OPITAL TOPOLOGY DESIGN OF BRACING SYSTEMS FOR MULTI-STORY STEEL FRAMES

By Qing Quan Liang,1 Yi Min Xie2 and Grant P. Steven3

ABSTRACT: This paper presents a performance-based optimization method for optimal topology design of bracing systems for multi-story steel building frameworks with overall stiffness constraint under multiple lateral loading conditions. Material removal criteria are derived by undertaking a sensitivity analysis on the mean compliance of a structure with respect to element removal. A performance index is proposed to evaluate the performance of resulting bracing systems in the optimization process. In the proposed method, unbraced frameworks are initially designed under strength constraints using commercial standard steel sections from databases. The optimal topology of a bracing system for the multi-story steel building framework is then generated by gradually removing inefficient materials from a continuum design domain that is used to stiffen the framework until the performance of the bracing system is maximized. Two design examples are provided to illustrate the effectiveness of performance-based design optimization method proposed for the conceptual layout design of lateral bracing systems for multi-story steel building frameworks.

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INTRODUCTION

The design of a multi-story steel building under lateral loads is usually governed by system performance criteria (overall stiffness) rather than by component performance criteria (strength). An important task in the design of a tall steel building for structural designers is to select cost efficient lateral load resistance systems. Pure rigid frame systems alone are not efficient in resisting lateral loads for tall steel buildings due to associated high costs. Truss members such as diagonals are often used to brace steel frameworks to maintain lateral drifts within acceptable limits. In the absence of an efficient optimization technique, the selection of lateral bracing systems for multi-story steel frameworks is usually undertaken by the designer based on a trial-and-error process and previous experience. The optimal layout design of bracing systems is a challenging task for structural designers because it involves a large number of possibilities for the arrangement of bracing members.

Stiffness-based sizing techniques have been developed for the minimum-weight design of lateral load resistance systems in multi-story steel buildings by various researchers. Baker (1990) has presented a sizing technique using energy methods for lateral load resistance systems in multi-story steel buildings. The discretized optimality criteria method proposed by Zhou and Rozvany (1992) is shown to be efficient for sizing large structural systems with a large number of design variables subject to stress and displacement constraints under multiple loading conditions. The automatic resizing technique developed by Chan et al. (1995) for the optimal design of tall steel building frameworks under lateral loads is very practical due to the use of commercial standard steel sections. In these approaches, all members of a lateral load resistance system are resized based on uniform strain energy density criteria. Kim et al. (1998) has presented an alternative to the design of tall steel
buildings where steel frameworks are designed based on strength criteria and only bracing members are resized for stiffness requirements. However, all of these sizing techniques only work for lateral bracing systems with fixed topologies. The efficiency of a sized structural system in resisting lateral loads is limited by the chosen topology of bracing systems.

Shape and topology optimization of continuum structures has been an active research area for few decades (Haftka and Gandhi 1986; Rozvany et al. 1995). The homogenization-based optimization method (Bendsøe and Kichuki 1988; Suzuki and Kichuki 1991; Tenek and Hagiwara 1993; Bendsøe et al. 1995), density function approaches (Meljnek et al.1993; Yang and Chuang 1994) and material removal methods (Rodriguez-Velazquez and seireg 1985; Atrek 1989; Xie and Steven 1993,1997) can be used to generate topologies and shapes for the design of continuum structures. These continuum topology optimization methods focus mainly on theoretical aspects rather than practical applications. In addition, further work on developing performance-based optimization criteria for obtaining globally optimal designs in continuum topology optimization is needed as addressed by Liang et al. (1999a).

The potential of continuum topology optimization methods is increasingly being realized by civil engineering industry. Continuum topology optimization can be used to find optimal reinforcement layouts which maximize the natural frequency of a given plane stress continuum structure (Diaz and Kikuchi 1992). Walther and Mattheck (1993) have used the soft kill option method to generate efficient frameworks for supporting floor systems in construction. The layout design of bracing systems for multi-story steel building frames under one lateral load case has been attempted by Mijar et al. (1998) using a topology optimization approach based on classical Voigt-Resuss mixing rules. The performance-based evolutionary optimization method has been proposed by Liang et al. (2000) as a rational and
efficient tool for automatically generating optimal strut-and-tie models in structural concrete. However, the potential of continuum topology optimization methods in solving practical civil engineering problems is far from being exploited.

In this paper, a performance-based optimization method formulated on the basis of system performance criteria is proposed for the topology design of bracing systems for multi-story steel building frameworks under multiple lateral load cases. The performance-based optimization criterion in terms of the performance index is developed for identifying the global optimum from the optimization process. In the proposed procedure, unbraced frameworks are firstly designed for component performance requirements by using standard steel sections. The optimal topology of a bracing system is produced by gradually removing inefficient material from a continuum design domain that braces the framework. The main features of the proposed design optimization method are outlined. Two design examples are given to demonstrate the practical applications of the proposed method. Results obtained by the present study are compared with existing solutions.

**PERFORMANCE-BASED TOPOLOGY OPTIMIZATION**

**Topology Design Problem Formulation**

The performance-based optimal design is to design a structure or structural component that can perform physical functions in a specified manner throughout its design life at minimum cost or weight. In the proposed formulation, a continuum design domain under plane stress conditions is used to stiffen a multi-story steel building framework. The framework itself with a fixed topology is treated as a non-design domain in which beam elements are not
removed during the optimization process. Therefore, the performance objective of optimal
topology design for bracing systems is to minimize the weight of the continuum design
domain while maintaining the overall stiffness constraint of the braced structure within an
acceptable limit. The performance objective can be expressed as follows:

$$\text{minimize } W = \sum_{e=1}^{n} w_e$$

subject to $$C - C^* \leq 0$$

where $$W$$ is the total weight of the continuum design domain, $$w_e$$ is the weight of the $$e$$th
element, $$C$$ is the mean compliance of a braced framework, $$C^*$$ is the prescribed limit of $$C$$
and $$n$$ is the total number of elements in the discretized continuum design domain. The mean
compliance is usually used as an inverse measure of the overall stiffness of a structure.
Maximizing the overall stiffness of a structure is equivalent to minimizing its mean
compliance.

**Material Removal Criteria**

Some regions of a continuum reinforcing system are not efficient in resisting lateral loads,
and are thus removed from the continuum design domain. Material removal criteria are used
to identify these underutilized regions in the optimization algorithm, and can be derived by
undertaking a sensitivity analysis. The sensitivity analysis in the proposed method is to study
the effects of element removal on the change of the mean compliance of a structure.

In finite element formulation, the equilibrium equation for a static structure can be written by
\[
\mathbf{Ku} = \mathbf{P} 
\]  
(3)

in which \( \mathbf{K} \) is the stiffness matrix of a structure, \( \mathbf{u} \) is nodal displacement vector and \( \mathbf{P} \) is nodal load vector. When the \( \ell \)th element is removed from a discretized continuum design domain, the stiffness and displacements will be changed accordingly and (3) can be rewritten as

\[
(\mathbf{K} + \Delta \mathbf{K})(\mathbf{u} + \Delta \mathbf{u}) = \mathbf{P} 
\]  
(4)

where \( \Delta \mathbf{K} \) is the change of stiffness matrix and \( \Delta \mathbf{u} \) is the change of nodal displacement vector. The change of the stiffness matrix is

\[
\Delta \mathbf{K} = \mathbf{K}_r - \mathbf{K}_e = -\mathbf{k}_e 
\]  
(5)

in which \( \mathbf{K}_r \) is the stiffness matrix of the resulting design and \( \mathbf{k}_e \) is the stiffness matrix of the \( \ell \)th element. The change of displacement vector can be obtained approximately by subtracting (3) from (4) and neglecting higher order terms as

\[
\Delta \mathbf{u} = -\mathbf{K}^{-1} \Delta \mathbf{Ku} 
\]  
(6)

The mean compliance or strain energy of a structure is represented by

\[
C = \frac{1}{2} \mathbf{P}^\top \mathbf{u} 
\]  
(7)

The change of the strain energy of a structure due to the removal of the \( \ell \)th element can be
approximately expressed by

\[ \Delta C = \frac{1}{2} P^T \Delta u = -\frac{1}{2} P^T K^{-1} \Delta K u = -\frac{1}{2} u^T \Delta K u = \frac{1}{2} u^T k_e u_e \]  

(8)

where \( u_e \) is the displacement vector of the \( e \)th element. It is seen from (8) that the change of the strain energy of a structure due to the removal of the \( e \)th element can be approximately calculated by the strain energy of the \( e \)th element. Therefore, the element strain energy can be used as a measure of the efficiency of an element in contribution to the structural stiffness and is denoted as

\[ c_e = \frac{1}{2} u_e^T k_e u_e \]  

(9)

To achieve the performance objective, elements with the lowest strain energy should be gradually removed from the continuum design domain. If a continuum design domain is divided into different size elements, the lowest strain energy density of elements referred to mass should be used as material removal criteria. The strain energy density of the \( e \)th element is calculated by \( \gamma_e = c_e / w_e \). For a structure under multiple load cases, a logical AND scheme is employed in the proposed method to take account of the effect of different load cases. In the logical AND scheme, an element is removed from the design domain only if its strain energy density is the lowest for all load cases. This can be expressed as follows:

\[
\begin{align*}
\text{Load case 1} : & \quad \gamma_e^1 \in Q^1 \\
\text{Load case } m : & \quad \gamma_e^m \in Q^m
\end{align*}
\]  

(10)
in which $\gamma^1_e$, ..., $\gamma^m_e$ are the strain energy density of the $e$th element under load case 1, ..., $m$ respectively, $Q^1$, ..., $Q^m$ are the vectors of the lowest strain energy density of elements under load case 1, ..., $m$ respectively, and $m$ is the total number of load cases. Elements with the lowest strain energy density are counted by a loop until they make up the specified amount, which is the element removal ratio times the number of elements in the initial design domain. The Element Removal Ratio (ERR) for each iteration is defined by the ratio of the number of elements to be removed to the total number of elements in the initial continuum design domain. In order to obtain a smooth solution, only a small number of elements are deleted from the continuum design domain at each iteration in the optimization process.

**Performance-Based Optimization Criterion**

In the proposed method, braced systems are gradually modified by removing elements from the continuum design domain. Although a structure can also be modified by removing or adding materials from or to the structure (Querin et al. 1998), removing underutilized materials from a design domain is a straightforward, realistic and general approach, especially for structures with complicated geometry and loading conditions. In order to determine the optimum from the optimization process, the performance of resulting topology at each iteration must be assessed. Performance-based optimization criteria in terms of performance indices have been proposed by Liang et al. (1999a, 1999b, 1999c) using the scaling design concept (Kirsch 1982) for identifying global optimum from the optimization history of continuum structures. These performance indices can also be used to compare the performance of structural topologies and shapes generated by different optimization methods.

For the topology design problem of bracing systems, a continuum design domain is
structurally connected to the framework. The stiffness of the braced framework is not a linear
function of the thickness of the continuum design domain. As result of this, element thickness
cannot be linearly scaled to keep the mean compliance constraint active at each iteration.
However, it is known that the best structure is the one that has the maximum stiffness and
minimum weight, as pointed out by Hemp (1973). Therefore, to evaluate the performance of
a bracing system for a framework under mean compliance constraint, the performance index
can be proposed as

$$PI = \frac{C_0 W_0}{C_i W_i}$$

(11)

where $C_0$ and $C_i$ are the mean compliance of the initial braced framework and current
braced framework at the $i$th iteration, respectively, and $W_0$ and $W_i$ are the weight of the
initial continuum design domain and the current continuum structure respectively. Because
beam elements of the steel framework are not removed in the optimization process, the
weight of the framework is not taken into account in the calculation of the performance
index.

The performance index is a dimensionless number that indicates the performance of a lateral
bracing system in terms of the efficiency of material use and overall stiffness in resisting
lateral deflections. The performance of a structural topology is improved when elements with
the lowest strain energy density are systematically removed from the design. The goal is to
seek the most uniform distribution of strain energy density within a continuum design
domain. However, in some cases the uniformity of strain energy density may not be achieved
even if the mean compliance constraint is active. As a result of this, the uniform strain energy
density condition is not used in optimization algorithms as a termination criterion. In the proposed method, the performance-based optimization criterion is maximizing the performance index of a braced framework. The performance index is used in the optimization algorithm to monitor the optimization process, from which the optimal topology can be identified.

**Design Optimization Procedure**

The design optimization process is generally iterative in nature in order to obtain a sound optimal design, and is divided into two main stages. In the first stage, the members of a steel framework are designed by using commercial standard steel sections from databases based on component performance criteria after undertaking a finite element analysis on the unbraced framework. In the second stage, a repeated finite element analysis and material removal cycle is carried out for the braced framework until the termination criterion is satisfied. The design optimization procedure is summarized as follows:

1. Model the unbraced framework with beam elements;
2. Analyze the unbraced framework;
3. Size the members of the framework using standard steel sections based on strength constraints;
4. Divide the continuum design domain into fine finite elements;
5. Analyze the braced framework;
6. Evaluate the performance of resulting bracing system using (11);
7. Calculate element strain energy density \( \gamma_e^m \);
8. Remove \( ERR \) (%) elements with the lowest \( \gamma_e^m \) from the design domain;
9. Repeat 5 to 8 until the performance index is less than unity.

When the performance index of a resulting structure is less than unity, its performance is lower than that of the initial braced framework without any element removal. Therefore, the iterative optimization process can be terminated. This termination criterion ensures that the optimum is obtained in the optimization history. It is desirable that the mean compliance constraint in terms of the top drift of the building is active at the optimum. However, it may not always be the case because the thickness of the continuum design domain also affects the performance of resulting systems. To deal with this problem, shape and sizing optimization techniques can be used to further optimize the structure until lateral drifts reach prescribed limits. Another way to handle this is to uniformly change the element thickness that leads to the satisfaction of the required system performance level. Continuum topology optimization methods may or may not result in truss-like structures, they are in this sense more general than truss topology optimization based on the ground structure approach.

**ILLUSTRATIVE DESIGN EXAMPLES**

**Example 1**

In this example, the performance-based optimization method proposed is used to find the best layout of a bracing system for a six-story steel building framework under multiple lateral loading conditions and the result is compared with that given by Mijar et al. (1998). A two-bay six-story plane steel framework as shown in Fig. 1(a) is to be designed to control the lateral drift. This unbraced framework was initially designed by Huang (1995) using standard steel sections under stress constraints according to the American Institute of Steel
Construction design code. In Huang’s design, the uniformly distributed load applied to floor beams was 14.59 kN/m and wind loads of 40.05 kN were applied as horizontal point loads at each floor level. The wide flange sections are used for 14 member groups that are listed as W 8 × 21, W 8 × 28, W 10 × 26, W 12 × 26, W 14 × 26, W 14 × 19, W 10 × 17, W 8 × 10, W 12 × 19, W 12 × 14, W 14 × 22, W 16 × 26, W 16 × 31 and W 24 × 62. The strong lateral wind loads shown in Fig. 1(a) were used by Mijar et al. (1998) to find the bracing layout for this framework. Under this lateral loading, stresses in the members of the unbraced framework may exceed the allowable stress.

In the present study, two lateral load cases are considered because wind loads are often reversible. Lateral bracing systems are mainly designed to resist lateral loads. Floor loads that are carried by beams and columns have a negligible effect on the layout of bracing systems so that they are not included in the present study. The framework itself is modeled using 342 linear beam elements with all moment connections and is treated as a non-design domain in which beam elements are not removed during the optimization process. The continuum design domain is modeled using 1620 four-node plane stress elements as shown in Fig. 1(b). The supports of the framework at points A, B and C are fixed. The Young’s modulus of material $E = 200$ GPa, Poisson’s ratio $\nu = 0.3$ and a uniform thickness $t = 0.0254$ m are adopted for the continuum design domain. The maximum lateral displacement of the unbraced framework is 0.56 m. The element removal ratio $ERR = 1\%$ is used in the optimization process.

The topology optimization history of the bracing system obtained by the present study is shown in Fig. 2(a) to (c). It can be seen that when inefficient materials are removed from the continuum design domain, the bracing system gradually evolves towards a discrete-like
structure. The topology shown in Fig. 2(b) is similar to that shown in Fig. 2(c). The optimized topology of the bracing system for this six-story steel building framework given by Mijar et al. (1998) is regenerated herein as shown in Fig. 2(d). This topology was obtained by minimizing the mean compliance of the braced framework under a volume constraint that allowed the solid material to occupy up to 30% of the initial design domain. The maximum lateral displacement of the optimized bracing system shown in Fig. 2(d) is 0.07 m, which was obtained by simply performing a linear finite element analysis. The material volume of the bracing system for the framework shown in Fig. 2(c) is 22% of the initial continuum design domain, but its maximum lateral displacement is only 0.024 m. By using (11), the performance index of the bracing layout shown in Fig. 2(c) is 1.15 whilst it is only 0.32 for the topology presented in Fig. 2(d). This indicates that the layout of bracing systems for multi-story steel building frameworks significantly affects the structural performance of lateral resistance systems. It is possible to achieve minimum-weight designs for bracing systems by using topology optimization techniques while lateral drifts are maintained within acceptable limits. The topology of the bracing system shown in Fig. 2(c) is interpreted as the layout arrangement of bracing members illustrated in Fig. 3. This bracing system can be constructed by using available standard steel sections from databases.

**Example 2**

A 3-bay 12-story tall steel building framework with fixed supports at points A, B, C and D is to be designed to resist lateral loads as shown in Fig. 4. Two lateral wind-loading cases are considered, i.e. one from the left and the other from the right. Gravity loads are not included in the analysis. All beams and columns are rigidly connected. The Young’s modulus $E = 200$ GPa, shear modulus $G = 7690$ MPa, and the density $\rho = 7850$ kg / m$^3$ are used for steel
sections. A linear elastic finite element analysis is performed for the unbraced framework. The BHP hot rolled standard steel sections are selected from databases to size the members of the framework based on section strength requirements. For practical purposes, beams are grouped together as having a common section for each floor whilst columns are grouped for every two stories. Sized members are summarized in Table 1. The continuum design domain with a uniform thickness of \( t = 0.025 \text{ m} \) is used to brace the framework. The Young’s modulus \( E = 200 \text{ GPa} \) and Poisson’s ratio \( \nu = 0.3 \) are used for the continuum design domain. The framework itself is modeled using 684 linear beam elements whilst the continuum design domain is divided into a \( 45 \times 108 \) mesh using four-node plane stress elements. The \( ERR = 2\% \) is adopted in the optimization process.

The maximum lateral displacement at the top of the unbraced framework is \( 0.618 \text{ m} \) that exceeds the drift limit \( h / 400 \) (\( h \) is the total height of the building). The overall behavior of a braced framework is like that of a cantilever structure with beams and columns as stiffeners. The performance index history of the bracing system obtained by the proposed method is shown in Fig. 5. When a small number of elements with the lowest strain energy density are removed from the continuum design domain, the performance index is gradually increased from unity to the maximum value in the optimization process. After reaching the peak, the performance index will be decreased if further elements are removed from the structure. The maximum performance index is 1.51, which corresponds to the optimal topology shown in Fig. 6(a). The optimal layout of the bracing system exhibits a large-scale discrete structure, which can be represented by using standard members. It can be seen from Fig. 6(a) that the result obtained provides very useful information for structural designers on which member of the framework should be stiffened by resizing. Exterior columns from the ground level up to the fifth level are needed to be resized. The optimal topology of the bracing system for this
12-story steel building framework can be interpreted as the bracing layout illustrated in Fig. 6(b), where columns that need to be resized are not shown. Since the mean compliance constraint in terms of the lateral drift does not reach the actual limit at the optimum, sizing techniques can be employed to further optimize the design using available standard steel sections.

**CONCLUDING REMARKS**

A performance-based optimization method for the minimum-weight and maximum-stiffness topology design of bracing systems for multi-story steel building frameworks under multiple lateral loading conditions has been presented. A performance index has been proposed for determining the optimal topology of a bracing system from the optimization process and can be used to compare the performance of structural topologies obtained by different optimization methods. The proposed method allows for an unbraced steel building framework to be initially sized based on component performance criteria by using commercially available standard steel sections from databases. The optimal topology of the bracing system is obtained by systematically removing inefficient materials from a continuum design domain that is used to stiffen the framework while the performance-based optimization criterion is satisfied.

The performance-based topology optimization method proposed has attractive features such as clear in concept and simple in mathematical formulation compared to other continuum topology optimization methods. Moreover, it can produce novel design proposals for bracing systems for multi-story steel building frameworks. The proposed performance index is a useful tool for structural designers in assisting the selection of the best topology for lateral
bracing systems when considering the structural performance, aesthetic and construction requirements. Examples presented have demonstrated that the design method presented can generate efficient topologies, which provide structural designers with useful information on bracing and stiffening multi-story steel building frameworks. The topology design method is effective and suitable for use in engineering practice in the conceptual layout design of bracing systems for multi-story steel building frameworks under lateral loads. Further research is needed to incorporate shape and sizing optimization techniques into the topology design method so that an integrated design tool for the optimal layout design of lateral bracing systems would be available for structural designers.

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APPENDIX. REFERENCES


FIG. 1. Two-Bay, Six-Story Steel Framework: (a) Geometry and Loading; (b) Design and Non-design Domains of Braced Framework
FIG. 2. Topology optimization history of Bracing Systems for the Six-Story Steel Framework:
(a) Topology with $V = 63\%V_0$; (b) Topology with $V = 30\%V_0$;
(c) Topology with $V = 22\%V_0$; (d) Topology with $V = 30\%V_0$ by Mijar et al. (1998).
FIG. 3. Layout of Bracing System for the Six-Story Steel Framework

FIG. 4. 3-Bay, 12-Story Steel Building Framework
FIG. 5. Performance Index History of Bracing System

FIG. 6. Optimal Bracing System for the 12-Story Steel Framework
FIG. 7. Layout of Bracing System for the 12-Story Steel Framework

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