Piecewise Adaptive Controller Design for Position Control of Magnetic Levitation System

A dissertation submitted by

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Abstract

Magnetic levitation is a new technology gaining popularity for use in many applications, the most notable being rail transportation. Magnetic levitation, or maglev, utilises high powered magnets to suspend objects off the ground. Electromagnets are known to exhibit severe nonlinear characteristics and providing precise control of position is no simple task.

The ECP Model 730 magnetic levitation development system was studied in this project as the basis for the design of a suitable control system. The system model was developed and the system nonlinearities were identified. A linearised approximation of the system model was developed and a PID controller designed. The designed controller was simulated in Simulink and managed to force the magnet position to settle in 790 ms with 5.6% overshoot. It was found that nonlinearities caused the controller effectiveness to degrade as the magnet position moved away from the desired operating point.

A piecewise model of the system was developed and controllers were designed to designed to work over the entire range of operation. An adaptive control strategy based on gain scheduling was implemented into the system and an improvement of 200 ms and 2.47% overshoot was observed for a 0.5 cm change in operating condition. The controller managed to switch between operating points but was shown to exhibit poor disturbance rejection when its position was continuously changed.

Results suggested that an adaptive PID controller is capable of adapting to changes in its operating condition, but tuning it to achieve desired performance specifications is difficult, and other controllers may be more appropriate.
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0061032788
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Nomenclature

CHR Cohen-Hrones-Reshwick
DSP Digital Signal Processor
ECP Educational Control Products
EDS Electrodymamic Suspension
EMI Electromagnetic Interference
EMS Electromagnetic Suspension
IAE Integral Absolute Error
ISE Integral Squared Error
ITAE Integral Time Absolute Error
JNR Japan National Railways
$K_d$ Derivative Gain
$K_i$ Integral Gain
$K_p$ Proportional Gain
MIMO Multiple Input, Multiple Output
MRAC Model Reference Adaptive Control
NdFeB Neodymium
PD Proportional + Derivative
PI Proportional + Integral
PID Proportional + Integral + Derivative
QFT Quantitative Feedback Theory
SAC Simple Adaptive Control
SIMO Single Input, Multiple Output
SISO Single Input, Single Output
SMC Sliding Mode Control
$T_i$ Integral Time
$T_d$ Derivative Time
Chapter 1

Introduction

1.1 Preamble

Global overpopulation has been a major concern for the best part of the last decade. Historical estimates have shown that this concern is reasonably justified. Since 1950, the earth’s population has risen from around 2.5 billion people to the latest estimate of around 7.2 billion. Sullivan & ClimateWire (2013) have even suggested that by 2050, the global population could reach a staggering 9.6 billion according to recent United Nations estimates. With such overcrowding, particularly in large metropolises, comes inevitable transportation issues.

Rail transportation has for a long time been considered the most effective way to transport mass commuters from point A to point B as quickly as possible with minimal fuss. The trouble is, traditional locomotives are somewhat outdated and as the demand grows, the need for more advanced technology becomes important. Fortunately though, research and implementation (with moderate success) has been undertaken with trains that utilise magnetic levitation technology.

German multinational company ThyssenKrupp has been a world-leader in innovative rail technology for several years, and are the innovative minds behind the fabled Shanghai Transrapid rail system. The Shanghai Transrapid system, a high speed locomotive that was commissioned in January 2004, is capable of travelling at speeds up to 500 km/h (International 2004). Currently the only commercial vehicle of its kind in regular use, the
1.2 Project Aims and Objectives

30 km journey from Pudong International Airport to the terminus at Long Yang Road Station is completed in an average time of just 8 minutes. Fast, yet safe and comfortable, the magnetic levitation technology that it runs on is clearly the way of the future.

Developing a control system for such a device is no trivial pursuit. To help students and professionals alike understand the complexities involved with the levitation of magnetic elements, Educational Control Products, an American company, have developed the Model 730 apparatus. Its operation is simple, yet effective. It contains a copper coil which when energised induces a magnetic field parallel to a magnet on a guide rod. As is the case with any electromagnet, the magnitude of the induced magnetic field is proportional to the amount of current flowing through the coil.

This very useful instrument has 1-degree of movement along the vertical axis and can be configured as either Single Input Single Output (SISO), Multiple Input Multiple Output (MIMO) or even Single Input Multiple Output (SIMO). There is an on-board real-time controller that has the ability to execute any user-defined control algorithm, meaning a wide variety of control schemes can be tested and implemented. Its usefulness is further enhanced by the ability to study both EMS and EDS levitation schemes.

Such physical systems have been known to exhibit nonlinear characteristics between input and output. The force applied to the magnet is not proportional to the distance of the magnet from the coil. By definition, a linear system is one in which the system output is directly proportional to any order of derivative of the input. Such systems are invariably much easier to control. Standard linear control theory does not apply to nonlinear systems, and therefore special techniques must be applied. Finding the optimal response is not an easy task, and determination of an appropriate control algorithm is critical to successful implementation.

1.2 Project Aims and Objectives

1.2.1 Aims

This project aims to investigate, develop and implement the use of a suitable adaptive control strategy for use in position control of the ECP Model 730 Magnetic Levitation System.
1.3 Motivation

The design and optimisation of the controller will be performed using MATLAB and Simulink Control Toolbox.

1.2.2 Objectives

In completing this project, the following broad objectives will be met:

- Investigate the ECP Model 730 Hardware and Software
- Test the system and study the existing demo program and experiments
- System model identification and linearisation
- Design and simulation of a PID controller for the SISO Maglev system
- Design piecewise adaptive controller for the SISO Maglev system

1.3 Motivation

The concept of magnetic levitation is not new. As far back as 1914, an American engineer by the name of Emilie Bachelet first demonstrated the use of EDS technology (Ota 2008), and in 1936 the idea of an electromagnetically levitated train system was put forward by German Hermann Kemper. As innovative and ground-breaking as it was at the time, it wasn’t until the early 1970s when research work in the area began to take place. Issues regarding feasibility, money and of course, safety prevented the idea from becoming a reality. Research work in the area was arduous, and it took a further 30 years before the first commercial maglev train was commissioned, the fabled Shanghai Maglev in China.

Before a prototype can be commissioned for public use, there needs to be surety that it is going to be safe. This is no truer than in the case of a magnetically levitated rail system. Travelling at such high speeds (current Japan prototypes have reached speeds of 603 km/h), safety becomes paramount. Whilst assured to be safe, the Transrapid has already seen fatalities in the form of a 2006 accident in Germany, in which the vehicle collided with a maintenance car, killing 23 people. A control system for a system of this nature is invariably important. Operating at such close tolerances with minimal margin for error requires precise control.
1.4 Overview of the Dissertation

A brief overview of the dissertation composition is given below:

**Chapter 2** presents the essential background theory and review of relevant literature.

**Chapter 3** states the methodology utilised in undertaking this research project.

**Chapter 4** discusses the system dynamics, development of the system model and system linearisation.

**Chapter 5** documents the development and simulation of a suitable PID controller for the system.

**Chapter 6** provides a discussion of the design and simulation of a proposed adaptive controller.

**Chapter 7** concludes the dissertation and provides recommendations and potential future work in the area.
Chapter 2

Background and Literature Review

2.1 Chapter Overview

The following chapter examines the relevant background information and contains a review of the literature applicable. It discusses magnetism and the concept of magnetic levitation. Control system design and PID controllers are mentioned, and the tuning methods considered. Nonlinear system theory is then introduced and means for nonlinear adaptive control are discussed. It then further discusses magnetic levitation relating to the ECP Model 730 before considering previous work in the area.

2.2 Magnetism

Magnetic fields are used to describe forces at a distance from electric currents. These currents may be Amperian currents as drawn from a power supply, or bound currents contained in permanent magnetic materials (Parks 1999).

These forces have been known to exist in three variations:

1. Forces between electric currents
2. Forces between currents and permanent magnets
2.2 Magnetism

or

3. Forces between permanent magnets

2.2.1 Electromagnetism

A time-varying magnetic field induces voltages in a closed loop according to Faraday’s Law: \( E = N \frac{\partial \phi}{\partial t} \) (Parks 1999), the direction of which is determined by Lenz’s Law (van Dyke 2002), in which the magnetic field is perpendicular to the direction of the current flow.

Numerically, this is given by:

\[
E = -N \frac{\partial \phi}{\partial t}
\]  \hspace{1cm} (2.1)

Figure 2.1: Lenz’s Law - induced magnetic field perpendicular to current (van Dyke 2002)

Similarly, the Biot-Savart Law defines the magnetic field strength at a distance from a current-carrying wire as \( dB = \frac{\mu_0 I dl \sin \theta}{4\pi r^2} \). The net magnetic field will always have both horizontal and vertical components. For a circular coil, the current elements will always follow the conductor path, thus the horizontal components cancel (Parks 1999). The
magnetic field strength in the vertical direction is then given by:

$B_z = \mu_0 \frac{2\pi r I}{4\pi z^2 + r^2} \cos \theta = \frac{\mu_0 I R^2}{2(z^2 + r^2)^{3/2}}$ (2.2)

The field strength decreases exponentially as the height $z$ above the plane of the coil increases. Off axis, the field strength is seen to decrease by a factor proportional to the fourth power of height (Fitzpatrick 2008).

### 2.2.2 Ferromagnetism

All magnetic fields are created by moving charged particles. André-Marie Ampère first hypothesised that the intrinsic magnetic properties of certain materials were due to current loops caused by the molecular currents of orbiting electrons. This was later proven to be inaccurate due to the large currents required. However, in 1928 Paul Dirac suggested that the total magnetic effect observed was also produced by a quantum phenomenon known as electron spin, an intrinsic property of electrons. (Sandeman et al. n.d.).

At the atomic level, all matter experiences electron spin. A rotating electrically charged body orbiting the nucleus creates a magnetic dipole with poles of equal magnitude and opposite polarity (Mahajan & Rangwala 1988). This is known as the electron magnetic moment.

In certain materials, a peculiar characteristic exists in which the presence of an external magnetic field causes the moments to align parallel creating a region of large net magnetisation, known as a domain. (Nave 2001a). When the external field has been removed the domains align, allowing the material to retain its magnetic properties.
Such materials are termed ferromagnetic and are used in permanent magnets. Typical ferromagnetic elements include iron, nickel, cobalt and many of their respective alloys. Other naturally occurring rare-earth metals such as neodymium and samarium are capable of producing some of the strongest magnetic fields known.

Permanent magnets have two magnetic poles, one north (N) and one south (S), as defined by the direction of the magnetic flux lines (flux lines will always travel from north to south). Opposite poles will attract, whilst like poles repel.
2.2 Magnetism

Furthermore, it has been shown that the magnetic moment of the electron spins acts as though it’s a current loop, circulating the perimeter of a volume of magnetised material (Parks 1999), and can be modelled as an infinite current sheet with multiple parallel wires (Hughes 2005b). The magnitude of these currents is proportional to the thickness of the materials, such that \( M = K = \frac{I}{dz} \), where \( K \) is current density in A/m.

2.2.3 Forces due to Magnetic Fields

The force acting on a charged particle due to external electric and magnetic fields is given by Lorentz’s Law: \( \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \), where \( \mathbf{E} \) is the electric field strength and
B the magnetic field strength (Hughes 2005a). This force always acts in a direction perpendicular to both the charge velocity (v) and the magnetic field (Nave 2001b).

\[ F = I L \times B \]  

(2.3)

Where I is the current through the wire and L is the wire length.

For the special case of a Neodymium Iron Boron permanent magnet positioned above a coil, the force of the magnet-coil interaction is shown to be:

\[ \frac{K}{(z + D)^n} \frac{I}{I_{\text{max}}} \]  

(2.4)

Where \( 3 < N < 4 \), depending on the height \( z \) above the plane of the coil.

### 2.3 Magnetic Levitation

Intuitively then, the force generated by the magnetic fields can then be used to lift or 'levitate' objects off the ground, provided that the magnitude of the magnetic force is large enough to overcome the effects of gravity and other counter accelerations. This principle is known as magnetic levitation, or maglev, and can be used to create frictionless and efficient technologies. It has already seen a large number of uses in a variety of different fields, such as transportation, chemical engineering and aerospace engineering just to name a few.

In a nutshell, magnetic levitation is defined as the suspension of an object using nothing other than magnetic fields. This is principally achieved by one of two ways, either:
2.3 Magnetic Levitation

1. Electromagnetic Suspension
   or

2. Electrodynamic Suspension

each with their own pros and cons.

EMS, as it is commonly referred to, is where levitation is achieved via means of attractive force between two (or more) magnetic fields. Typically, this configuration consists of a fixed upper ferromagnetic element with an actively controlled lower electromagnet as the levitated object. Precise current control is critical as to control the air-gap (Parks 1999). If the current in the electromagnet is too large, the two magnets will come in contact. If the current is too small however, insufficient force is available and the electromagnet fails (Lee, Kim & Lee 2006).

By far the more common of the two, this configuration is inherently unstable due to the nature of the induced magnetic field and precise control of position is therefore critical (Yaghoubi 2013).

EDS on the other hand, utilises the repulsive force between two magnets of the same polarity to levitate the object off of the ground. In this configuration, the lower magnetic element (either ferromagnet or electromagnet) is fixed. Currently, five (5) variations of this method exist (Parks 1999). The most common of which consists of either a permanent magnet or electromagnet situated above a normal copper coil.

This configuration however, is stable and precise air-gap control is not required (Lee et al. 2006).
2.3 Magnetic Levitation

2.3.1 Application - Trains

By far, the most well known and well documented use of magnetic levitation technology is in trains. Undoubtedly the most advanced vehicles currently available to railroad industries, the maglev train is the first real innovation in the field since the invention of the railroad (Yaghoubi 2013). In contrast to traditional wheel-on-rail technology, there is no physical contact between the maglev train and its guideway, meaning no friction and the possibility of ultra high speeds. In addition to the obvious advantages, maglev trains have numerous other traits that make them highly desirable, including minimal maintenance, low energy loss, lower vehicle weight and low noise emissions.

A maglev train can be seen to have three main functions, propulsion, levitation and guidance, all of which are controlled using magnetic force (Lee et al. 2006). Both EMS and EDS technologies have been proposed and (to some extent) developed. The EMS system, like the Shanghai TransRapid, consists of electromagnets on the train body that interact with levitation rails on the rail guideway (Yaghoubi 2013). It has been stated by Lee et al. (2006) that the typical air-gap between train and guideway is only around 10 mm, which is controlled using highly accurate and precise control systems. This becomes difficult to maintain at high speeds.

![Figure 2.8: EMS Train (Foundation 2015)](image)

Japanese National Railways (JNR) have taken a different approach. Since 1962, they have been researching and developing a railway system that can link Tokyo and Osaka (a
2.3 Magnetic Levitation

distance of around 500 km) in around one hour. The research undertaken has primarily been focused on utilising EDS technology with super-cooled, superconducting electromagnets on the guideway as the driving force. This system provides enough levitation that the train is able to levitate at a height of 10 cm above the track (Lee et al. 2006).

![Figure 2.9: EDS Train (Foundation 2015)](image)

2.3.2 Application - Rockets

Engineers and researchers at NASA’s Marshall Space Flight Center in Huntsville have designed and tested the possibility of using magnetic levitation as a means for launching rockets. Utilising similar EDS technology to JNR, the prototype known as MagLifter, uses high-powered electromagnets to levitate the vehicle above the track and propel it forward with high acceleration to a speed of up to 885 km/h (Yaghoubi 2013). At the end of the track the rocket separates from the guide vehicle and ascends into a low earth orbit (Poleacovschi 2011).
2.4 Control Systems Design

Control systems are an integral part of modern society (Nise 2011). Without them, there would be no manufacturing, no vehicles, no machines. No technology that the world takes for granted (Doyle et al. 1990). The design of control systems is a complex and challenging task that requires a great deal of ingenuity that is quite often overlooked.

The simplest definition, a control system serves the purpose of making the output of a device, $y$, behave in a desired way by manipulating the input $u$ in some way. In one case it might be desirable to keep $y$ at a certain point, irrespective of the input conditions and operating environment. Other situations exist where the goal is to make the output track a certain reference signal $r$, and changes in conditions are compensated for accordingly (Doyle et al. 1990). In any case, the design of control systems relies heavily on appropriate knowledge of system dynamics and transient behaviour, that is, how the system is likely to respond when exposed a certain condition. There are certain performance criteria which can describe and dictate how well a control system performs to its desired specification, which can be broken down into two distinct sections, namely:

- Transient Response
- Steady State Error

Transient response information details how a system responds with time to any change in its equilibrium. There are three parameters that are usually used to describe this: rise
2.4 Control Systems Design

Rise time is a measure of how quickly the system can go from 10% to 90% of its final value. The overshoot describes how much the system overshoots desired target; typically expressed as a percentage of the difference between the peak response and the desired response, and is generally considered an undesirable trait, especially in position control systems with very close tolerances. The final transient response characteristic is the settling time, and it is a measure of how quickly a system can reach within some predefined percentage of the desired final value, typically taken as 2%.

Steady State Error (sometimes referred to as just error), is a measure of how far the system output is away from the desired value at the steady state. In general, it is the function of the control system to keep this error under control, ideally zero, but at least within some predefined tolerance values. As such, most control systems utilise the concept of a feedback loop in order to get information about the system performance and provide it with the necessary control. A simple feedback loop is used to calculate the difference between the actual value and the desired value. The control system then works to minimise this error.

![Figure 2.11: Illustration of Feedback in a Control System (Doyle et al. 1990)](image)

Clearly then, the appropriate transient behaviour depends entirely on the system to be controlled. In some cases it may be necessary to reach steady state as quickly as possible with no overshoot and zero steady state error, whereas in other cases it may be deemed unnecessary and even detrimental for a system to exhibit such aggressive behaviour and a more gradual response may be required.

Design of an appropriate control system is often as much a trial and error process as it is astute design, and there is no single one size fits all solution. A good control system for one plant, may be completely inappropriate for another. Because of this, there are virtually endless possibilities for providing control and the applicability to the system
2.5 PID Control

to be controlled lies in the required performance. However, certain methods have been tried and tested and can perform admirably for a variety of systems, one such is the PID controller. This is discussed in the next section.

2.5 PID Control

PID control is one of the most effective and widely used linear control strategies, controlling about 90% of all closed-loop processes in industry. A PID controller is a constant-gain feedback controller that generates a control input based on the error between the desired input \( r \) and calculated output \( y \). It consists of a proportional gain (P) that is directly proportional to the error, an integral gain (I) proportional to the accumulated error signal and a derivative gain (D) proportional to the rate of change of the error. All three components work in tandem to provide a system response that meets the required performance specifications.

The effects of the three controller parameters are neatly summarised in the below table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Rise Time</th>
<th>Overshoot</th>
<th>Settling Time</th>
<th>Steady State Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_p )</td>
<td>Decrease</td>
<td>Increase</td>
<td>Small Change</td>
<td>Decrease</td>
</tr>
<tr>
<td>( K_i )</td>
<td>Decrease</td>
<td>Increase</td>
<td>Increase</td>
<td>Eliminate</td>
</tr>
<tr>
<td>( K_d )</td>
<td>Small Change</td>
<td>Decrease</td>
<td>Decrease</td>
<td>No Change</td>
</tr>
</tbody>
</table>

In the time domain, the equation representing the output of a PID controller is:

\[
u(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{de}{dt}
\]  \hspace{1cm} (2.5)

By taking the Laplace Transform of the above the equation, the controller transfer function can be determined.

\[
C_{PID}(s) = \frac{K_ds^2 + K_ps + Ki}{s}
\]  \hspace{1cm} (2.6)

2.5.1 PID Tuning

Before implementation, it is necessary to tune the PID parameters to achieve the desired closed-loop performance. Several techniques exist for this, each with varying degrees
of complexity and suitability. Despite this, finding the appropriate parameters is still a difficult task (Salen & Rashed 2013). There is no single solution, and several permutations for the one physical system may be available.

Such techniques include (the list is not exhaustive):

- Design techniques utilising the root locus
- Ziegler-Nichols Tuning Formula
  - Open-Loop method
  - Closed-Loop method
- Cohein-Hrones-Reshwick Method
- Optimisation based tuning (IAE, ITAE, ISE)
- Manual Tuning (Trial and Error)

**Root Locus Design Technique**

The root locus design method of PID tuning is a simple technique that utilises the information contained within the open loop response of the system. Salen & Rashed (2013) suggest four (4) steps for PID design using this method.

1. Construct (or obtain) an accurate root locus plot
2. Design a PD controller to meet the transient performance specifications (overshoot, settling time and rise time)
3. Design a PI controller to eliminate the steady-state error
4. Find the proportional gain $K_p$ by applying angle criterion

**Ziegler-Nichols Tuning Formula**

The most well known, and probably most widely used, PID tuning methods are the Ziegler-Nichols tuning rules. Developed in 1942 by John Ziegler and Nathaniel Nichols
these tuning rules are based on the required transient specification of the plant and the plant dynamic behaviour. Systems tuned using these methods are typically known to exhibit between 10 and 60% overshoot (Salen & Rashed 2013). Two methods exist, the open loop or process curve method and the closed loop method.

The open loop tuning method is suitable if and only if the step response of the system exhibits a first-order response or a response similar to an s-curve, with no overshoot. Shown in Figure 2.12, a tangent line is drawn parallel to the steepest change in the output signal. The properties of this tangent are used to determine the appropriate parameters.

Typically used for systems with considerable dead-time, the following table summarises the formula used to calculate the required P, I and D gains.

<table>
<thead>
<tr>
<th>Controller Type</th>
<th>( K_p )</th>
<th>( T_i )</th>
<th>( T_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>( \frac{T}{L} )</td>
<td>( \infty )</td>
<td>0</td>
</tr>
<tr>
<td>PI</td>
<td>( \frac{0.9T}{L} )</td>
<td>( \frac{L}{0.3} )</td>
<td>0</td>
</tr>
<tr>
<td>PID</td>
<td>( \frac{1.2T}{L} )</td>
<td>2L</td>
<td>0.5L</td>
</tr>
</tbody>
</table>

Where, \( L \) is the time that the tangent line crosses the time-axis and \( T \) is the time taken for the tangent line to reach steady-state.

The closed-loop method is based upon where the stability limit of the system lies. It
2.5 PID Control

involves adding a proportional controller to a closed-loop system and increasing the gain until the system exhibits marginal stability (sustained oscillations). The value of $K$ at this point is known as the ultimate gain $K_u$ and the period of oscillation is the ultimate period $T_u$ (Patra et al. 2013). Based upon these parameters the P, I and D gains can be calculated.

Table 2.3: Rules for Ziegler-Nichols Closed-Loop Tuning Formula

<table>
<thead>
<tr>
<th>Controller Type</th>
<th>$K_p$</th>
<th>$T_i$</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.5$K_u$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>0.45$K_u$</td>
<td>$\frac{T_u}{T_i}$</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>0.6$K_u$</td>
<td>$\frac{T_u}{\tau}$</td>
<td>$\frac{T_u}{N}$</td>
</tr>
</tbody>
</table>

However suitable, these tuning rules are only a very rough ‘rule of thumb’ and additional tuning may still be required.

Cohein-Hrones-Reshwick Method

The CHR method is a modification on the Ziegler-Nichols closed-loop method just discussed, and emphasises set-point regulation and disturbance rejection, of which there is usually a trade-off (Salen & Rashed 2013). In calculating the PID gains, it requires knowledge of the time-constant $\tau$ and dead-time $\tau_d$ of the system. This method enables design of the controller to achieve either a 0% overshoot or a 20% overshoot, depending on the requirements of the controlled system response.

For set-point regulation:

Table 2.4: CHR Method for 0% Overshoot for Set-Point Regulation

<table>
<thead>
<tr>
<th>Controller Type</th>
<th>$K$</th>
<th>$T_i$</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$\frac{0.3R}{K_p}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>$\frac{0.35R}{K_p}$</td>
<td>1.2$\tau$</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>$\frac{0.6R}{K_p}$</td>
<td>$\tau$</td>
<td>$0.5\tau_d$</td>
</tr>
</tbody>
</table>

Where, $R = \frac{\tau}{\tau_d}$ and $K_p = 0.2K_u$.

For disturbance rejection:
2.5 PID Control

Table 2.5: CHR Method for 0% Overshoot for Disturbance Rejection

<table>
<thead>
<tr>
<th>Controller Type</th>
<th>$K$</th>
<th>$T_i$</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$0.3 R / K_p$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>$0.6 R / K_p$</td>
<td>$4 \tau$</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>$0.95 R / K_p$</td>
<td>$2.4 \tau$</td>
<td>$0.42 \tau_d$</td>
</tr>
</tbody>
</table>

Strictly speaking however, the overshoot is not always 0%, the response is simply labelled as such, and is in fact, "the quickest response with minimal overshoot.”

Optimsation Based Tuning - IAE, ITAE and ISE Criterion

A heavy research area is the use of mathematical optimisation algorithms to calculate PID parameters with respect to a minimisation cost function. A complicated field of research, the most prominently used in PID design are the Integral Absolute Error (IAE), Integral Time Absolute Error (ITAE) and the Integral Squared Error (ISE). Such methods are computationally intensive and are considerably more complex than traditional means.

The IAE criterion, as the name suggests, calculates the sum of the absolute error such that:

$$\int_0^\infty |e(t)|dt$$  \hspace{1cm} (2.7)

The ITAE criterion sums the time $\times$ the absolute error:

$$\int_0^\infty k|e(t)|dt$$  \hspace{1cm} (2.8)

The ISE criterion calculates the sum of the error squared:

$$\int_0^\infty |e^2(t)|dt$$  \hspace{1cm} (2.9)

2.5.2 Manual Tuning - Trial and Error

Several physical systems are unable to be tuned using any of the above mentioned techniques, and must be tuned manually. Typically an initial estimate for the PID parameters is made, and using an educated trial and error process the P, I and D gains are altered to achieve the desired result. In the majority of industries, this is nothing more than a
simple knob turning exercise. Computer software such as MATLAB and Simulink however, eliminate a lot of the tedium associated with this and have in-built PID tuners that allow manual tuning of gains to achieve a desirable response.

2.6 Nonlinear Systems Theory

Few physical systems in the world are truly linear (Hedrick & Girard 2010). Because of this, the need for an understanding of nonlinear system behaviour is critical. By definition, a linear system is classified as any system that has an output directly proportional to the input, or any order of its derivative (Clarke 2012). Such systems are known to possess two characteristic properties: homogeneity and superposition (Nise 2011). The homogeneity principle states that for any system input \( r(t) \) that yields an output \( c(t) \), multiplication by a scalar \( A \) on the input will yield an output that is multiplied by that same scalar. Likewise, the superposition theorem states that the output response of a system to the sum of the inputs is the sum of the responses to the individual inputs (Nise 2011). These two characteristics ensure that a linear system is always deterministic, meaning no randomness occurs and the output response can always be correctly predicted. It then follows intuitively that if the output can be predicted, control can be easily applied.

Any system that does not satisfy the properties of homogeneity and superposition is therefore nonlinear (Hedrick & Girard 2010). Because the superposition theorem doesn’t hold, unlike in linear systems, a nonlinear system cannot be separated into smaller parts and solved individually by traditional means (Laplace Transform, Fourier Analysis etc.). In addition to this, nonlinear systems have been known to exhibit certain peculiar behaviours, namely multiple equilibria, limit cycles and chaos.

2.6.1 Nonlinear Behaviour 1 - Multiple Isolated Equilibria

By definition, the equilibrium point \( x_0 \) of a dynamic system is any point where the system can stay for all future time without moving. In the case of linear systems, there is only one such point. Nonlinear systems however, can have anywhere between zero and \( \infty \) equilibrium points. Furthermore, the behaviour of the system around these equilibrium points is very different, and as such they cannot be described accurately by a single linearised model (Ding 2013).
2.6.2 Nonlinear Behaviour 2 - Possibility of Limit Cycles

It is well known that for a linear system to exhibit periodic oscillations of constant amplitude, it must have a pair of poles located on the imaginary axis (Khalil 1996). In this case, the amplitude of oscillation is determined solely by the system’s initial conditions (Gallestey & Al-Hokayem 2015). However, due to the presence of unwanted disturbances, be it environmental or otherwise, sustained oscillations in linear systems are difficult to maintain and instability quite often occurs. Nonlinear systems have been known to go into a state of oscillation irrespective of the initial conditions. Such a state is known as a limit cycle and is a complex phenomenon that is not uncommon. Due to their uncontrollable nature, limit cycles are known to cause fatigue in physical systems, especially those that involve a mechanical component (Hedrick & Girard 2010). In general, limit cycles are a highly undesirable trait in control system design and great care must be taken to ensure they are avoided and adequate control is achieved (Shimkin 2009).

2.6.3 Nonlinear Behaviour 3 - Chaos

Whilst being relatively deterministic in nature, nonlinear systems have been known to exhibit chaotic behaviour. Chaos is random and is a much more complicated steady-state behaviour than say, equilibrium or periodic oscillations (Khalil 1996). It is nearly impossible to predict, and therefore much harder to control.

2.7 Nonlinear Control

Research in the field of nonlinear control systems is wide and varied. As such, several techniques have been developed. In any case, nonlinear control strategies rely heavily on the concept of linearisation and developing an approximate linear model of the system.

2.7.1 Linearisation Around an Operating Point

It has been shown that for small perturbations about an operating point, a nonlinear system can be approximated as being linear. Typically taken as the equilibrium point, the linearisation process is done by applying a Taylor Series Expansion approximation at
that point, as is given below.

\[
f(x) - f(x_0) \approx \frac{df}{dx} \bigg|_{x-x_0} (x - x_0)
\]  

(2.10)

The resultant equation yields a linear approximation that is tangential to the equilibrium point. This is a sufficient approximation provided that the system is operated within a reasonably small region either side of the linearisation point.

![Linearisation Around an Operating Point](image)

Figure 2.13: Linearisation Around an Operating Point (Clarke 2012)

The problem is, as the system operation moves away from the equilibrium point, the error in approximation is large; the system approximation is no longer accurate.

It was previously mentioned however, that a nonlinear system can have anywhere between zero and \( \infty \) equilibrium points. By utilising this principle, a more accurate system model can be achieved by linearising over a large range of operating points. Nonlinear systems that can approximated this way are known as piecewise linear systems (Nise 2011).

### 2.7.2 Adaptive Control

As the need for fully-automated intelligent systems grows, so has the depth of research undertaken in the field of adaptive control. The principle behind adaptive control is to have the control law adapt its own behaviour and update its parameters in response to a change in the operating environment (Friedland 1996), whether it be a change in operating
condition or in the presence of a disturbance. Such a scheme is deemed appropriate when a design model of the system is not available, or a model is available, but the parameters are either unknown or subject to wide variation.

Consider a set of critical performance parameters ($\theta$) that are subject to variation. An adaptive control scheme can be designed that depends entirely on the value of $\theta$. The controller parameters will be tuned appropriately according to $\theta$.

Two important questions immediately arise from this. Can the parameters be measured directly, or must they be estimated by some means?

**Gain Scheduling**

In the simple case that the parameters can be measured, or at least inferred, a gain scheduling technique can be applied. Gain scheduling is an open-loop compensation, and can be viewed as a system with feedback control in which the feedback gains are adjusted by feedforward compensation (Åström & Wittenmark 1989). This technique has been used successfully over the last few decades for control of complex systems such as autopilots and chemical processes (Tsay 2013).

The simplest form of adaptive control, gain scheduling consists of a variety of linear controllers designed at various operating points to cope with large parameter variations. For piecewise linear systems, this is a good approach. Once again, consider a nonlinear system linearised around a single equilibrium point. It can be shown that this system is linear only for small perturbations around this point. A controller that is designed based on such an approximation is only going to work effectively so long as the plant operation is constrained to that region. Instead of a single point linearisation, it is required to consider the system behaviour relative to family of operating points which spans the envelope of operation (Leith & Leithead 2000). Changes in operating points are modelled as switches between linearised systems (Sang & Tao 2012).

Linear controllers are then designed at each operating point. The family of linear controllers corresponding to each operating point is then implemented as a single controller whose parameters are changed by monitoring the scheduling variable (Gain Scheduling 2009). Typically, gain scheduled controllers are fixed single-loop controllers that use look-
up tables or a parametric gain surface to specify gain values as a function of the scheduling variables (Rugh & Shamma 2000). Information inside the controller can be used as the scheduling variables (de Oliveira & Karimi 2012). A gain scheduled PID controller is shown in Figure 2.14.

![Figure 2.14: Block Diagram of a Gain Scheduled PID controller (Tsay 2013)](image)

**Model Reference Adaptive Control**

On the other hand, if the values of $\theta$ cannot be directly measured, a self-tuning controller must be employed. These techniques make use of parameter estimation and the relationship between the uncertain parameters and other relevant system quantities (Friedland 1996).

Model-reference adaptive control is one such method. An MRAC approach consists of a system reference model which produces the desired output, and the difference between the plant output and the reference output is used to adjust the control parameters and the control input (Ding 2013). The advantage of this technique is that any linear controller can be applied (PID, PI etc.).

The reference input is applied to both the controller and the reference model. The output of both the plant and the model are combined to generate an error signal.

$$
\epsilon = y_m(t) - y(t)
$$

The magnitude of this error signal is then used to adjust the controller parameters accordingly. If the model dynamics are chosen correctly, it should be possible to drive this error towards zero. As the error approaches zero and the closed-loop behaviour of the plant approaches that of the model, the signal that drives the output of the adaptive loop vanishes (Friedland 1996).
The reference model output is said to be:

\[ y_m(t) = W_m(s)r(t), W_m(s) = 1/P_m(s) \]  \hspace{1cm} (2.12)

where \( P_m(s) \) is the desired stable closed-loop characteristic polynomial (Sang 2012).

### 2.8 ECP Model 730 Magnetic Levitation System

Developed by American company Educational Control Products, the ECP Model 730 is a specially designed development tool used specifically for nonlinear control system development and implementation, when related to magnetic levitation. As is the case with all magnetic levitation systems, this system exhibits a nonlinear relationship between input and output. Systems of this nature are termed nonlinear and as such, traditional linear systems theory and control methods do not apply.

The device itself consists of numerous components.

Two high flux drive coils located at the top and bottom of the maglev plant are responsible for generating the magnetic force required for levitation. The levitated objects are two rare neodymium earth magnets. When a coil is energised, a magnetic field is induced parallel to it, forcing the NdFeB magnet(s) up or down the vertical guide rod. Two high precision non-contact laser sensors are situated at both ends of the plant to provide accurate position measurements. Both magnets and coils are capable of providing up to 6 cm of controlled levitation range (Parks 1999).
This system is extremely versatile and numerous configurations are possible. SISO is achieved with one coil (input) and one magnet (output), MIMO is achieved with both coils and both magnets, and SIMO is possible with one coil and two magnets. Each configuration will exhibit different properties. These are summarised in Figure 2.17.
By energising the lower coil and aligning the magnet with an opposite polarity, repulsive levitation or EDS can be studied. Such a configuration is seen to be open-loop stable. Energising the upper coil and aligning the magnet so that their poles attract allows EMS to be studied, as was noted however, such a configuration is very unstable. Likewise, MIMO and SIMO operation can be studied by following similar principles.

The operation of the system is performed via a software/hardware interface. The system hardware consists of a real-time controller unit that encompasses a DSP based real-time controller, servo/actuator interfaces, servo amplifiers and auxiliary power supplies should a failure occur. The on-board DSP is capable of executing control laws at high sampling rates, allowing any continuous or discrete control algorithm to be applied. The software utilised by this system is specific to the equipment, in terms of an 'Executive' program. Run on a host Windows PC, this program is the connection to the system hardware. Control laws are developed and implemented on the plant using the C programming language, in an intuitive, easy-to-understand way (Parks 1999).
In recent times, there has been considerable research conducted into the control of the ECP Model 730 System. Primarily, this research has been aimed towards applying robust control to the system. It is only much more recently that adaptive control has been considered as a suitable, and most likely better, option.

A system is said to be robust, if it can adequately cope with changes in its operating conditions (Åström & Wittenmark 1989). Hence, it follows intuitively that certain control strategies exist to provide this so-called robustness. Unlike adaptive control, robust control is static, it doesn’t adapt its parameters in response to variations in operating conditions: the controller is designed in a way such that it is insensitive to a range of parameter variations (Williams 2008). PID controllers are known to exhibit a certain amount of robustness, and this can be exploited.

In 2009 Nataraj & Patil (2009) published a report detailing their findings on using Quantitative Feedback Theory to design a robust controller for use on the ECP Model 730 system. QFT, developed by Isaac Horowitz, involves the feedback being tuned to the amount of plant and disturbance uncertainty present, and to the system’s required operating specifications. According to Nataraj & Patil (2009), this technique is rapidly gaining popularity in the design of robust feedback systems due to its ability to work with both SISO and MIMO systems. The system dynamics were found and the plant was linearised around a 2 cm operating point.

After ascertaining the linearised system model for the plant, the authors then proceed to design the controller. The results from their experiment yielded promising results; the rise time was estimated to be 0.42 seconds with no overshoot and had a peak control effort of 2.8 volts.

Clarke (2012) proposed the use of a robust deadbeat controller for this application. The control technique devised, utilised a conventional PID controller with a cascaded gain, adds state variable feedback to the plant and adds a zero in the feedback loop of the plant/controller combination as shown. It is stated that this design has the ability to withstand up to 50% displacement from the linearisation point. When linearised at a point of 2 cm, the simulation showed the plant to settle in 181 ms. As was expected, when the position is moved more than 1 cm away from the operating point, oscillation
began to appear. Furthermore, when the position is moved even further away violent oscillations were exhibited and the plant becomes almost unstable. The final results showed that this design consistently displayed a 60% improvement in settling time and 66% in disturbance rejection, as well as a 30% increase in bandwidth.

![Robust Deadbeat Control Technique](image)

The first reported attempt at an adaptive control strategy for this system belonged to Yasser et al. (2006). The authors researched and discussed the suitability of an adaptive sliding mode control strategy for the ECP Model 730 plant. The main feature of sliding mode control is that it can quickly switch the control law to drive the system states from any initial state onto a specified sliding surface (Yasser et al. 2006). It is stated that a difficulty exists when applying SMC to a system with uncertain or unknown parameters and changing characteristics. To overcome this, the authors have utilised a simple adaptive control strategy to construct a reference control input of which to apply sliding mode control. For the purposes utilised above, the plant parameters are considered unknown and the system responses modelled against the reference input. When compared to SAC only, the simulated results show that the SMC/SAC control system reduced the system error markedly.
Following on, Junior, Schnitman & de Souza (2010) experimented with the possibility of utilising an exact linearisation technique with state feedback combined with a pole placement control method. A Monte Carlo method based on a cost function was used to estimate the unknown system parameters. The control scheme involved a transformation of the plant nonlinearities into an equivalent linearised form by applying a feedback law that cancels out the nonlinearities through the addition of nonlinear compensators in the control loop. The pole placement control technique discussed, allocates closed-loop poles at any desired position by means of state feedback. In this study the authors selected the poles to be at -1 and -2 respectively. After simulation, the system settled at the desired position of 4 cm in about 0.6 seconds. Junior et al. (2010) concluded that the control scheme applied was suitable at one particular reference signal and future work would involve an adaptive control scheme. Later, Torres et al. (2010) followed on from this by applying a fuzzy logic control solution combined with exact linearisation. The controller was found to produce some overshoot but it adequately tracks a reference input from 0 to 3 cm.
From the literature study, it is clear that this area is broad and there are many areas which need to be considered. The study has also revealed that there are several possible solutions to this problem, some of which have been tried already with varying degrees of success. The search for the most appropriate control strategy is ongoing, and this forms the basis of this project.

2.10 Chapter Summary

This chapter has discussed the background theory behind this project, in particular the physics relating to magnetism and the principle behind magnetic levitation. It also discussed the theory that applies to nonlinear control and the ECP Model 730 Magnetic Levitation System, as well as considering previous research work in the area.
Chapter 3

Methodology

3.1 Chapter Overview

This chapter highlights the research methodology undertaken in performing this project. The consequential effects and ethical responsibilities are discussed, along with a discussion on the project planning. A brief account of each section of this project along with justification will also be mentioned. It concludes by conducting a risk assessment and resource analysis.

3.2 Methodology

3.2.1 Assessment of Consequential Effects and Ethical Responsibilities

Engineers, whether professionally accredited or students in training are required by Engineers Australia to perform an assessment of consequential effects prior to undertaking any major technical task. This project is no different. As such, a thorough assessment of the consequential effects and potential drawbacks associated with the development of this project was performed.
Assessment of Consequential Effects

The Institution of Engineers Australia has outlined ten aspects of sustainability that need to be considered for every task undertaken that has any impact (big or small) on society. For this project, all the resources required were either supplied (the ECP machine) or could be readily sourced (from the library, electronic databases etc.), so no manufacturing was required. This means minimal (or zero) wastage of important global resources. As this system deals with powerful electromagnets, electromagnetic interference may be a problem. There are strict guidelines regarding EMC in Australia and all equipment that may produce, or be subject to, EMI must abide stringent standards to ensure the wellbeing of society. Nevertheless, this system should be operated away from potentially susceptible devices, such that unwanted EMI effects (however small) aren’t prevalent.

As has been previously stated, magnetic levitation technology, in particular when applied to trains, greatly minimises pollution. There is minimal friction losses, so no degradation of expensive infrastructure. The operation of the system contains no harmful pollutants (petroleum, oil etc.) that should the system breakdown, will be released into the environment. The only environmental drawback of the operation of this system, and all magnetic levitation system in general, is that a current source is required to drive the actuators. With the ECP 730 system this typically small, but driving a train with 200 passengers and numerous amounts of cargo at 500 km/h this can no longer be ignored.

As technology has advanced, as wonderful as it is, a significant problem has arisen in the form of e-waste has arisen. According to CleanUpAustralia (2009), e-waste is one of the fastest growing contributors to Australia’s waste stream. This a problem because most (if not all) electronic equipment contains materials which are potentially dangerous to humans. Chemicals such as arsenic, lead and mercury are found in abundance in landfills across the country. In addition to this, many electronic components contain valuable resources that can be recycled and reused in the future. In regards to this project, it is imperative that when the development system comprising of the maglev plant and the host PC reach their useful end-of-life that they are disposed of correctly, to ensure environmentally sustainability in the long term.

Cost effectiveness is another important aspect to consider when undertaking project work. The cost of acquiring one of the maglev devices is not insignificant. It is hoped that the initial outlay will be greatly outweighed by cost benefit in the future. Costs have already
3.2 Methodology

caused significant conjecture around the world with Japan’s superspeed maglev train reportedly costing around US$100 billion to build (McCurry 2015), while still being at least 12 years before the system becomes commercially available. Such information is obviously a concern that must be considered.

Despite the relative lack of knowledge regarding magnetic levitation technologies, it is important going forward that projects such as this one are continually undertaken so that society can eventually benefit from it. It is important to learn from others mistakes and use this as a building block to improve.

Ethical Responsibilities

As required by Engineers Australia, all practising engineers must abide by a strict code of ethics and uphold the integrity of the profession. As outlined in the code, all engineers must:

1. Demonstrate Integrity
2. Practise Competently
3. Exercise Leadership
4. Promote Sustainability

In the execution of this project all of these guidelines were followed.

3.2.2 Project Planning - Methodology

Any major technical task will invariably require considerable time and effort. As such, a well defined methodology is required to ensure seamless transition from one stage to the next. Sadly though, in a not-so-perfect world, things will not always go according to plan, and sometimes the reason is unclear. On this basis alone, the project was separated into clear stages, with goals set along the way.
Before conducting a technical task of this nature, a thorough understanding of the requirements is required, not only to prevent failure, but also to gain an appreciation of potential benefits and future uses.

The ECP Model 730 software, whilst simple to use, contains a lot of components. The technical manual provided contains a thorough walk-through of all operations relating to the equipment and even contains numerous examples and exercises that may be undertaken. As such these must be read and understood to prevent any future problems from occurring.

An initial literature search and review was deemed appropriate to gain an understanding and appreciation of previous tasks that are similar in nature. The information gained there was used to make justified design decisions regarding future project tasks.

**System Model Identification and Linearisation**

Prior to any control system being designed, a system model must be developed. For this system, a full model is not readily available and must be determined experimentally. Clarke (2012) undertook a similar project and performed the experimentation to obtain such data. After much discussion, it was deemed unnecessary to repeat such a process. In light of this, the data obtained by Clarke (2012) was used for modelling purposes, saving precious time.

Using the data obtained, a system model can be determined. To match the experimental data to a mathematical model, a regression was performed using MATLAB software. Initial research suggested that a 4th order polynomial approximation be appropriate here.

The system model also required linearisation so that an appropriate control strategy can be applied. Based on discussions in numerous research papers, the simplest and most effective means of doing this was by using a Jacobian linearisation method based on a Taylor Series Expansion.
3.2 Methodology

Design and Simulation of PID and Adaptive Controllers

To prevent damage to the equipment due to a faulty design, before any physical implementation and testing was done, the proposed controllers needed to be tested using a simulation software. MATLAB and Simulink packages contain an in-built control toolbox add-on that allows easy determination of appropriate parameters for control algorithms by means of PID tuning and the like. The response to certain parameters could be studied and the best solution proposed. Mathworks most recent version of MATLAB and Simulink (R2015B) was downloaded and installed with the control design add-on enabled to ensure all the required software was up and running.

Testing and Implementation on ECP Model 730 System

The final stage in this task (if time permits) is to test the designed control schemes on the physical system. Once the simulations show adequate results, and the chance of catastrophic equipment damage minimised, the algorithms can be implemented. The ECP software is written in a easy-to-understand 'C-like' programming language with examples given in the provided manual. If required (and time permits), the control law can be tested again in the simulation stage to be further improved. This will be repeated as necessary.

3.2.3 Risk Assessment

What is risk? As defined, a risk arises when there is a possibility that a hazard can cause harm to people, the environment or other equipment. Strictly speaking, a hazard is anything in the workplace that has the potential to harm people, whether it be operators, customers or bystanders. Risk management and hazard identification is a broad area that involves development of techniques to minimise risk and propose safety measures and an action plan in the event an accident does occur. A risk assessment was performed for this project and the information documented in the tables at the end of the document.
3.3 Resource Analysis

To successfully undertake this task, several resources were sourced and utilised. The following resources were required.

- Resource Material
- ECP operators manual
- ECP 730 equipment
- MATLAB and Simulink simulation software
- Document processing software (MS Word or LaTeX)
- PC

3.3.1 Resource Material

As this project requires numerous amounts of background information, a large amount of reference material was required. This is in the form of published articles, technical manuals, textbooks, specialist reference books and internet sources such as webpages and library databases. Most of this information was freely available, either online or via the USQ library (inside or online).

3.3.2 ECP Operators Manual

To complete this task fully, as desired, the ECP manual was a critical resource. It contains all the information for successful operation of the ECP machine, as well as background information and examples of system modelling, linearisation and the like. A hard copy of this manual is located in the control laboratory along with the machine. An electronic copy was also obtained should one be misplaced.

3.3.3 ECP Model 730 Equipment

The basis of this project, the ECP Model 730 is critical to successful completion of this project. The equipment, which is housed in the control systems laboratory, consists of
the maglev plant, interface equipment and host software. USQ has sourced two of these machines, so should one have been inoperable, the second one can be used and the project continue as per normal. In the highly unlikely event that both machines were out of order, depending on how long they took to fix, other means may have had to be explored.

3.3.4 MATLAB and Simulink Simulation Software

A critical component of this project was the use and availability of the MATLAB and Simulink simulation software. Prior to the commencement of this project, all necessary programs and toolboxes add-ons were purchased, downloaded and installed. An account has been setup on Mathworks so that if the program crashed, the product can be re-downloaded and configured as is required. In addition to this, USQ has MATLAB and Simulink on most of the computers on-campus and specifically in the Engineering building. If all else failed, these computers could have been utilised to perform the simulations required in this task.

3.3.5 Document Processing Software

For the entire compilation of this research project, a document preparation system was required. There are two reasonable options for this task: Microsoft and LaTex. LaTex was the preferred option and was used for the duration of the dissertation compilation. In addition to this, preparing the documents was very time-intensive and loss of data could have proven catastrophic. In light of this, several backup copies were made: on an external hard drive, USB device and the computer hard drive.

3.3.6 PC

Most, if not all of the tasks related to this project required the use of a computer. Presently, it has all been undertaken on a personal laptop. Should this laptop fail, a backup (although a little outdated) could have been used. This could have proven time-consuming however, as a lot of the required software (MATLAB, LaTeX etc.) would have required download and installation. As previously stated, USQ has numerous machines available for use with most of the software readily available for use. This could have been
3.4 Timelines

To assist in progress monitoring, and so that the task does not become overwhelming, specific timelines have been developed. The project was divided into milestones with a completion date for each. The time expected for each one to be completed was developed. See Appendix D.

3.5 Chapter Summary

This chapter outlined the methodology behind the research conducted and discussed the consequential effects and ethical responsibilities. A safety and resource analysis was conducted and the results were documented. It also briefly mentioned the project planning and required project timelines.
Chapter 4

System Model Development and Linearisation

4.1 Chapter Overview

This chapter will discuss the dynamics of the ECP Model 730 Magnetic Levitation System and develop a suitable system model. It also documents the plant linearisation and system response. The majority of the information provided is drawn from the supplied manual by ECP.

4.2 System Dynamics

Before any control of the system is attempted, a suitable model of the plant dynamics must be obtained. As previously stated, several modes of operation are possible (SISO, MIMO and SIMO).

Consider the following simplified free-body diagram of the system.
It can be seen that the coil 1 exerts a repulsive magnetic force on magnet 1 and attractive force on magnet 2. Likewise, coil 2 exerts a repulsive force on magnet 1 and attractive force on magnet 2. There is also a repulsive force generated by the magnetic fields between the magnets. In addition, there is a weight force pushing down on each magnet. A downward force on each magnet also exists due to viscous friction effects, or air resistance.

It is clear then, that for levitation to occur, the magnetic field must generate enough force to overcome the effects of gravity and air resistance.

If $y_1$ represents the levitation height in cm of the lower magnet from 0 and $y_2$ the levitation height of the upper magnet from the upper coil. Performing a force balance on the system yields:

$$m\ddot{y}_1 + c_1\dot{y}_1 + F_{m1,2} = F_{u1,1} - F_{u2,1} - mg \tag{4.1}$$

For the first magnet.

And,

$$m\ddot{y}_2 + c_2\dot{y}_2 - F_{m1,2} = F_{u2,2} - F_{u1,2} - mg \tag{4.2}$$

For the second magnet.
The following table neatly summarises the variable definitions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$, $c_2$</td>
<td>Air viscosity coefficient</td>
</tr>
<tr>
<td>$F_{m1,2}$</td>
<td>Force between the two magnets</td>
</tr>
<tr>
<td>$F_{u1,1}$</td>
<td>Force applied to magnet 1 from coil 1</td>
</tr>
<tr>
<td>$F_{u1,2}$</td>
<td>Force applied to magnet 2 from coil 1</td>
</tr>
<tr>
<td>$F_{u2,1}$</td>
<td>Force applied to magnet 1 from coil 2</td>
</tr>
<tr>
<td>$F_{u2,2}$</td>
<td>Force applied to magnet 2 from coil 2</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of magnetic disc</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational Acceleration</td>
</tr>
<tr>
<td>$y_1$</td>
<td>Distance between magnet 1 and coil 1</td>
</tr>
<tr>
<td>$y_2$</td>
<td>Distance between magnet 2 and coil 2</td>
</tr>
<tr>
<td>$y_c$</td>
<td>Distance between coils</td>
</tr>
<tr>
<td>$i_1$</td>
<td>Input current to coil 1</td>
</tr>
<tr>
<td>$i_2$</td>
<td>Input current to coil 2</td>
</tr>
</tbody>
</table>

For the remainder of this project, only the single input, single output system with repulsive levitation will be considered. That is, only the bottom coil will be energised as the input and one magnet used as the output.

An unenergised coil exerts zero force, so the differential equation representing the SISO system with repulsive is therefore:

$$m \ddot{y}_1 + c_1 \dot{y}_1 + 0 = F_{u1,1} - 0 - mg$$  \hspace{1cm} (4.3)

Which simplifies to:

$$m \ddot{y}_1 + c_1 \dot{y}_1 = F_{u1,1} - mg$$  \hspace{1cm} (4.4)

Parks (1999) states that for the ECP Model 730 system, viscous friction is typically small enough to be negligible, and can be ignored.

The fully simplified differential equation is therefore:

$$m \ddot{y}_1 + mg = F_{u1,1}$$  \hspace{1cm} (4.5)
Furthermore, it has been shown that the magnetic force generated by the coil takes the form of:

\[ F_{u1,1} = \frac{i_1}{a(y_1 + b)^n} \]  \hspace{1cm} (4.6)

Where \( a, b \) and \( n \) are constants that must be found empirically by experimentation. Typically however, a fourth order approximation of the actuator characteristic is considered appropriate (Parks 1999), and \( i_1 \) can be replaced with \( u_1 \), the control effort output of the controller, that is proportional to \( i_1 \) by a factor of \( 10^4 \).

The equation for the magnetic force then becomes:

\[ F_{u1,1} = \frac{u_1}{a(y_1 + b)^4} \]  \hspace{1cm} (4.7)

Equation 4.7 clearly shows the nonlinearity of the system. As the distance of the magnet from the coil increases, so does the required input. This relationship is described by a second-order nonlinear differential equation.

### 4.2.1 Actuator Characteristics

Torres et al. (2010) found \( b \) to be equal to approximately 6.2. This value will be used.

To find \( a \), Parks (1999) suggests that the coil should be energised with different control efforts and corresponding levitation height recorded. This experimental data was obtained by Clarke (2012). A plot of the results is shown below.
The equation described by (4.7) describes the relationship between the input and the nonlinear actuator. It was stated above that this is best represented by a 4th order polynomial. To adequately describe the system, the values $a$ and $b$ must be also found. The values are constants that relate to the physical properties of the coil.

Based on the data obtained, a MATLAB script (shown in Appendix C) was written that calculates a 4th order regression equation to represent the system.

The results from the calculation yield a value of $a$ to be 1.631.

Equation 4.7 then becomes:

$$F_{u1,1} = \frac{u_1}{1.631(y_1 + 6.2)^4}$$

To test the validity of this approximation, the above equation was used to calculate the corresponding control effort for a range of displacements.

The calculated values were found to have an average error of 3.28%. The model can therefore be deemed valid.
4.2 System Dynamics

Figure 4.3: Comparison of Approximated and Measured Actuator Characteristics

4.2.2 Simulink Model Development

A basic working model of the plant dynamics was developed using Simulink. The model converts an input current in Amperes into magnet position in cm. The differential equation representing the maglev plant can be rearranged to yield the following:

\[ \ddot{y} = \frac{i_1}{ma(y_0 + b)^4} \]  

(4.9)

A function block was developed that accepts both the input current and the magnet position, and calculate the value of \( \dot{y} \) based on Equation 4.9. The output of the function block is connected to two integrators that then determines the value of \( y \).
4.3 System Linearisation

It is important to note that for simulation purposes a scaling factor of $\frac{1}{10^4}$ needs to be applied to the value $a$ to correspond to the units of $u_1$.

For control purposes however, it is necessary to represent the system as a block diagram in either the conventional transfer function or state-space form. For the remainder of this paper, the figure below will be considered as the standard representation of the plant.

Neither the system transfer function nor the state space model for the standard nonlinear system are available; its linear characteristics must be approximated by linearisation. This is discussed in the next section. Simulink control toolbox has a useful linearisation feature for nonlinear systems represented like the model illustrated in Figure 4.4. This feature was used to verify the validity of the transfer function obtained in the following sections.

4.3 System Linearisation

As the system is nonlinear standard control theory cannot be applied. In order to apply control to this system it must first be linearised. A linearisation method based on a Taylor’s Series expansion around an operating point as described in Chapter 2 was applied.
From above, the nonlinear differential equation that describes the system is:

\[ m\dddot{y}_1 + mg = \frac{u_1}{1.631(y_1 + 6.2)^4} \]  

(4.10)

The operating point is taken as the equilibrium point that satisfies the equation:

\[ F_{u1,1} - mg = 0 \]  

(4.11)

Clearly then, several points are possible. \( y_0 \) and \( u_0 \) must therefore be selected accordingly.

Parks (1999) has stated in the ECP manual, that the initial desired operating point should typically be between 2-3 cm. For this project a value of 2.5 cm was chosen as \( y_0 \) because it gives a \( u_0 \) of exactly 11000 counts.

For this nonlinear system with two variables, the Taylor’s Series expansion is:

\[ \frac{\partial f(y_e, u_e)}{\partial y} \delta y + \frac{\partial f(y_e, u_e)}{\partial u} \delta u = m \frac{d^2(y_e + \delta y)}{dt^2} \]  

(4.12)

Substituting 4.8 into 4.12 and performing the differentiation yields:

\[ -4u_0 \frac{a(y_0 + b)^5}{a(y_0 + b)^6} \delta y + \frac{1}{a(y_0 + b)^4} \delta u = m \ddot{y} \]  

(4.13)

Now letting, \( \frac{4}{a(y_0 + b)^5} = k_0 \) and \( \frac{1}{a(y_0 + b)^4} = k_1 \).

The linearised differential equation for the system is therefore:

\[ (-k_0 u_0) \delta y + k_1 \delta u = m \ddot{y} \]  

(4.14)

The system transfer function can be easily found by taking the Laplace Transform of both sides.

\[ G_p(s) = \frac{k_1}{m s^2 + \frac{k_0}{m}} \]  

(4.15)

The above plant model shows that magnetic levitation system models a second order undamped system.

It is clear from the above plant transfer function, that the system will display oscillatory behaviour. This response is far from ideal when control of position is crucial.
The above model, given by equation 3.16 represents the dynamics of the magnetic levitation plant only. The system also consists of a sensor with its own gain equal to $10^6$ counts/meter (Parks 1999). The total system gain then, is the product of the plant gain, as given above, and the sensor gain.

This gives a total system transfer function as:

$$G_s(s) = \frac{891.8}{s^2 + 451}$$  \hspace{1cm} (4.16)

This has a very similar form to the system transfer function found by Clarke (2012) and Nataraj & Patil (2009) for the linearisation point of 2 cm.

### 4.4 Open-Loop Step Response

An important property of any dynamic system is how it behaves in response to a step change input. Figure 4.6 below shows the system behaviour when a constant input of 2.5 is applied to it.

![Open Loop Step Response of Linearised System](image)

Figure 4.6: Open-Loop Step Response of Linearised System at 2.5 cm
It can be seen from Figure 4.6, that the system oscillates periodically between 0 and 10 cm with a period of about 0.4 seconds. Considering the input is to 2.5 cm, this is considerable steady-state error and reinforces the need for a suitable position controller. The system should be forced to settle at 2.5 cm.

4.4.1 Feedback

Almost all useful control techniques are applied in closed-loop, that is they utilise negative feedback. For the ECP 730 control system, feedback is required to provide the controller with information about the position of the magnet, so as to provide appropriate control. The feedback loop will take the system output, feed it back to the input and calculate an error signal based on the desired position.

The closed-loop transfer function of a system is defined as:

\[ T(s) = \frac{G(s)}{1 + G(s)H(s)} \] (4.17)

Where \( H(s) \) is the feedback law. For this project, only unity feedback was considered.

Therefore, the closed-loop transfer function for the maglev system with unity feedback is:

\[ T(s) = \frac{891.8}{s^2 + 1343} \] (4.18)

Stability

A major design consideration of any control system is its stability. By definition, a system is said to be stable if every bounded input yields a bounded output (Nise 2011). The stability of the linearised system is considered briefly here.

The closed-loop poles of the system are defined as any values of \( s \) such that \( 1 + G(s)H(s) = 0 \). For this system, \( 1 + G(s)H(s) = s^2 + 1343 \), and therefore:

\[ s = \pm \sqrt{-1343} \] (4.19)

\[ s = \pm j36.65 \] (4.20)
Now consider the root locus of the system shown in Figure 4.7. It can be seen that the uncompensated system has two poles located on the imaginary axis at $\pm j36.65$. The system is marginally stable. This is clearly problematic. A marginally stable system is very sensitive to disturbances, and any change in environmental conditions can cause catastrophic instability. The first design criterion then is to ensure asymptotic stability for any input.

![Root Locus](image)

Figure 4.7: Root Locus of Magnetic Levitation System

A suitable control technique is clearly required to not only stabilise the system, but also to force it to settle at the required level when the step input is applied.

### 4.5 Chapter Summary

This chapter discussed the nonlinear dynamics governing a magnetic levitation system and in particular the ECP Model 730 Apparatus. A working model was developed in Simulink and by application of Taylor’s Series Expansion, a linear approximation around an operating point for the system was found. The transfer function representation of the system was described and the system behaviour was analysed. The chapter was concluded with a brief consideration of system stability.
Chapter 5

PID Controller Design

5.1 Chapter Overview

The following chapter documents the design and simulation of a PID controller for the linearised system described in Chapter 3.

5.2 Design of PID Controller

Based on the discussion in the previous chapter, it is clear that control is needed to:

1. Control the position of the magnet and
2. Provide system stability

When observing the response of the uncompensated system, it is clear that many of the PID tuning rules discussed will not be applicable. The system is a second order undamped system with constant natural oscillations, automatically ruling out the use of either of the Ziegler-Nichols methods. A numerically optimised solution could be used, but the computation time would be expected to be significant. Instead, a procedure was developed based on an error minimisation technique and an initial guess with logical reasoning. The following section describes the procedure used.
5.2 Design of PID Controller

5.2.1 PID Tuning Method

It can be easily deduced that the PID action described by equation 2.5 will add an $s$ term in the denominator of the closed-loop transfer function, dampening the system and adding a pole. Because of this, it is reasonable to assume that the value of $K_I$, the integral gain, will need to be relatively large to compensate for the large steady-state error.

The ideal response for this system will be close to a deadbeat response, that is, it will have a small and finite settling time with no overshoot. It is not possible to select parameters to achieve a desired response using mathematical techniques; it is a trial and error process. Due to the continuous nonlinear nature of the plant, it may not be possible to completely remove the overshoot, without making the system unstable, so a value of 3% was deemed suitable.

To perform the tuning, a MATLAB script was written that initially calculates the steady-state error for a controller with parameters $K_p = 1, K_i = 0$ and $K_d = 0$. A loop was structured to increment the gain values until steady-state error is 0, $K_p$ was incremented by 0.2, $K_i$ by 0.5 and $K_d$ by 0.001. These values were found to give the best response.

It was found that it was not possible to remove the error completely, the MATLAB script didn’t converge, so a value of 0.0005 (0.05%) was considered small enough. The derivative gain was then increased by a further 0.01 to reduce the overshoot below 3%. The code can be found in Appendix B.

After simulation, the optimal tuned parameters were found to be: $K_p = 11.600, K_i = 26.500$ and $K_d = 0.553$.

The generated PID response is shown.

The initial simulation results were promising. The designed PID controller has managed to remove the steady state error and caused the system to settle with 2.75% overshoot in 406 ms.
5.2 Design of PID Controller

5.2.2 Stability

It is now worth considering the stability of the system with PID control. The pole location can be seen from the root locus below.

Figure 5.1: Response of Linearised Magnetic Levitation System with PID Control
The transfer function is:

\[ G_c(s) = \frac{493.2s^2 + 10350s + 23630}{s^3 + 493.2s^2 + 10800s + 23630} \]  (5.1)

with poles at \(-470\), \(-20\) and \(-2.5\).

The PID controller action has transformed the system from marginally stable to stable; it has shifted the poles away from the unstable right-half plane. This has satisfied the first design criterion of the controller, to ensure stability under the desired operating conditions.

### 5.3 Simulation

To prevent damage to the ECP Model 730 system, a working model of the controller and plant at the desired operating point was developed in Simulink. The ECP manual (Parks 1999) states that, to prevent damage to the equipment the input current should be limited to 4 A. To incorporate this into the model, a constant block of 20 000 counts of control effort, and another block of -20 000 counts were added in to clamp the current
5.3 Simulation

The block diagram of the model is shown.

The switches provided act as the limiter. If the value of input current exceeds the threshold of ± 20 000 counts, the switch clamps the input at that threshold, preventing excess current spikes. The figure below illustrates how the controller behaves with this limitation applied.

![Figure 5.3: Simulink Model of Maglev with PID Control](image1)

![Figure 5.4: Control Effort Required to Bring Magnet to Desired Position](image2)

It can be clearly seen that the required control effort (proportional to current) to bring the
5.3 Simulation

magnet to 2.5 cm, fluctuates wildly in both the positive and negative directions, before stabilising at 11000 counts as the magnet reaches its desired position. The initial spike that can be seen as the magnet moves from rest is needed to overcome the equilibrium forces.

This result is comparable to data obtained by Clarke (2012).

After simulation, the following PID response for 2.5 cm was obtained.

![PID Response for 2.5 cm Input](image)

**Figure 5.5: Simulated PID Response**

It can be seen that due to the saturation limits, the system is a little more sluggish to respond and settles with a fraction more overshoot (790 ms and 5.6% respectively) than was designed for, but is still suitable. Figure 5.6 shows the result of a similarly designed controller by Fan (2013) for a 2 cm linearisation point and step input. Unfortunately, the P, I and D gains of this controller were not published and as such are not readily available for comparison.
The response obtained closely matches the response obtained by Fan (2013) and can therefore be considered suitable.

Because this controller has been designed specifically for operation at 2.5 cm, it is expected, and justified by theory, that as the magnet position changes, the linearisation error will increase and the PID response will change. The system transfer function at each point is different, thus requiring different controller parameters.

Figure 4.7 shows the simulation result of moving the magnet to a position of 2 cm.
It can be seen that this response exhibits slightly more overshoot than the 2.5 cm response, but still settles reasonably well, definitely within the acceptable limits. It has a peak response of 2.12 cm and settles in 810 ms with 6.3% overshoot. Although it is difficult to see, this is definitely larger than the 2.5 cm point. However, the difference is almost negligible. Research has shown that well designed PID controllers, especially those with relatively large integral gains like this one does, are known to have a reasonable amount of internal robustness and can resist small changes in the operating condition.

Also notice the effect of the linearisation error at about 0.05 seconds. A notch can be seen on the graph, where the linearisation error is causing a slight jitter in the magnet response.

At 1 cm, 1.5 cm away from the desired operating point, this linearisation error is more readily observed. This jitter is more pronounced. The response is clearly a lot rougher than at the 2.5 cm, or even the 2 cm point. Whilst this error is not yet large enough to hinder the controller performance significantly, it is reasonable to assume that any further movement away from the desired operating point will cause failure. In fact, Clarke (2012)
observed that after 2 cm beyond the linearisation point, the magnet starts to exhibit violent oscillations and instability starts to occur. The controller effectiveness degrades as distance from operating point increases.

Consider now what occurs when the magnet is levitated to a position of 3.5 cm. It can be seen that magnet position has gone into an uncontrolled state of oscillation. The maximum control limits have been exceeded and the controller was unable to provide enough control effort to control the position adequately.

Figure 5.8: Simulated PID Response for 1 cm Operating Point - Oscillations Occurring
This is confirmed on the figure below, which shows the output of the controller capped to the upper maximum limit.
The above discussion has highlighted the need for a suitable adaptive control strategy for the ECP Model 730 Magnetic Levitation System. The system nonlinearities cause controller errors when the magnet is moved more than 1 cm from the linearisation point. A means therefore is required so that the controller will update it parameters in response to the change in desired magnet position. This is discussed in the next chapter.

Also highlighted was the fact that the magnet cannot be levitated above 4 cm. In fact it was shown that even at 3.5 cm, the magnet was unable to be adequately controlled.
This chapter has documented on the design and simulation of a feedback control system for the Magnetic Levitation plant based on PID control techniques. The effect of the linearisation error was shown and discussed, and the model limitations verified. It was also proven that a PID controller can resist small changes in operating condition, but as the distance is increased the controller’s effectiveness is reduced.
Chapter 6

Piecewise Adaptive Controller Design

6.1 Chapter Overview

The previous chapter designed and discussed the use of PID control algorithm for position control of the magnetic levitation plant. It was seen that a PID controller designed around a certain operating point will only provide appropriate control within a small excursion away from that point (between 0.5 and 1 cm). Any larger excursion reduces the controller effectiveness. To counteract this problem, an adaptive controller must be designed. For this system, the critical performance parameter that determines the operating conditions, is the desired magnet height, or linearisation point. As this is known, or can be defined, a gain scheduling technique can be used.

The following chapter will therefore develop a piecewise linear model of the system, design PID controllers at each desired operating point and implement a gain schedule as a means of controlling the magnet position.

6.2 Piecewise Linear Model Development

Several classes of nonlinear systems exist that are able to be considered linear over a wide variety of operating points. Such systems are termed as piecewise linear systems. The
development of a piecewise model is straightforward. It involves determining a suitable range of operation, linearising the system at suitable intervals along this range, and then developing a suitable means of switching between functions.

It has been shown already that the current limiters on the ECP Model 730 prevent the magnet from being levitated at a height greater than 3.5 cm. Thus, the safe range of operation for the SISO repulsive levitation configuration can be seen to be from 0 - 4 cm. This logically, then becomes the range for which the system will be linearised.

By linearising at a large number of operating points, the entire system can be accurately modelled. The accuracy of the piecewise linear model depends on the amount of linear segments. However, it was stated in the above chapter that the linearisation error remains acceptable until at least 0.5 cm away from the desired linearisation point. Because of this, the piecewise model will be developed at intervals of 0.5 cm, and no real benefit would be gained by considering intervals smaller than this.

A MATLAB function and script was developed to perform the linearisation at the selected operating points, and output the system transfer function at each point. The linearisation points were 0.5 cm, 1 cm, 1.5 cm, 2 cm, 2.5 cm, 3 cm, 3.5 cm and 4 cm.

The following table shows the transfer functions at each point.

<table>
<thead>
<tr>
<th>Operating Point</th>
<th>Linearised Transfer Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 cm</td>
<td>( \frac{2536}{s^2+585.7} )</td>
</tr>
<tr>
<td>1 cm</td>
<td>( \frac{1901}{s^2+545} )</td>
</tr>
<tr>
<td>1.5 cm</td>
<td>( \frac{1453}{s^2+509.6} )</td>
</tr>
<tr>
<td>2 cm</td>
<td>( \frac{1130}{s^2+478.5} )</td>
</tr>
<tr>
<td>2.5 cm</td>
<td>( \frac{891.8}{s^2+451} )</td>
</tr>
<tr>
<td>3 cm</td>
<td>( \frac{713.2}{s^2+426.5} )</td>
</tr>
<tr>
<td>3.5 cm</td>
<td>( \frac{577.1}{s^2+404.5} )</td>
</tr>
<tr>
<td>4 cm</td>
<td>( \frac{472}{s^2+384.7} )</td>
</tr>
</tbody>
</table>

The piecewise model of the system has been implemented in Simulink. To do this, a model has been developed that will accept an input signal of control effort and select the appropriate transfer function approximation of the linearised system. It makes the
6.3 Adaptive Controller Design

selections based on a series of if else statements implemented in an if else action block that match the required criteria (see Appendix C).

Figure 6.1: Piecewise Model Implementation

Figure 6.1 shows the simulation results for the piecewise implementation. The model appears to work as intended. Initially, an 11000 count input is applied, signalling that the desired position is 2.5 cm, the model selects the system transfer function to be:

\[ G(s) = \frac{891.8}{s^2 + 451} \]

representing the linearised system at this point. At 0.5 seconds, the input is increased to 16998 counts and the transfer function representing the 3.5 cm point is selected. At 1 second the input changes to 5159.8 counts corresponding to a height of 1 cm, and at 1.5 seconds the input is increased to 8680.8 counts for a 2 cm height. The switching between systems is clearly observed as the frequency of oscillation is changing at the switching points.

6.3 Adaptive Controller Design

The implemented piecewise model was used as the adaptive switching block between linearised systems. For the design of the adaptive controller scheme, PID controllers were designed corresponding to each linear operating condition. The tuning method described
6.3 Adaptive Controller Design

in chapter 5 was again used. A MATLAB algorithm was written that accepts an input of desired magnet position and outputs the respective Proportional, Integral and Derivative gains to match the design criteria.

The following table shows the desired PID gains for the controller at each operating condition.

<table>
<thead>
<tr>
<th>Operating Point</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 cm</td>
<td>2</td>
<td>2.5</td>
<td>0.055</td>
</tr>
<tr>
<td>1 cm</td>
<td>6.4</td>
<td>13.5</td>
<td>0.287</td>
</tr>
<tr>
<td>1.5 cm</td>
<td>7.8</td>
<td>17</td>
<td>0.364</td>
</tr>
<tr>
<td>2 cm</td>
<td>9.8</td>
<td>22</td>
<td>0.454</td>
</tr>
<tr>
<td>2.5 cm</td>
<td>11.8</td>
<td>27</td>
<td>0.554</td>
</tr>
<tr>
<td>3 cm</td>
<td>13.8</td>
<td>32</td>
<td>0.674</td>
</tr>
<tr>
<td>3.5 cm</td>
<td>16.8</td>
<td>37.5</td>
<td>0.813</td>
</tr>
<tr>
<td>4 cm</td>
<td>18.8</td>
<td>44.5</td>
<td>0.969</td>
</tr>
</tbody>
</table>

The gains are shown to increase linearly. This is due to the linear system approximation, and shows that greater control effort is required as the distance from zero increases.

6.3.1 Adaptive Controller Simulation and Implementation

The adaptive control scheme chosen is a gain schedule based on techniques described by Rugh & Shamma (2000). It is a single loop feedback controller that utilises look up tables to select the appropriate PID gain parameters based on the scheduling variable, which in this case is the desired magnet height.

A simulation was performed in Simulink that utilises this principle and the piecewise model described above. A PID controller was designed with its Proportional, Integral and Derivative gains defined externally and stored in 1-D look-up tables. The data stored in the tables are defined such that the breakpoints represent the desired operating conditions and each breakpoint has a corresponding gain value. The appropriate gain values are selected based on the desired position of the magnetic levitation plant.
6.3 Adaptive Controller Design

Full details can be found in Appendix C.

Several performance tests were undertaken to see how the system responds to various parameters.

The first simulation tested the controller response to a 2.5 cm step input. As can be seen, the response is very similar to the response obtained earlier. It did however, settle with less overshoot (4%). The settling time was unchanged.
For a 2 cm step input. It can be seen the response is much better than the one obtained in the previous chapter. This time the magnet position is settling with 3.83% overshoot in 610 ms, an improvement of 2.47% and 200 ms respectively.

![Adaptive Controller Response for 2 cm Height](image)

Figure 6.4: Step Response of Adaptive Controller for 3 cm Input

Compare now the results from the adaptive controller with the PID controller designed in Chapter 5 for a 1 cm input.

![Adaptive Controller for 1 cm Input](image)  ![PID Response for 1 cm Input](image)

(a) 1 cm Adaptive Controller Response  (b) 1 cm PID Response

Figure 6.5: Comparison of Adaptive Controller and PID Responses for a Height of 1 cm

The response is a lot better, it settles much faster and with less overshoot.

For a 1.5 cm input, the response is much the same. It exhibits a nice critically damped
response with minimal overshoot.

The next set of simulations tested the model for small step changes in operating condition. The input signal steps from 2 cm to 2.5 cm, 2.5 cm to 3 cm, 3 cm to 2.5 cm and from 2.5 cm back to 2 cm. The results can be seen in the below figure. It can be seen that the controller performs better when the step change is in the positive direction, that is, the height increases. It settles much more gradually and with an overshoot of less than 1%. The peak position is only around 0.2 mm above the desired position. When stepping down however, the overshoot is not insignificant, and the position goes past the target by about 12%. In both cases however, the system is much quicker to respond, it reaches within 2% of the desired position in less than 100 ms.
The system response to a large step change was also simulated. Figure 6.8 shows the results of stepping from 0 cm to 3 cm, 3 cm to 1.5 cm, 1 cm to 1.5 cm and from 1.5 cm to 3 cm. For larger step changes, the system takes longer to settle. This suggests that the controller has a poor disturbance rejection and cannot cope with large parameter variations after it has settled. This is not unusual, the controller was designed to meet a certain transient behaviour, with no thought for how disturbances would impact it. A PID controller will always have a trade-off between transient response and disturbance rejection, and the design should be made meet the requirements of both as much as possible, and this was not considered.
The result doesn’t appear to be that much better than for the single PID controller.

It is also worth noting now how the control effort reacts in response to these changes in operating conditions. As can be seen from Figure 6.9, the required control effort to bring the magnet to position, changes with each change in input. It also never reached the limits of unsafe operation.
6.4 Summary of Results

The results are slightly contrary to what was expected. It was expected that the system response would display characteristics similar to the PID response obtained in chapter 5, at each of the operating conditions. The system responds better in terms of settling time, and the controller fares much better when the desired position increases rather than decreases. For the most part, the controller performs admirably at all operating conditions, but has poor disturbance rejection and cannot cope with large changes in its operating conditions, which suggests that a PID controller is probably not the best option. They have a fixed robustness that enables them to resist small changes in operating condition, but it is nearly impossible to achieve a particular response with a good degree of precision, as is required here, and for most position control problems. Also, as was stated, what a PID controller can achieve in transient behaviour is compromised in its disturbance rejection qualities.

Other possible controller options are discussed in the concluding chapter.

There is always a hint of uncertainty in results of simulation, and the accurateness cannot be determined until appropriate experimentation is performed.
To fully determine the suitability of this controller, and to verify the simulation results, the algorithm would need to be implemented on the ECP Model 730 system and tested. It was originally intended that testing of the design would occur, but due to time constraints, issues with the simulations and the expected complexity of implementing this controller it was not able to be performed. This would be a logical ground for immediate future work.

6.5 Chapter Summary

This chapter detailed the development and implementation of a piecewise linear model for the nonlinear magnetic levitation system. The simulation was performed and the piecewise model successfully implemented. An adaptive controller based on a gain scheduling technique was also developed and the simulation performed. The results were discussed and possible explanations were provided.
Chapter 7

Conclusions and Future Work

7.1 Conclusions

This research project set out to investigate magnetic levitation and possible adaptive control schemes for controlling the position of magnetically levitated systems. The project focussed much of the research on the ECP Model 730 Development System and its features were studied. Two potential controllers were designed and simulated using Simulink and the results presented.

Initially, a broad literature review was conducted to gain an understanding of relevant theory, applications and to identify what previous work in the area had already been done. The literature review covered magnetism and magnetic field theory, the principle of magnetic levitation and possible applications of magnetic levitation technologies. Also covered in depth was the concept of nonlinear systems theory, and the real world behaviour of such systems. This then identified the need for adaptive control and means for achieving this were studied and discussed. The research emphasised the sizeable gap in knowledge of adaptive control and that nonlinear control theory is a complex and challenging field. Finally, it highlighted what attempts had been made at providing suitable position control to the ECP Model 730 magnetic levitation system. Research uncovered two potential control strategies that may be appropriate.

The first steps at designing a controller were then undertaken. This included finding the nonlinear differential equation that describes the dynamics of the system and developing
Future Work

A suitable system model was developed in Simulink. The system was then linearised around a suitable operating point using Taylor’s Series Expansion and the system transfer function was found at this point. The uncompensated system response mirrored that of an undamped second order system and reiterated the need for suitable position control.

The first controller designed was a regulation PID controller. The parameters were tuned using a technique in MATLAB that was based on error minimisation. The designed system and controller were modelled in Simulink, and showed reasonable performance characteristics; forcing the system to settle at 2.5 cm in 790 ms with 5.6% overshoot. Based on the research conducted, it was found that the controller designed was only going to provide adequate control for small perturbations around this operating point. This characteristic was modelled and verified to an extent. PID controllers do have a certain amount of robustness that means they are able to resist small changes in operating environment. The designed controller was found to be accurate to around 0.5 to 1 cm beyond the desired operating condition, beyond which linearisation errors started to occur.

It was also found that the system is piecewise linear, and that it can linearised at a wide range of operating points and be accurately approximated. The system was then linearised over a safe range of operation and a method of implementing this linear approximation was designed in Simulink.

The second controller designed followed on from the PID controller and piecewise implementation and consisted of devising a gain schedule to update the PID parameters based on the desired operating condition. A total of 8 different PID controllers were designed and implemented in Simulink by means of a Look-Up Table algorithm. Simulation of the designed system found an initial improvement in settling time and overshoot. Further tests uncovered found that the controller can adapt itself to operating point changes, but the responsiveness is not much better than a single PID controller, and for big step changes the overshoot was significant.

Future Work

It was initially hoped that the designed control strategy could be implemented and tested experimentally on the ECP Model 730, but strict time constraints and unforeseen problems in simulation prevented this from occurring. Therefore the logical next step for this
7.2 Future Work

project would be to develop the code to implement this strategy onto the physical system and examine its performance. If errors were found than the model could be reworked as appropriate. After testing of this occurred, it would be interesting to see if the design could hold up for EMS levitation, and how it might perform under the MIMO mode of operation.

Additionally, it would be worthwhile designing an MRAC control system as the next step and see how that performs. The controller could be made track a certain reference model to say, a deadbeat response and adapt its gains according to the error present.

Furthermore, the Robust Deadbeat Controller designed by Clarke (2012) could be implemented in the gain schedule instead of a standard PID controller. They are known to exhibit a certain amount of robustness as it is, so it would be interesting to see the performance improvement if the gains were to be adapted.
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Appendix A

Project Specification
TOPIC: Piecewise Adaptive Controller Design for Position Control of a Magnetic Levitation System

SUPERVISOR: Assoc Prof Paul Wen

PROJECT AIM: To investigate, develop and implement control strategies for controlling the position of the ECP 730 Magnetic Levitation Development System.

PROGRAMME: ISSUE A 18 March 2015

1. Investigate the ECP Model 730 hardware and software
2. Test the system and study the existing demo program and experiments
3. System model identification and linearisation
4. Design and simulation of a PID controller for the SISO MagLev system
5. Piecewise system model development and implementation
6. Design piecewise adaptive controllers for the levitation and guideline subsystems
7. Simulate and improve the design using MATLAB and Simulink
8. Implement, test and evaluate the design using the ECP Model 730 Development System
9. Demonstrate the designs and recommend experiments for USQ control courses ELE2103 and ELE3105 (Optional, as time permits)

AGREED:
<table>
<thead>
<tr>
<th>Name</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student</td>
<td></td>
</tr>
<tr>
<td>Supervisor</td>
<td></td>
</tr>
<tr>
<td>Examiner/Co-Examiner</td>
<td></td>
</tr>
</tbody>
</table>
Appendix B

MATLAB Code for Simulation
function [i] = calc(y)

% This function will accept an input of desired magnet height in cm output the
% required control effort in counts
F = 0.12*9.81;
b = 6.2;
n = 4;
a = 1.631;
i = F*(a*(y+b).^n);
end

%% This script will plot the nonlinear approximation of the actuator - calls the calc function
clear all
close all
clc

%% System Constants
m = 0.12;%mass of magnet, given in manual
g = 9.81;%gravitational acceleration
F = m*g;%magnet weight
b = 6.2;%taken from ECP manual
n = 4;%typical value (3<n<4.5)

%% Experimental data
y_measured = [0.5 0.9 1.2 1.9 2.2 2.7 3.2 3.7 4.2];%measured disk displacement
i_measured = [4000 5000 6000 8000 10000 12000 14000 18000 22000];%measured control effort (current)

%% Variable approximations
a = mean(i_measured./(F.*(y_measured+b).^n));%4th order approximation using measured values

%% Calculated values
y = 0:0.001:4.2;%calculate displacements from 0 - 4.2
i_calculated = F*(a*(y+b).^n);%fourth order approximation

%% Plot results
plot(y_measured,i_measured,'x'),title('Measured Actuator Characteristics'),xlabel('Magnet position from rest (cm)'),ylabel('Control effort (counts)')%plot measured points
figure
plot(y,i_calculated,y_measured,i_measured,'x'),title('Nonlinear actuator characteristics'),xlabel('Magnet position from rest (cm)'),ylabel('Control effort (counts)')%compare measured v approximated
legend('4th Order Approximation','Measured Values')
B.2 Code for PID Optimisation

%% This script will determine a suitable PID controller for the system

clear all
close all
clc

%% System Constants
a = 1.631;
b = 6.2;
m = 0.12; % mass of magnet

g = 9.81; % gravitational acceleration
y0 = 2.5; % linearisation point 2.5 cm
u0 = 11000; % input at linearisation point

opt = stepDataOptions('StepAmplitude',2.5); % set step parameters - input 2.5

k1 = (4*u0)/(a*(y0+b)^5); % 1st linearisation constant
k2 = 1/(a*(y0+b)^4); % 2nd linearisation constant - actuator gain

ksensor = 1*10^6; % sensor gain, taken from ECP manual
ksystem = k2*ksensor; % product of all gains in system except controller
B.2 Code for PID Optimisation

```matlab
%% Linearised Transfer function
num = (ksystem/m);
den = ([1 0 k1*100/m]);

G = tf(num,den) %open loop transfer function
step (G,2,opt) %obtain step response
title('Open Loop Step Response of ECP 730')
ylabel('Levitated Magnet Height (cm)')
figure
rlocus(G);

H = 1; %unity feedback

T = feedback(G,H) %closed loop transfer function
figure
step(T,2,opt) %closed loop step response
figure
rlocus(T);

%% PID Design
Kp = 1; %Proportional gain - initially 1
Kd = 0; %Integral gain - initially 0
Ki = 0; %Derivative gain - initially 0

C = pid(Kp,Ki,Kd); %define controller
TC = feedback(C*G,H); %closed loop step response of plant plus controller

[y,t] = step(TC,opt); %initialise array of values for plotting
sserror = abs(2.5-y(end)); %calculate steady state error

while sserror > 0.0005 %error close to 0 - loop until true
    Kp = Kp + 0.2; %increase Kp a little
    Kd = Kd + 0.001; %add small amount of derivative
    Ki = Ki + 0.5; %add more integral - dampen system
    C = pid(Kp,Ki,Kd); %recalculate controller
    TC = feedback(C*G,H); %recalculate closed loop transfer function
    [y,t] = step(TC,opt); %output step response
    sserror = abs(2.5-y(end)); %calculate steady state error
end
```

```
characteristics = stepinfo(y,t); %output response characteristics
ST = characteristics.SettlingTime; %define settling time from characteristics
OS = characteristics.Overshoot; %define overshoot from characteristics

if OS > 3 %design criterion - overshoot less than 3%
    Kd = Kd + 0.01; %add some derivative gain
    TC = feedback(C*G,H); %recalculate transfer function
end
end

TC = feedback(C*G,H)

%% Plot
figure
step (TC,2,opt) %PID response
figure
rlocus(TC)

display 'The PID parameters are:', disp(Kp),disp(Ki),disp(Kd) %output values to command window

function [G] = piecewise(y0)
%This function will accept a linearisation point and output the
%corresponding system transfer function at that point
%
 a = 1.631;
b = 6.2;
m = 0.12;%mass of magnet
g = 9.81;%gravitational acceleration

u0 = calc(y0);

k1 = (4*u0)/(a*(y0+b)^5);%1st linearisation constant
k2 = 1/ (a*(y0+b)^4);%2nd linearisation constant - actuator gain

ksensor = 1*10^6;%sensor gain, taken from ECP manual
ksystem = k2*ksensor;%product of all gains in system except controller
num = (ksystem/m);
den = ([1 0 k1*100/m]);

G = tf(num,den); %open loop transfer function
end

%% This script will determine a linearised transfer function for each operating point
clear all
close all
clc

G_A = piecewise(0.5)
G_B = piecewise(1.0)
G_C = piecewise(1.5)
G_D = piecewise(2.0)
G_E = piecewise(2.5)
G_F = piecewise(3.0)
G_G = piecewise(3.5)
G_H = piecewise(4.0)

B.4 Code for Adaptive Controller Design

function [TC] = apid(y0)
%% This function will output the P,I,D parameters for any input y0

G = piecewise(y0);

step (G,2); %obtain step response

H = 1; %unity feedback

T = feedback(G,H); %closed loop transfer function

step(T,2); %closed loop step response

Kp = 1; %Proportional gain - initially 1
Kd = 0; %Integral gain - initially 0
Ki = 0; %Derivative gain - initially 0
opt = stepDataOptions('StepAmplitude',y0); %set step parameters - input 2

C = pid(Kp,Ki,Kd); %define controller
TC = feedback(C*G,H); %closed loop step response of plant plus controller

[y,t] = step(TC,opt); %initialise array of values for plotting
sserror = abs(y0-y(end)); %calculate steady state error
%step(TC,opt)

while sserror > 0.0005 %error close to 0 - loop until true

Kp = Kp + 0.2; %increase Kp a little
Kd = Kd + 0.001; %add small amount of derivative
Ki = Ki + 0.5; %add more integral - dampen system

C = pid(Kp,Ki,Kd); %recalculate controller
TC = feedback(C*G,H); %recalculate closed loop transfer function

[y,t] = step(TC,opt); %output step response
sserror = abs(y0-y(end)); %calculate steady state error

characteristics = stepinfo(y,t); %output response characteristics
ST = characteristics.SettlingTime; %define settling time from characteristics
OS = characteristics.Overshoot; %define overshoot from characteristics

if OS > 3 %design criterion - overshoot less than 3%
    Kd = Kd + 0.01; %add some derivative gain
    %TC = feedback(C*G,H); %recalculate transfer function
end
end

TC = feedback(C*G,H);

%% Plot
step (TC,2,opt) %PID response

display 'For a operating point of', disp(y0)
display 'The PID parameters are:', disp(Kp),disp(Ki),disp(Kd) %output values to command window

end
This script will calculate the PID controller parameters at each point:

```matlab
clear all
close all
clc

apid(0.5) %PID controller at 0.5 cm point
figure
apid(1.0) %PID controller at 1 cm point
figure
apid(1.5) %PID controller at 1.5 cm point
figure
apid(2.0) %PID controller at 2 cm point
figure
apid(2.5) %PID controller at 2.5 cm point
figure
apid(3.0) %PID controller at 3 cm point
figure
apid(3.5) %PID controller at 3.5 cm point
figure
apid(4.0) %PID controller at 4 cm point
```
Appendix C

Simulink Files

C.1 Dynamic Model of Maglev Plant
C.2 PID Controller System Model

Full PID Controller Model - Over Page
C.2 PID Controller System Model
C.3 Piecewise Adaptive Controller System Model

Piecewise Model used in the Adaptive Controller Model - Over Page
Appendix D

ECP Model 730 Datasheet
ECP’s unique MagLev apparatus dramatically demonstrates closed loop levitation of permanent and ferromagnetic elements. The apparatus includes laser feedback and high flux magnets to affect large displacements and provide visually stimulating tracking and regulation demonstrations. The system is quickly set up in the open loop stable and unstable (repulsive and attractive fields) configurations shown. By adding a second magnet and driving both actuators, MIMO control is studied. The inherent magnetic field nonlinearities may be inverted via provided real-time algorithms for linearizing the system in SISO operation. An optional turntable accessory provides graphic demonstration of induced field levitation - the principal used in high speed bullet trains!

**Provides A Variety Of SISO & MIMO, Stable & Unstable Plant Configurations**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Repulsive Levitation</th>
<th>Attractive Levitation</th>
<th>MIMO Levitation, Locally Open Loop Stable</th>
<th>MIMO Levitation, Open Loop Unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ic</td>
<td>Ic</td>
<td>Ic</td>
<td>Ic</td>
<td>Ic</td>
</tr>
<tr>
<td>Equations of Motion</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = \frac{m g}{k y} + c y = F_{nl} - mg</td>
<td>y = \frac{m g}{k y} + c y = F_{nl} - mg</td>
<td>m_1 y + c y_1 = F_{nl1} + F_{nl2} + F_{nl3} + m_1 g</td>
<td>m_1 y + c y_2 = F_{nl1} + F_{nl2} + F_{nl3} + m_1 g</td>
<td></td>
</tr>
<tr>
<td>Linearized Forms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = \frac{m g}{k y} + c y + k y' = k_1 y</td>
<td>y = \frac{m g}{k y} + c y + k y' = k_1 y</td>
<td>m_1 y + c y_1 = F_{nl1} + F_{nl2} + F_{nl3} + m_1 g</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transfer Function</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y = \frac{k y}{I_c} + \frac{m g}{k y} + c y</td>
<td>Y = \frac{k y}{I_c} + \frac{m g}{k y} + c y</td>
<td>m_1 y + c y_1 = F_{nl1} + F_{nl2} + F_{nl3} + m_1 g</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Notation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F_{nl1} = \frac{k_1}{(y+c y)N_{cm}^2}</td>
<td>F_{nl2} = \frac{k_1}{(y+c y)N_{cm}^2}</td>
<td>F_{nl3} = \frac{k_1}{(y+c y)N_{cm}^2}</td>
<td>F_{nl4} = \frac{k_1}{(y+c y)N_{cm}^2}</td>
<td></td>
</tr>
<tr>
<td>F_{nl5} = \frac{k_1}{(y+d y)N_{cm}^2}</td>
<td>F_{nl6} = \frac{k_1}{(y+d y)N_{cm}^2}</td>
<td>F_{nl7} = \frac{k_1}{(y+d y)N_{cm}^2}</td>
<td>F_{nl8} = \frac{k_1}{(y+d y)N_{cm}^2}</td>
<td></td>
</tr>
<tr>
<td>&quot;c denotes value relative to some equilibrium pt. k_{cm}, d, &amp; N_{cm} are positive constants, (N_{cm} = 4)</td>
<td>&quot;c denotes value relative to some equilibrium pt. k_{cm}, d, &amp; N_{cm} are positive constants, (N_{cm} = 4)</td>
<td>&quot;c denotes value relative to some equilibrium pt. k_{cm}, d, &amp; N_{cm} are positive constants, (N_{cm} = 4)</td>
<td>&quot;c denotes value relative to some equilibrium pt. k_{cm}, d, &amp; N_{cm} are positive constants, (N_{cm} = 4)</td>
<td></td>
</tr>
</tbody>
</table>

**Diagram Description:**

- **Model 730 MagLev Apparatus Fact Sheet**
- **Laser Conditioning Electronics** provides high resolution position measurement.
- **Rare Earth Magnets** provide magnetic fields for levitation.
- **Drive Coils** generate repulsive and attractive fields.
- **Top & bottom Laser Feedback** and **Sensors** ensure accurate tracking.
- **Low Friction Guide** allows smooth motion.
- **Guide Magnet Storage** safely stores high strength magnets.
- **Attachment For Turntable** demonstrates induced field levitation.
- **MIMO Levitation, Open Loop Unstable** configuration shows the apparatus dramatically demonstrating closed loop levitation of permanent and ferromagnetic elements.
Appendix E

Milestones and Timelines
<table>
<thead>
<tr>
<th>Milestone</th>
<th>Expected Time To Complete</th>
<th>Timeline</th>
<th>Progress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investigate the ECP Model 730 hardware and software</td>
<td>2 - 3 weeks</td>
<td>Complete by end of March</td>
<td>Done</td>
</tr>
<tr>
<td>Study the existing demo program and experiments</td>
<td>2 - 3 weeks</td>
<td>Complete by mid-April</td>
<td>Done</td>
</tr>
<tr>
<td>System Model Identification and Linearisation</td>
<td>4 weeks</td>
<td>Complete by mid-May</td>
<td>Done</td>
</tr>
<tr>
<td>Design and Simulation of a PID controller for maglev system</td>
<td>4 -5 weeks</td>
<td>Complete by end of June</td>
<td>Done</td>
</tr>
<tr>
<td>Piecewise System Model development and implementation</td>
<td>3 weeks</td>
<td>Complete by mid-July</td>
<td>Done</td>
</tr>
<tr>
<td>Design and Simulation of Piecewise Adaptive Controller</td>
<td>5 - 6 weeks</td>
<td>Complete by end of September</td>
<td>Done</td>
</tr>
<tr>
<td>Implement, test and evaluate the designs using the ECP Model 730 developement system</td>
<td>4 weeks</td>
<td>Complete by mid-October</td>
<td>Not Done - Out of time</td>
</tr>
<tr>
<td>Further simulation and improvement of designs using MATLAB and Simulink</td>
<td>3 weeks</td>
<td>Complete as time permits</td>
<td>If time permits</td>
</tr>
<tr>
<td>Compile Dissertation</td>
<td>4 - 6 months</td>
<td>Complete by 29 October</td>
<td>Done</td>
</tr>
</tbody>
</table>
Appendix F

Risk Assessment
<table>
<thead>
<tr>
<th>Identified Hazard</th>
<th>Electric Shock from maglev plant and associated equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Level</td>
<td>Significant - if the system is over-energised the electromagnets may give off a mild electric shock</td>
</tr>
<tr>
<td>Likelihood of Occuring</td>
<td>Significant - possible, if system is incorrectly operated</td>
</tr>
<tr>
<td>Exposure</td>
<td>Regularly - equipment will be utilised once or twice a week</td>
</tr>
<tr>
<td>Consequences</td>
<td>Major equipment/component damage Major injury - electric shock if large enough can cause problems with vital organs e.g. heart and brain</td>
</tr>
<tr>
<td>Mitigation Strategies</td>
<td>1. Restrict access and use to trained personnel only (locked in laboratory) 2. Only operate equipment as it was designed for (as instructed in the manual) 3. Follow all OH&amp;S procedures regarding tag-outs etc.</td>
</tr>
</tbody>
</table>
**Table F.2: Risk of Hand Injury**

<table>
<thead>
<tr>
<th>Risk Assessment</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Identified Hazard</td>
<td>Unstable operation of ECP 730 equipment - magnet may oscillate uncontrollably</td>
</tr>
<tr>
<td>Risk Level</td>
<td>Slight - possible but unlikely if operated correctly</td>
</tr>
<tr>
<td>Likelihood of Occurring</td>
<td>Significant - may occur during initial testing</td>
</tr>
<tr>
<td>Exposure</td>
<td>Regularly - equipment will be utilised once or twice a week</td>
</tr>
<tr>
<td>Consequences</td>
<td>Minor equipment/component damage if left operating in that state. Minor injury - bruising and pinching to hands may occur if placing hands near equipment during operation</td>
</tr>
<tr>
<td>Mitigation Strategies</td>
<td>1. Restrict access and use to trained personnel only (locked in laboratory) 2. Keep hands away from system when operating. 3. Switch off power immediately if unstable operation occurs</td>
</tr>
</tbody>
</table>
Table F.3: Risk of Laser Radiation

<table>
<thead>
<tr>
<th>Risk Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identified Hazard</td>
</tr>
<tr>
<td>Laser Radiation side-effects</td>
</tr>
<tr>
<td>Risk Level</td>
</tr>
<tr>
<td>Slight - possible but unlikely, low powered laser source</td>
</tr>
<tr>
<td>Likelihood of Occuring</td>
</tr>
<tr>
<td>Slight - follow all safety regulations when operation, refer to manual</td>
</tr>
<tr>
<td>Exposure</td>
</tr>
<tr>
<td>Regularly - equipment will be utilised once or twice a week</td>
</tr>
<tr>
<td>Consequences</td>
</tr>
<tr>
<td>Major injury - lasers can cause severe damage to eyes, and have been known to cause cancer</td>
</tr>
<tr>
<td>Mitigation Strategies</td>
</tr>
<tr>
<td>1. Restrict access and use to trained personnel only (locked in laboratory)</td>
</tr>
<tr>
<td>2. Wear eye protection when operating equipment.</td>
</tr>
<tr>
<td>3. Never look directly at laser source</td>
</tr>
<tr>
<td>4. Limit usage as much as is practical</td>
</tr>
</tbody>
</table>
## Table F.4: Risk of Eye Fatigue

<table>
<thead>
<tr>
<th>Risk Assessment</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Identified Hazard</td>
<td></td>
</tr>
<tr>
<td>Eye Strain from computer work - simulations, research, writting.</td>
<td></td>
</tr>
<tr>
<td>Risk Level</td>
<td></td>
</tr>
<tr>
<td>Significant - numerous amounts of work involved in front of a computer</td>
<td></td>
</tr>
<tr>
<td>Likelihood of Occuring</td>
<td></td>
</tr>
<tr>
<td>Substantial - as deadlines draw closer more time is expected (in excess of 8 hours per day)</td>
<td></td>
</tr>
<tr>
<td>Exposure</td>
<td></td>
</tr>
<tr>
<td>Occasionally - whilst computer work will always be done, long periods are only expected closer to deadlines</td>
<td></td>
</tr>
<tr>
<td>Consequences</td>
<td></td>
</tr>
<tr>
<td>Minor injury - eye strain, blurred vision, fatigue, headaches are expected</td>
<td></td>
</tr>
<tr>
<td>Mitigation Strategies</td>
<td></td>
</tr>
<tr>
<td>1. Limit usage to a few hours per day</td>
<td></td>
</tr>
<tr>
<td>2. Plan time better so less required at end</td>
<td></td>
</tr>
<tr>
<td>3. Take time out to get fresh air - rest eyes, refresh brain</td>
<td></td>
</tr>
<tr>
<td>4. Get plenty of sleep</td>
<td></td>
</tr>
</tbody>
</table>