delta u_j = -\{F_j\}^T [K]^{-1}[\Delta K]\{u\} = \{u_{ej}\}^T [k_e]\{u_e\} \tag{15}$$

in which $\{u_{ej}\}^T$ is the nodal displacement vector of the *e*th element under the virtual unit load and $\{u_e\}$ is the displacement vector of the *e*th element under the real loads. It is seen from Eq. (15) that the change in the constrained displacement due to the elimination of the *e*th element can be calculated by the virtual strain energy of the *e*th element, which is represented by

$$c_e = \left|\{u_{ej}\}^T [k_e]\{u_e\}\right| \tag{16}$$

To obtain the maximum stiffness and minimum weight design, it is obviously that elements with the lowest virtual strain energy should be eliminated from the structure. If a continuum structure is divided into different size elements, the lowest virtual strain energy density of elements referred to mass should be used as the element elimination criteria. The virtual strain energy density of the *e*th element is defined as $\gamma_e = c_e / w_e$.

For a structure subject to multiple displacement constraints under multiple load cases, the virtual strain energy density of the *e*th element for element removal can be evaluated by using the weighted average approach as

$$\gamma_e^m = \sum_{l=1}^{p} \sum_{j=1}^{m} \beta_j \gamma_e \tag{17}$$

in which the weighting parameter β_j is defined as u_j^l / u_j^{l*}, which is the ratio of the *j*th constrained displacement to the prescribed limit under the *l*th load case, and p is the total number of load cases. If the constrained displacement is far from its prescribed limit, it will be less critical in the optimization process.

Since the virtual strain energy density of elements are approximately evaluated by neglecting the higher order terms in the sensitivity analysis, only a small number of elements with the lowest virtual strain energy density are allowed to be removed from the structure at each iteration. The Element Elimination Ratio (*EER*) for each iteration is defined by the ratio of the number of elements to be removed to the total number of elements in the initial design domain and is kept constant during the whole optimization process. The accuracy of the solution is improved by using a smaller element removal ratio but the computational cost will be increased. This will be illustrated in details in the first example for single displacement constraint.

5. Performance optimization procedure

The displacement constraints have been used as the termination criterion in continuum topology optimization to determinate the optimal design [17,18]. However, constrained displacements are significantly affected by the element thickness because the stiffness matrix of a plane stress continuum structure is a linear function of the element thickness. This means that the element thickness can be uniformly changed to satisfy the prescribed displacement limit. The resulting topology that satisfies this termination criterion may not be the global optimum in the design space. This problem can be overcome by incorporating the proposed performance index into the optimization procedure, which is given as follows:

Step 1:Model the continuum structure with fine elements;

Step 2: Analyze the structure for applied loads and virtual unit loads;

Step 3: Calculate the performance index *PI* using Eq. (9);

Step 4: Calculate the virtual strain energy density γ_e^m for each element;

Step 5: Eliminate *EER* (%) elements with the lowest γ_e^m from the design;

Step 6: Repeat Steps 2 to 5 until the *PI* is less than unity.

The performance index is used to monitor the performance history of the resulting topology at each iteration and as a termination criterion in the above optimization procedure. Since the solution depends on the element removal ratio that is similar to the step size in the optimality criterion method [28], the smooth convergence of the performance index to the maximum value cannot always be guaranteed in the optimization process. Moreover, the design can always be modified by removing elements from the structure according to the virtual strain energy density of elements. Therefore, the prescribed tolerance for the relative change in the performance index cannot be used as the termination criterion in the optimization procedure. If the performance index is less than unity, the efficiency of the corresponding topology is less than the initial design domain. In the proposed procedure, therefore, the iterative process can be terminated when the performance index is less than unity. This ensures that the optimal topology that corresponds to the maximum performance index is obtained during the optimization.

In the optimality criteria method [28], a Lagrangian function is formed for the objective function and constraints to obtain the optimality condition. A recurrence formula derived from this optimality condition is used to modify the design variables in order to generate the next structural layout. The optimality condition provides information on how to modify the design but it does not indicate which topology generated in the optimization process is the optimum. In contrast, in the performance-based optimization approach presented here, the structure is always modified by removing *EER* (%) elements with the lowest virtual strain energy density from the design at each iteration. Maximizing the performance index is used as the optimization criterion. The main advantage of the proposed approach is that the performance-based optimization criterion can be used to monitor the optimization process and to select the optimum from the optimization history.

6. Examples

The performance-based optimization approach is applied to several example problems in this section. Various structures with single and multiple displacement constraints and height constraints are optimized using the proposed method. The performance index considering the most critical displacement constraint is employed to select the optimum from the optimization history. The efficiency of structural topologies obtained by different optimization methods in the literature is evaluated using Eq. (9). The results obtained by the present study are verified by the classical solutions.

6.1 Structure with single displacement constraint

The design domain for a deep cantilever beam subject to a displacement constraint imposed on the loaded point in the vertical direction is shown in Fig. 1. The design domain is divided into a 32 × 72 mesh using four-node plane stress elements. The support of the cantilever beam is fixed. A concentrated load of 200 N is applied to the centre of the free end. The Young's modulus E = 200 GPa, the Poisson's ratio v = 0.3 and the thickness of the beam t = 1 mm are used. Plane stress conditions are assumed. The Element Elimination Ratio EER = 1% is used in the optimization process.

Figure 2 presents the performance index history for the cantilever beam. While gradually removing elements with the lowest virtual strain energy density from the structure, the performance index gradually increases from unity to the maximum value of 2.08. This means that the weight of the initial scaled design domain is 2.08 times that of the optimal design, which is also scaled to satisfy the prescribed displacement limit. It is observed from Fig. 2 that the performance index may jump in the optimization process. This is because the element elimination ratio of 1% used is still high. A smoother solution can be obtained by using a smaller *EER* of 0.5% for this cantilever beam as shown in Fig. 3, but the computational time is approximately double of that using a removal ratio of 1%. The topologies obtained at different iterations are presented in Fig. 4. The performance index of the optimal topology generated using *EER* = 0.5% as shown in Fig. 4(e) is 2.05. Hence, it can be observed that the two optimal topologies obtained using different element removal ratios are similar and the element removal ratios do not affect much the weight of the optimal design for this example. The optimal topology evolves to a two-bar truss-like structure, where its optimal height is two times of its span. This optimal truss layout may be obtained by the analytical method. A comparison of material volumes required for the initial design and four topologies shown in Fig. 4 (a) to (d) for various displacement limits is given in Table 1. It can be seen from the table that the volumes of the optimal topologies are always less then those of other four topologies for each displacement limit. It also shows that the material layouts in an optimal topology are independent of the magnitude of prescribed displacement limits.

6.2 Structure with multiple displacement constraints

Figure 5 illustrates the design domain of a simply supported transverse beam with multiple displacement constraints of the same limit imposed on points A, B and C, where three concentrated loads of 10 kN are applied to these points. The design domain is represented using a 96 × 32 mesh of plane stress elements. The Young's modulus E = 200 GPa, Poisson's ratio v = 0.3 and thickness of elements t = 5 mm are assumed. The *EER* = 1% is adopted in the optimization process.

The performance index history of the simply supported beam is presented in Fig. 6. Due to symmetry, the displacements at points A and C are the same. The performance index curves shown in Fig. 6 are obtained by using Eq. (9) for the design with the constrained displacements at points A and B, respectively. The maximum performance indices are 1.46 and 1.43, which are calculated using the constrained displacements imposed at points A and B, respectively. The optimal topology that corresponds to the maximum value of the *PI* occurs at iteration 76 for both of displacement constraints. It is obvious that the weight of the optimal design is dominated by the critical displacement constraint imposed at point B. The optimization history for this beam is demonstrated in Fig. 7.

6.3 Structures with height constraints

The design space in practical engineering design problems is often limited and has a significant effect on the efficiency of structural topology. This example is to investigate the effect of height constraints imposed on the initial design domains on the efficiency of final optimal topologies. The design domain for structures with fixed supports subject to various height constraints is shown in Fig. 8. In case (a), the design domain with h/L = 1/2 is divided into 100 × 50 mesh using plane stress elements. In case (b), the design domain with h/L = 1/4 is divided into 100 × 25 mesh. In case (c), the design domain with h/L = 1/8 is divided into 100 × 13 mesh. In case (d), a 100 × 9 mesh is used for the structure with h/L = 1/12. The Young's modulus E = 200 GPa, Poisson's ratio ν = 0.3 and thickness of elements t = 2 mm are used for all cases. A point load P = 400 N is applied to the structure. A displacement constraint is imposed on the loaded point in the vertical direction. The *EER* = 1% is used for these plane stress problems.

Figure 9 shows the optimal topologies selected using the performance index from the optimization history for continuum structures with various height constraints. It is well known that case (a) is the Michell type structure with fixed supports. The optimal topology generated by the performance-based optimization approach shown in Fig. 9(a) agrees extremely well with the analytical solution given by the Australian inventor A.G.M. Michell [29]. The maximum performance indices for cases (a) to (d) are 1.89, 1.67, 1.73 and 1.58, respectively. It is seen that case (b) with a larger height has a lower performance index than case (c) that has a smaller height. This is because the performance index for each structure is determined by the relative weight of the initial design to the current design. The efficiency of optimal topologies for different structures such as structures with different heights cannot be evaluated by comparing the performance indices. The suitable method is to compare their scaled weight or volumes with respect to the same displacement limit such as 0.5 mm imposed on the loaded point. Table 2 presents a comparison of material volumes required for each optimal design while satisfying the same displacement limit. It is seen from the table that the volume required for the optimum design increases with the decrease in the height constraints. Therefore, the efficiency of the optimal topology for a structure is improved when increasing the height of the initial design domains.

6.4 Comparison of the efficiency of structural topologies

For a same design problem, using different optimization methods usually results in different final designs. In order to select the best topology for the design of continuum structures, the proposed performance index is used to compare the performance of structural topologies generated by different structural optimization methods. A short cantilever beam with fixed support subject to a displacement constraint imposed on the loaded point in the vertical direction shown in Fig. 10 is optimized by using proposed method. The design domain is divided into 32 × 20 plane stress elements. A concentrated load of 3 kN is applied to the center of the free end. Young's modulus E = 207 GPa, Poisson's ratio v = 0.3 and the thickness of the structure t = 1 mm are adopted in the study. The EER = 2% is used in this problem.

The performance index history for this short cantilever beam carried out by the proposed approach is shown in Fig. 11. For the initial design without any hole, the performance index is equal to unity whilst the maximum performance index of 1.20 occurs at iteration 27. The optimal topology obtained by this study is presented in Fig. 12(a). The topology shown in Fig. 12(b) is given by Chu et al. [17]. The performance index corresponding to Fig. 12(b) is 1.11. The topology given by Zhao et al. [20] is presented in Fig. 12 (c), where the model is regenerated. The volume of this topology is 8550 mm^3 and the vertical displacement at the load point is 0.518 mm, which is easily obtained by undertaking a finite analysis analysis. Therefore, the performance index for the topology shown in Fig. 12(c) is obtained as 1.18 using Eq. (9). The performance index of the topology given by Suzuki and Kikuchi [4] using the homogenization-based optimization method as shown in Fig. 12(d) is found to be 1.04. It is shown that the structural performance of the optimal topology obtained by the present study is higher than those presented by other researchers.

7. Conclusions

A performance-based optimization method has been presented in this paper for the optimal topology selection of continuum structures with displacement constraints. The continuum topology optimization is treated as the problem of improving the structural performance of the design in terms of the material use in effectively resisting deformations by gradually eliminating inefficient materials from the design domain. The performance-based optimization criterion is maximizing the performance index. The proposed performance index is used to monitor the optimization process and as a termination criterion in the optimization process.

It has been demonstrated that the performance-based optimization approach presented can effectively generate optimal structural topologies, which have been verified by the analytical solutions. It is shown that a smoother solution can be achieved by using a smaller element elimination ratio in the performance optimization process but at the expense of a higher computational cost. The results indicate that increasing the height of the initial design domain usually improves the efficiency of the final optimal design. Furthermore, the proposed performance index can be used to compare the efficiency of structural topologies produced by different structural optimization methods.

The performance-based design concept has been extended to solve the practical topology optimization problem of civil engineering structures, in which the structural weight is chosen as the objective function and the mean compliance is treated as the constraints [25]. It is expected that the performance-based optimization criteria given in this paper and Ref. [24,25] could be incorporated in existing continuum topology optimization methods to guarantee success in obtaining the global optimal designs with reasonable effort. Although the stiffness is the most important aspect that needs to be considered in the design of a structure, the stress constraints should be included in the proposed method as well to make it a general design tool for practicing engineers.

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Figures and Tables

Table 1 Material volumes required at different iterations for various displacement limits

u_j^* (mm)	V_0^s (mm^3)	V_{50}^s (mm^3)	V_{70}^s (mm^3)	V_{opt}^s (mm^3)	V_{90}^s (mm^3)	PI_{max}
0.5	740	468	405	355	390	2.08
0.75	493	312	270	237	260	2.08
1.0	370	234	203	178	195	2.08

Table 2 Effects of height constraints on the material volumes of optimal topologies

Height *h/L*	u_{optj} (mm)	u_j^* (mm)	V_{opt} (mm^3)	V_{opt}^s (mm^3)
1/2	0.023	0.5	28800	1342
1/4	0.036	0.5	20864	1481
1/8	0.244	0.5	7138	3483
1/12	0.428	0.5	9125	7811

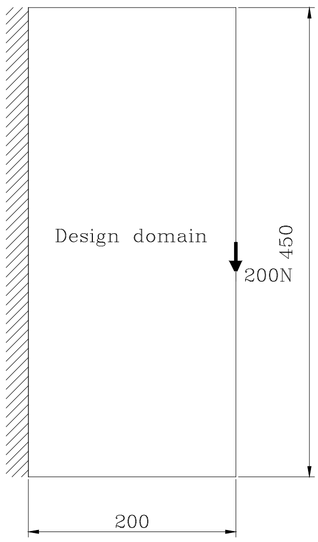


Fig. 1. Design domain of the deep cantilever beam.

Fig. 2. Performance index history of the deep cantilever beam (*EER* = 1%)

Fig. 3. Performance index history of the deep cantilever beam (*EER* = 0.5%)

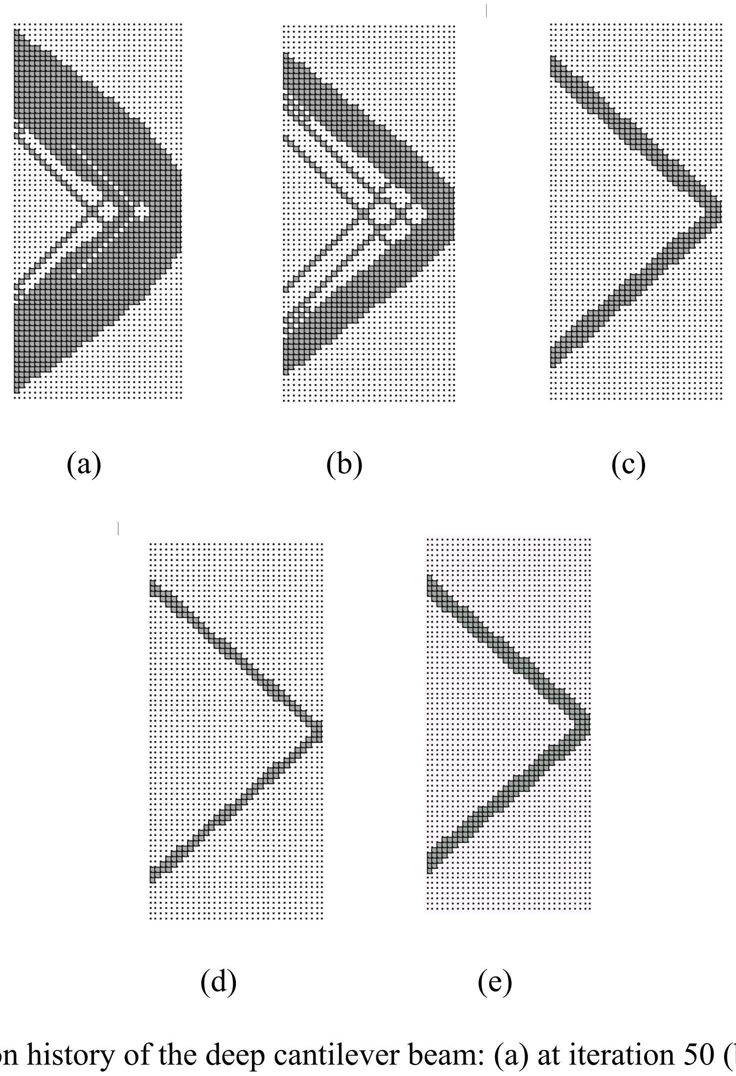


Fig.4. Optimization history of the deep cantilever beam: (a) at iteration 50 (b) at iteration 70 (c) optimum (*EER* = 1%) (d) at iteration 90 (e) optimum (*EER* = 0.5%)

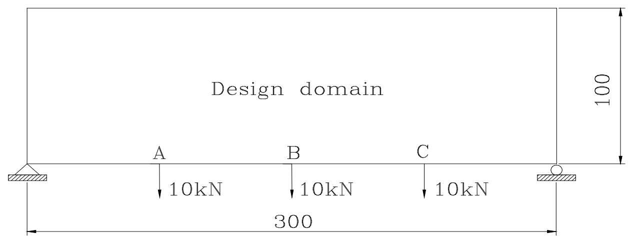


Fig. 5. Design domain of the structure with multiple displacement constraints

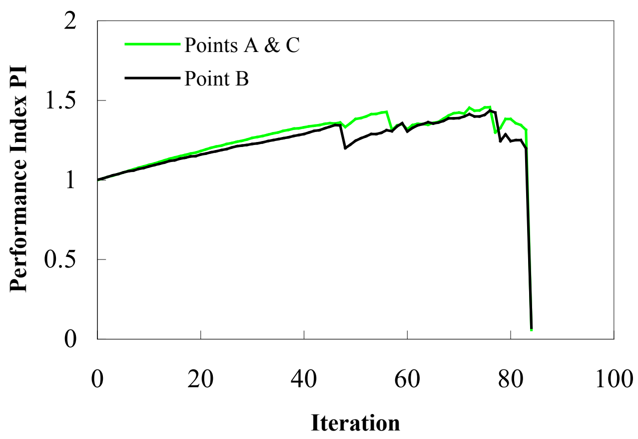
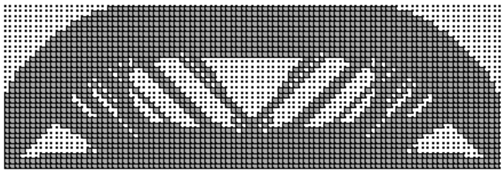
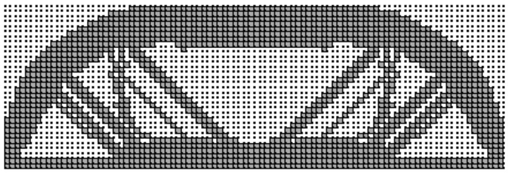


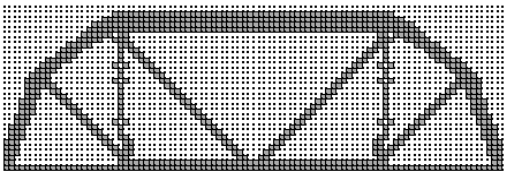
Fig. 6. Performance index history of the structure with multiple displacement constraints



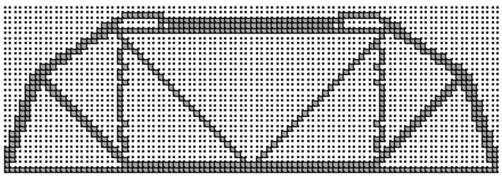
(a)



(b)



(c)



(d)

Fig. 7. Optimization history of the structure with multiple displacement constraints: (a) at iteration 30 (b) at iteration 50 (c) optimum (e) at iteration 83

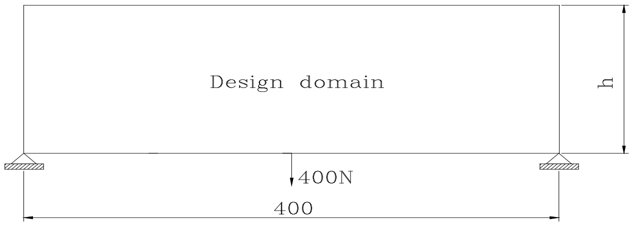
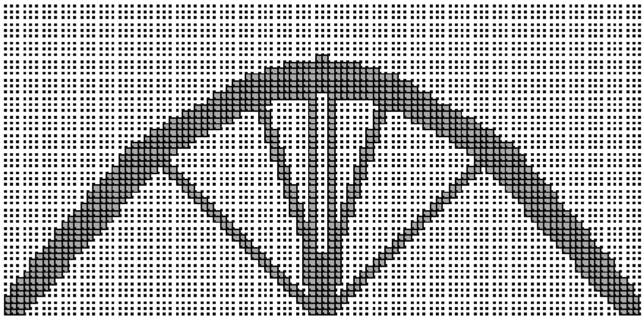
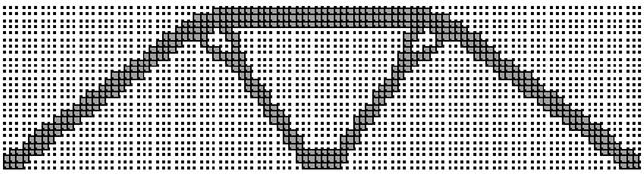


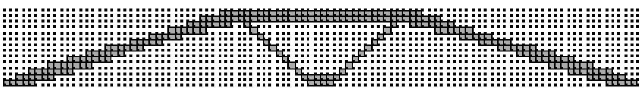
Fig. 8. Design domain of structures with various height constraints



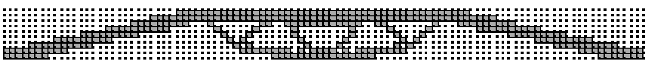
(a)



(b)



(c)



(d)

Fig. 9. Effects of height constraints on the optimal topologies: (a) h/L = 1/2 (b) h/L = 1/4 (c) h/L = 1/8 (d) h/L = 1/12

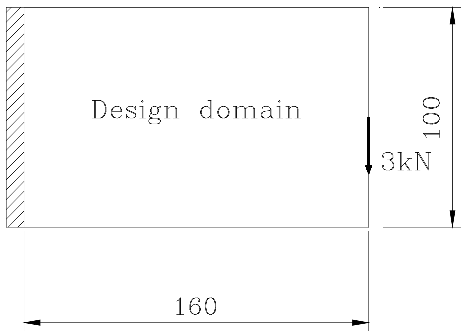


Fig.10. Design domain of the short cantilever beam

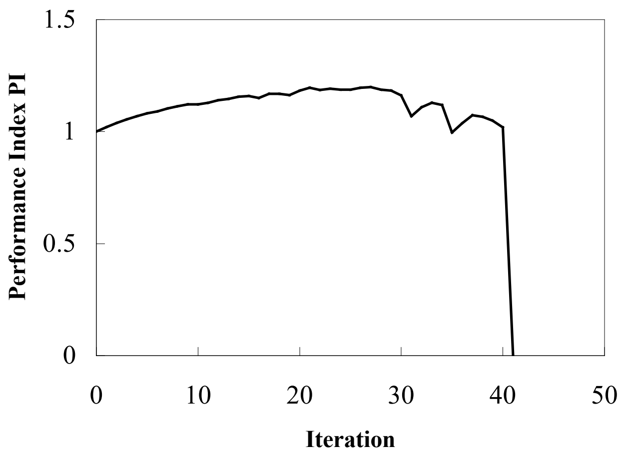


Fig. 11. Performance index history of the short cantilever beam

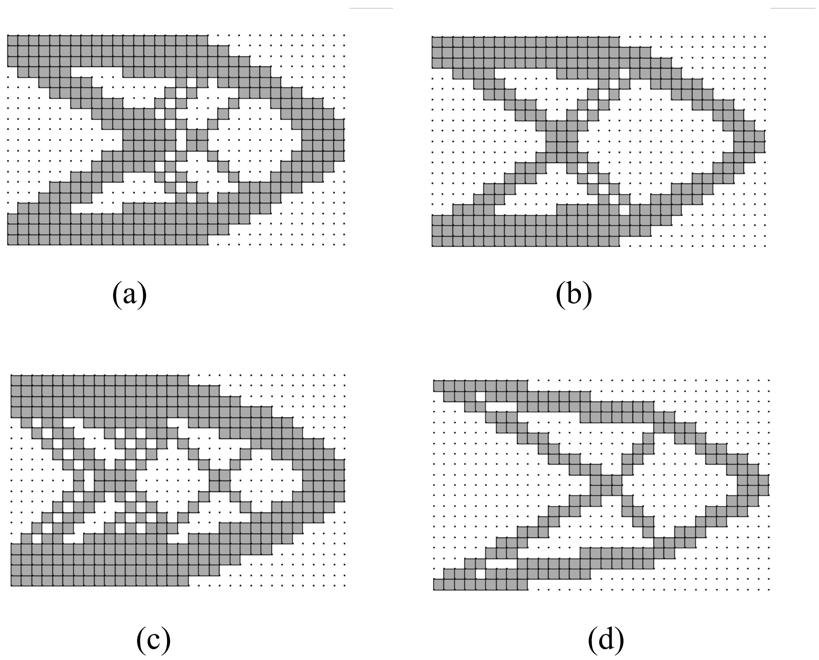


Fig. 12. Comparison of the efficiency of structural topologies: (a) topology by present study, PI = 1.20 (b) topology by Chu et al. [17], PI = 1.11 (c) topology by Zhao et al. [20], PI = 1.18 (d) topology by Suzuki and Kikuchi [4], PI = 1.04