TERRORIST CHOICE:
A STOCHASTIC DOMINANCE AND PROSPECT THEORY ANALYSIS

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The paper explores terrorist choice by applying two well-known theoretical frameworks: stochastic dominance and prospect theory. We analyse each pair of attack methods that can be formed from the RAND-MIPT database and the Global Terrorism Database (GTD). Instances of stochastic dominance are identified. Prospect theory orderings are computed. Attention is accorded to the identification of 'trigger points' and the circumstances that may lead to an increased likelihood that a terrorist will select an attack method associated with a higher expected number of fatalities, i.e. a potentially more damaging attack method.

Keywords: Terrorist Choice, Prospect Theory, Stochastic Dominance, Attack Methods, Trigger Points, Fatalities.

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I. Introduction

The attack methods from which a terrorist chooses are prospects, the most immediate outcomes of which are the fatalities that are inflicted. Hence, in this paper, a prospect is an attack method that is expected to inflict fatalities x_i with probability p_i, where $(p_1 + p_2 + \cdots + p_n = 1)$. There are at least several ways to predict which prospect a terrorist will choose when confronted with a pair of risky prospects. Cumulative prospect theory (CPT) is one approach. Others include expected value, expected utility and stochastic dominance. Each method has strengths and weaknesses. Each requires different information about the decision-maker and the prospects under consideration. Stochastic dominance has found widespread application in economics because of its generality (Hadar and Russell 1969; Hanoch and Levy 1969; Rothschild and Stiglitz 1970 & 1971; Meyer 1977a & 1977b; Levy 1992). CPT is consistent with stochastic dominance (Tversky and Kahneman 1992; Levy and Wiener 2013). Our analysis draws on both of these approaches and the common ground between them.

A law enforcement problem lies at the heart of the analysis and provides the motivation for it: When will a terrorist choose a higher risk, higher expected impact attack method? Or, more succinctly, where are the 'trigger points' at which the terrorist who initially selects the less risky attack method switches to the more risky alternative? We reduce this complex problem to some basic components and examine the terrorist's choice between a pair of attack methods where one attack method is more risky than the other[1]. Because there is a generally positive relationship between the average number of fatalities inflicted by terrorist attacks and the variability or risk of those outcomes, it is important to study the circumstances under which terrorists may choose the more risky of two alternatives. Two aspects of the decision-making process are singled out: (1) reference points; and (2) cumulative distribution intersection points. Both of these aspects of the decision-making process are fundamental to answering the questions posed above.

We start by analysing different pairs of attack methods by first-order (FSD) and second-order (SSD) stochastic dominance. We also apply CPT to each pair of attack methods. For each approach, basic orderings of the attack methods within each pair are determined. We draw on recent advances in the economic theory of risk-taking and incentives to explain why a reordering of the FSD-SSD rankings of prospects may be observed when certain (convex) incentive structures apply to terrorist actions. In particular, we identify the outcomes, x_i, to which these incentives must apply in order to compel a terrorist—the terrorist group member or members—to choose the potentially more (or less) damaging attack method. We note that in most cases where a trigger point exists, even a terrorist with a relatively low reference point will already have accorded a higher prospect value to the higher risk attack method. The terrorist will prefer it in the absence of any additional incentives.

[1] When is something more risky than before? And when is something more risky than something else? There is not a straightforward answer to these questions and, unfortunately, no definitive 'proof' that one definition of 'riskier' is better than another definition (Rothschild and Stiglitz 1970, p.226-227). According to Rothschild and Stiglitz (1970), possible answers to the question include (1) Y is riskier than X when Y is equal to X plus some *noise*; (2) Y is riskier than X if every person who is risk averse prefers X to Y; (3) Y is riskier than X if Y has more weight in the tails of its probability distribution; (4) Y is riskier than X if Y has a greater variance or standard deviation than X.

Built on the foundation provided by Becker (1968), Ehrlich (1973), Landes (1978) and Sandler *et al*. (1983), the accumulated results from the application of economic theory to the study of terrorist choice now encompass such important topics as: (1) the strategic behaviour of terrorists (Sandler *et al*. 1983; Atkinson *et al*. 1987; Lapan and Sandler 1988; Sandler and Scott 1987; Rosendorff and Sandler 2004; Siqueira and Sandler 2006); (2) explanations for the stylised facts characterising the time series of terrorist actions (Enders and Sandler 2002; Enders and Sandler 2005); (3) terrorist choice of attack targets and attack methods (Barros *et al*. 2007; Brandt and Sandler 2010; Santifort *et al*. 2013; Phillips 2009; 2011) and; (4) the implications of particular aspects of choice, especially substitution and deterrence (Frey and Luechinger 2003; Phillips 2013). Reviews are contained in Sandler and Enders (2004), Intriligator (2010) and Schneider *et al*. (2014). A sketch of the 'new frontiers' for terrorism research is contained in Sandler (2011). A summary of the application of 'rational choice' models to the analysis of terrorism is contained in Anderton and Carter (2005). Behavioural models, especially Tversky and Kahneman's (1992) CPT, have recently been applied in ways that complement the existing orthodox approaches (Butler 2007; Phillips and Pohl 2014). This paper uses both orthodox and behavioural approaches to contribute to the ongoing research program.

Our contribution is organised as follows. Section II presents the theory that underlies the analysis. This includes CPT and the stochastic dominance criteria. Section III presents an analysis of the RAND-MIPT and Global Terrorist Database (GTD) attack methods. Instances of FSD and SSD are identified. Section IV presents an analysis of the 'trigger points' or those outcomes that are associated with a higher likelihood that the decision-maker will select the more risky alternative from a given pair. These are identified from both the CPT and stochastic dominance perspectives. Section V concludes the paper.

II. Theory

Kahneman and Tversky (1979) developed prospect theory (PT) and later revised it (Tversky and Kahneman 1992). The revised version is called *cumulative* prospect theory (CPT) to highlight its use of the 'cumulative functional' which transforms cumulative probabilities rather than individual probabilities (Tversky and Kahneman 1992, p.298). The theory is based on behaviour observed in economics experiments where individuals are asked to choose between different risky prospects. Several violations of orthodox expected utility theory were observed in experiments. These violations are called *biases*. The main types of behavioural bias encompassed by prospect theory are:

(1) Decision-makers assess risky prospects relative to a reference point rather than in absolute terms. The decision is *reference dependent*.

(2) Decision-makers are risk-averse in the domain of gains and risk seeking in the domain of losses. They prefer to take risk to avoid a certain loss but will tend to avoid risk if a gain has been secured.

(3) Losses loom larger than gains. A loss of some magnitude will weigh more heavily on the decision-maker than a gain of the same magnitude. The decision-maker is *loss averse*.

(4) Outcomes that are a long way from the reference point have less impact on the assessment of a prospect's value. This is called *diminishing sensitivity*.

Formally, the theory depicts the decision-maker as assigning a prospect value, V, to each risky prospect by the application of the value function,

$$V = \sum v(\Delta x_i)\,\pi(p_i) \tag{1}$$

Values, v, are assigned to outcomes x_i by a decision-maker who assesses the outcomes relative to a reference point and whose behaviour exhibits risk seeking or risk aversion depending on whether an outcome is above or below the reference point,

$$v(\Delta x) = \begin{Bmatrix} v(\Delta x)^{\alpha} & \Delta x \geq 0 \\ -\lambda(-\Delta x)^{\beta} & \Delta x < 0 \end{Bmatrix} \tag{2}$$

Unlike expected utility theory, values are not directly weighted by their probability of occurrence. Rather, the cumulative probabilities themselves are weighted by π before being applied. The probability weighting functions are the most complex component of prospect theory,

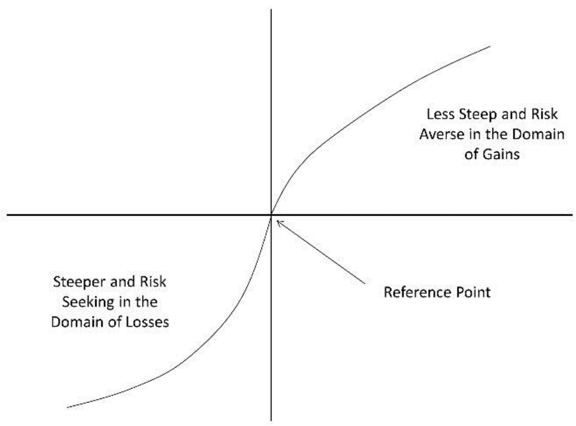
$$\pi_i = \begin{Bmatrix} \pi_i^- = w^-(p_1 + \cdots + p_i) - w^-(p_1 + \cdots + p_{i-1}) & \Delta x < 0 \\ \pi_i^+ = w^+(p_i + \cdots + p_N) - w^+(p_{i+1} + \cdots + p_N) & \Delta x \geq 0 \end{Bmatrix} \tag{3}$$

$$= \begin{Bmatrix} w^+(p) \coloneqq \frac{(p)^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}} & \Delta x \geq 0 \\ w^-(p) \coloneqq \frac{(p)^{\delta}}{\left(p^{\delta}+(1-p)^{\delta}\right)^{1/\delta}} & \Delta x < 0 \end{Bmatrix} \tag{4}$$

Prospect theory yields the S-shaped utility function depicted in Figure 1. It shows a utility function that inflects at the reference point such that a concave or risk-averse segment traverses the domain of gains and a convex or risk seeking segment traverses the domain of losses.

Figure 1

The S-Shaped Utility Function of Prospect Theory



The precise shape of the utility function depends on the values of the parameters in equations (2) and (4). Estimates of the parameter values differ. Tversky and Kahneman (1992, p.311) report a set of estimates. These are the values that we use in our analysis. Other estimates have been contributed by Camerer and Ho (1994), Tversky and Fox (1995), Wu and Gonzalez (1996), Gonzalez and Wu (1999) and Bleichrodt and Pinto (2000). That the values for the parameters must be determined experimentally or inferred from empirical data is perceived to be a weakness of the theory. Another is that the reference point is not observable (Pesendorfer 2006, p.713-714). However, it is possible that reference points may emerge in different contexts that can at least provide some basis for an operationalisation of the theory. For example, Phillips and Pohl (2014) explore the choices of lone wolf terrorists who might reference their actions on the outcomes of terrorist attacks carried out by predecessors. That is, the outcomes of previous attacks provide a plausible set of reference points.

The original version of prospect theory permitted violations of stochastic dominance. Kahneman and Tversky (1979) had assumed that decision-makers would rule out clearly dominated prospects during the 'editing phase'. Tversky and Kahneman (1992) developed CPT to remedy this shortcoming. Tversky and Kahneman (1992, p.299) note, with apparent dissatisfaction, that the assumption of stochastic dominance is one that theorists are 'reluctant to give up'. But this is not surprising given the widespread application of stochastic dominance in economics (see Levy 1992; Bawa 1982). According to Hart (2011, p.617), "Stochastic dominance is a partial order on risky assets (gambles) that is based on the uniform preference—of all decision-makers in an appropriate class—for one gamble over another." We state Levy's (1992, p.555) theorem as a formal expression of stochastic dominance: *Let F and G be the cumulative distributions of two distinct uncertain options (prospects or gambles), X and Y. F dominates G by FSD, SSD, and TSD if and only if*:

$$F(X) \leq G(X) \quad \textit{for all X (FSD)}, \tag{5}$$

$$\int_{-\infty}^{x}[G(t) - F(t)]dt \geq 0 \quad \textit{for all X (SSD)}, \tag{6}$$

$$\int_{-\infty}^{x}\int_{-\infty}^{v}[G(t) - F(t)]dtdv \geq 0 \quad \textit{for all X, and}$$

$$E_F(X) \geq E_G(X) \ \textit{(TSD)} \tag{7}$$

One of the desirable properties of the stochastic dominance criteria is that it can be visualised by plotting the cumulative distributions for the prospects under consideration. Tversky and Kahneman (1986, p.253) explain, "…for unidimensional risky prospects, A is preferred to B if the cumulative distribution of A is to the right of the cumulative distribution of B." For example, consider two risky prospects A and B, each with a cumulative distribution function F and G respectively. The two cumulative distributions are drawn in Figure 2. In this example, prospect A dominates prospect B by FSD and inequality (5) (above) is satisfied. The cumulative distribution for prospect A is located to the right of the cumulative distribution for prospect B. Second-order dominance is also reflected in the relative positions of the cumulative distributions for two prospects. In Figure 3 the cumulative distributions have been drawn such that they intersect each other. FSD does not apply because inequality (5) is violated. However, SSD may still apply. Since SSD may not be obvious from a visual inspection of the cumulative distributions alone, it can be useful to plot the constraint integral (6) as well. In Figure 3 we depict the constraint integral (6) as being always positive (as required for SSD). In this case, F dominates G by SSD.

Figure 2

Cumulative Distributions F and G for Prospects A and B

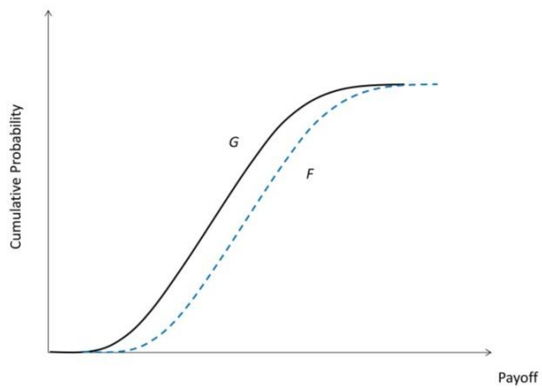
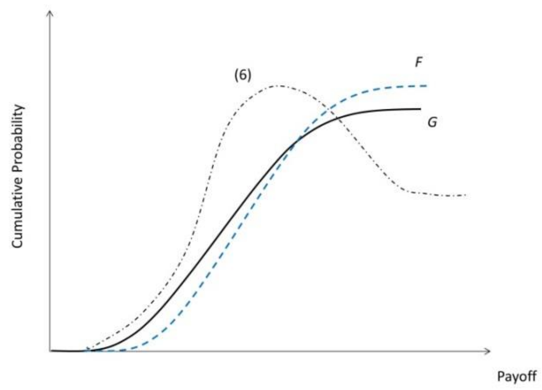


Figure 3

Cumulative Distributions F and G and the Constraint Integral (6)



The analysis presented in this paper concentrates on those points at which a terrorist's selection of the more risky and more potentially damaging attack method from a given pair is triggered. If the terrorist originally chooses the less risky alternative but switches to the more risky alternative at some point, this change cannot violate or be inconsistent with FSD. It will be recalled that CPT is consistent with stochastic dominance and no ordering of prospects by prospect value should violate FSD. It is necessary to show how trigger points may emerge within the decision-making frameworks that have been introduced in this section without violating any of their fundamental tenets. Trigger points can emerge within CPT in situations where the statistical properties of the attack methods are such that they do not permit a clear stochastic dominance ordering. That is, where there is no clear stochastic dominance to violate. We return to this later. Importantly, trigger points can emerge within the stochastic dominance framework even when one prospect clearly dominates another by SSD.

Consider once more the two prospects A and B with respective cumulative distributions F and G depicted in figure 3 and assume that F dominates G by SSD. In this case, G is more risky than F. However, it is easy to see that at points to the right of the intersection of the two cumulative distributions G dominates F by FSD. Past the intersection of the two distributions the cumulative distribution for G is located to the right of the cumulative distribution for F. An incentive structure that censures the two probability distributions such that positive payoffs are received only at or beyond the level of outcome associated with the intersection of the two cumulative probability distributions will ensure that the more risky of the two alternatives is always chosen, *regardless of the preferences of the decision-maker*. Braido and Ferreira (2006) provide a proof of this in their analysis of risk-taking behaviour and options-based compensation schedules, where those compensation schedules are defined by a particular 'strike price' at which payoffs become positive.

Braido and Ferreira (2006) show that there is a 'strike price' (trigger point) that will always entice the decision-maker who has been granted an option to choose the more risky of two alternatives. Braido and Ferreira (2006, p.516) state the familiar stochastic dominance criteria (above). For the case of two risky prospects with cumulative distributions F and G such that $F \neq G$ and F dominates G by SSD, $F \gtrsim_s G$, the authors state a weak technical assumption to rule out distribution functions that display 'pathological behaviours' in the neighbourhood of the upper-bound (T) of payoffs: Defining T_{FG} as the limit point at which the two distributions differ from each other:

$$T_{FG} = min\{\theta \in [0, T]: F(t) = G(t)\} \qquad for\ all\ t \in [\theta, T] \tag{8}$$

Now, there is a neighbourhood of T_{FG} in which the number of times $F(t) - G(t)$ changes sign is finite. Braido and Ferreira (2006, p.516) note that this assumption applies to all standard distributions. They follow with their main proposition, the proof of which is their main result. Their proposition is that under the technical assumption just delineated: there exists a strike price $\hat{s} \in (0, T)$ such that:

$$\int_0^T u(\max(t - \hat{s}, 0))dG(t) > \int_u^T u(\max(t - \hat{s}, 0))dF(t) \tag{9}$$

Braido and Ferreira's (2006, p.516-517) formal argument is illustrated by pointing out that the two cumulative distributions for F and G intersect and that F dominates G by SSD which implies $G(t) < F(t)$ for all $t \in (\hat{s}, T_{FG})$. They point out that… "G always yields higher payoffs than F when the domain is restricted to values higher than $\hat{s}$." That is, for pairs of risky prospects characterised by cumulative distributions F and G where F dominates G by SSD there is a point beyond which G dominates F by FSD.

We are interested in the circumstances under which a terrorist will choose the more risky and potentially more damaging attack method. Prospect theory provides one such clue about where to look for these 'trigger points'. Reference points provide a natural starting point. Because a higher reference point extends the loss domain and, by assumption the risk-seeking domain, more risky choices may be observed more often when decision-makers have relatively high reference points. Another clue is provided by stochastic dominance. Braido and Ferreira (2006) have pointed out the relevance of the intersection points of the two cumulative probability distributions that describe alternative prospects. Intuitively one might think that the trigger points would be the same in both theoretical-statistical frameworks. That is, one might suspect that the reference point beyond which more risky prospects might be accorded higher prospect values than their less risky alternatives would align with the point at which the two cumulative distributions for the two prospects intersect. In the analysis that follows we show that this is not the case. Trigger points within prospect theory, when they exist, occur sooner.

III. Attack Methods

The analysis draws on two different databases: *annual* RAND-MIPT data for the period 1968 to 2007 and *monthly* Global Terrorism Database (GTD) data for the period 2000 to 2008. The RAND database concentrates on the outcomes of *transnational* terrorist attacks. For example, a bombing in one country carried out by a terrorist group based in another country or an attack by a domestic group on an international target within the group's home country[2]. The GTD lists all attacks, including those with both domestic origins and targets. Each database covers a number of attack methods and each lists the outcomes for each attack: the fatalities and injuries that were inflicted. As explained above, the analysis treats an attack method as a risky prospect that is expected to inflict fatalities x_i with probability p_i, where $(p_1 + p_2 + \cdots + p_n = 1)$. Fatalities per attack (per year) are assumed to be log-normally distributed[3]. Using the log-normal hypothesis avoids truncating the distribution at 0 to allow for the fact that expected fatalities cannot be negative. Like other distributions that might be assumed for either of the established terrorism databases, the assumption of a log-normal distribution cannot on its own incorporate the weight that terrorists may apply in their decision-making to the possibility that the planned attack may be pre-empted and the plot foiled before any terrorist operation can be carried out. Collecting these cases and incorporating them into the analysis may be a task for future research[4]. For most attack methods, the log-normality hypothesis *cannot* be rejected when computed over the outcomes of successful attacks (attacks that inflicted at least one fatality). The analysis presented here holds for all standard distribution functions (see Braido and Ferreira 2006, p.516). Descriptive statistics for each of the two datasets are presented in Tables 1 and 2, including the Shapiro-Wilkes test-statistics (W) for log-normality. A Shapiro-Wilkes statistic $W < 0.05$ *rejects* the null hypothesis that the data are log-normally distributed.

[2] An example is an attack on an embassy.

[3] Assassination is an exception. However, the outcomes (fatalities) per 'assassination' attack are *normally* distributed. That is, the normality assumption cannot be rejected for 'assassination'. The Shapiro-Wilkes statistic is 0.245.

[4] The authors thank the referee for this suggestion. These are cases where fatalities are zero, not because the operation failed but because the operation was pre-empted. Such cases, if included in the dataset, would reduce the mean or expected number of fatalities for planned attacks. The potential effects on the preference orderings are unclear. If each type of attack method is subject to a similar proportion of pre-emptions, the relative number of expected fatalities may not be significantly impacted.

Table 1

RAND-MIPT: Fatalities and Variability of Outcomes

Attack Type	Average Fatalities Per Attack Per Year	Standard Deviation	Shapiro-Wilkes (p)
Armed Attacks	1.30	1.122	0.89
Arson	0.32	0.751	0.21
Assassination	1.04	0.387	< 0.001
Hostage	3.59	11.661	0.615
Bombing	1.28	1.545	0.71
Hijacking	1.97	4.994	0.02
Kidnapping	0.39	0.335	0.782

Table 2

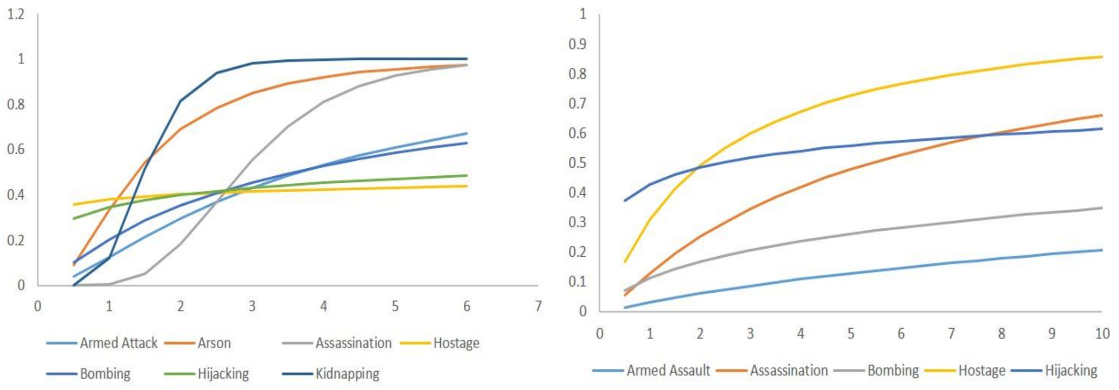
GTD: Terrorist Attack Methods 2000 to 2008

Attack Type	Average Fatalities Per Attack Per Year	Standard Deviation	Shapiro-Wilkes (p)
Armed Assault	50.22	7.94	0.028
Assassination	20.37	5.16	0.006
Bombing	41.97	9.75	0.165
Hostage-Taking	8.81	5.10	0.907
Hijacking	10.77	17.06	0.266

Figure 4 displays the log-normal cumulative distributions for each attack method. For the RAND-MIPT attack methods, few clear examples of FSD can be identified. For the GTD attack methods, 'armed assault' clearly dominates each attack method by FSD. Although 'bombing' is dominated by 'armed assault', it clearly dominates each other attack method by FSD and 'assassination' dominates 'hostage-taking'. Referring specifically to the type of terrorist activity that is encompassed by the GTD data where the dominance results are clearest, it is possible to reach tentative conclusions about the attack methods that would be preferred by risk-averse terrorist decision-makers whose utility functions, $u(x)$, belong to the set of all utility functions such that $u'(x) \geq 0$ and $u''(x) \leq 0$ for all $x \in [a, b]$ (Hadar and Russell 1969). This class of decision-makers will tend to favour 'armed assault' whenever it forms part a pair of attack methods. Similarly, 'bombing' will be preferred by this class of decision-makers when it is paired with any attack method other than 'armed assault' and 'assassination' will be preferred to 'hostage-taking'. In each case, switching to 'armed assault' represents a FSD enhancement.

Figure 4

Cumulative Distributions: (a) RAND-MIPT and (b) GTD



Tables 3 and 4 present the second-order stochastic dominance (SSD) matrices for each pair of attack methods formed from each database. Each SSD matrix can be interpreted as follows. First, each attack method is assigned the row and column position that corresponds to its place in the lists presented in Tables 1 and 2. As such, row 1 and column 1 in Table 3 correspond to 'armed attacks' and so on. Each element of the matrix, a_{ij}, where i and j denote the i^{th} row and j^{th} column, displays the SSD result for each attack method pair i and j. A value of 3 is present where there is no clear SSD ordering. A value of 1 is present when attack method i SSD dominates attack method j. A value of 2 is present when attack method i is SSD dominated by attack method j. A value of 0 simply identifies the fact that an attack method cannot be ordered against itself. For example, in Table 3, $a_{12} = 1$ says that 'armed attacks' SSD dominates 'arson' whereas $a_{15} = 3$ says that there is no clear SSD result between 'armed attacks' and 'bombing' and $a_{25} = 2$ says that 'arson' is SSD dominated by 'bombing'. Table 4 is interpreted in the same way. Although some attack methods dominate others by SSD, it is not true in general that unambiguous orderings are obtainable for each pair. In many cases, the constraint integral (6) is violated and neither attack method dominates. Of course, running into such a problem is one of the main shortcomings of the stochastic dominance method (Hart 2011, p.626). For the RAND-MIPT data, 'arson' and 'kidnapping' are the attack methods most consistently dominated by SSD. For most other pairs there is generally no clear SSD result. For the GTD data the matter is more clear-cut. The SSD results reflect the FSD results that were already apparent from the plots of the cumulative distributions. As we concluded above, 'armed assault' dominates by FSD (and SSD).

Table 3

RAND-MIPT: Stochastic Dominance (SSD) Matrix

	1	**2**	**3**	**4**	**5**	**6**	**7**
1	0	1	3	3	3	3	1
2		0	2	2	2	2	3
3			0	3	3	3	1
4				0	3	3	1
5					0	3	1
6						0	1
7							0

Table 4

GTD: Stochastic Dominance (SSD) Matrix

	1	**2**	**3**	**4**	**5**
1	0	1	1	1	1
2		0	2	1	3
3			0	1	1
4				0	3
5					0

IV. Trigger Points

FSD and SSD provide partial orders over the pairs of attack methods. In some cases, a clear result is obtained. In others, there is not a clearly defined dominance of one attack method over the other. Although this is a shortcoming of the stochastic dominance methodology, when clear dominance results are obtained they apply to a wide range of decision-makers (Fishburn 1964; Hadar and Russell 1969). As we have mentioned before, the original PT permitted violations of FSD. CPT ensures that violations of FSD are not observed. If one attack method dominates the other by FSD it will also be found to have the higher prospect value for all reference points (Levy and Wiener 2013). When there is no clear FSD result but a clear (or unclear) SSD result, prospect theory will accord prospect values that change as the reference point against which the prospects are assessed changes. That is, prospects are reordered as the reference point increases or decreases. CPT permits attack method switching when the terrorist's reference point changes as long as there is no violation of FSD. We want to identify those reference points at which the reordering of attack methods in favour of the more risky alternative takes place.

Table 5 reports the results of an application of CPT to selected attack method pairs formed from the RAND-MIPT data. The prospect values for each attack method are presented and the reference points at which the more risky alternative in a given pair is accorded the higher prospect value are highlighted. For most cases, the trigger points occur at relatively low reference points. That is, the less risky alternative rarely holds its position as having the higher prospect value over a range of more than a few reference points. From a law enforcement perspective, these results acquire some degree of practical significance when it is considered that a reference point may be shaped by the outcomes of previous or recent terrorist attacks. If low-outcome attacks generate low reference points among terrorists planning to carry out subsequent attacks—or 'copycat' attacks[5]—then the attack methods that terrorists select may be more targeted in nature. If, on the other hand, high-outcome attacks generate high reference points, higher risk attack methods might be more likely to be selected for subsequent attacks. This may be the case *regardless* of the attack method used in the earlier attack. Attack methods with more variable outcomes also have higher expected fatalities, in general. They are also, by nature, less targeted or discriminating. The implications are most stark for a pair such as 'assassination' and 'bombing'. For low reference points, assassination will be accorded the higher prospect value. This ordering is reversed for reference points higher than 2 fatalities. From a law enforcement preparation and response perspective, the scenes of these two types of attacks may be quite different. Which is more likely to be experienced may be determined to some extent by the terrorist's reference point. The outcomes of recent attacks might shine some light on this aspect of the terrorist's decision-making profile.

Table 5

RAND-MIPT: Prospect Values and 'Trigger' Reference Points

Attack Method Pairs (i, j)	Prospect Values (i, j)					
	(Ref = 1)	(Ref = 2)	(Ref = 3)	(Ref = 4)	(Ref = 5)	(Ref = 6)
Armed Attacks, Assassination	1.017, 1.172	0.982, 1.0499	0.944, 0.796	0.904, 0.562	0.865, 0.363	0.826, 0.144
Armed Attacks, Bombing	1.017, 1.008	0.982, 0.987	0.944, 0.965	0.904, 0.943	0.865, 0.921	0.826, 0.90
Armed Attacks, Hostage-Taking	1.017, 1.000	0.982, 0.999	0.944, 0.999	0.904, 0.999	0.865, 0.999	0.826, 0.999
Arson, Assassination	1.021, 1.172	0.865, 1.0499	0.732, 0.796	0.6072, 0.562	0.488, 0.363	0.373, 0.144
Assassination, Hostage-Taking	1.172, 1.000	1.0499, 0.999	0.796, 0.999	0.562, 0.999	0.363, 0.999	0.144, 0.999
Assassination, Bombing	1.172, 1.008	1.0499, 0.987	0.796, 0.965	0.562, 0.943	0.363, 0.921	0.144, 0.90
Bombing, Hijacking	1.008, 1.000	0.987, 0.9992	0.965, 0.998	0.943, 0.997	0.921, 0.996	0.90, 0.994
Hijacking, Kidnapping	1.000, 1.079	0.999, 0.5896	0.998, 0.239	0.997, −0.089	0.996, −0.405	0.994, −0.711

To complete the analysis we return to our discussion of stochastic dominance and, in particular, the cumulative distributions that describe the outcomes of each attack method. In our treatment of the relevant theory we highlighted Braido and Ferreira's (2006) result which centred on the conditions under which an application of a convex incentives schedule to risky prospects would compel the decision-maker to select the more risky alternative from a pair of prospects regardless of the decision-maker's preferences. When the cumulative distributions for two prospects intersect—and FSD is ruled out—an incentives schedule exists that will guarantee the selection of the more risky prospect from a pair even if that prospect is

[5] See Phillips and Pohl (2014).

dominated by SSD by the less risky prospect. Once pointed out, it is quite obvious that the cumulative distribution for a prospect dominated by SSD lies to the right of the cumulative distribution of its alternative past the point at which the two cumulative distributions intersect. This 'trigger point', however, remains dormant in the sense that it plays no direct role in shaping the choice of the risk-averse decision-maker. Although the decision-maker may be aware that potentially higher outcomes are in general attainable by the selection of a more risky prospect, there will be other overriding considerations (*i.e.* risk). Braido and Ferreira's (2006) analysis shows that a convex incentives schedule that censures the cumulative distributions at the point of intersection serves to activate the trigger point. When positive payoffs are restricted to outcomes beyond this point, the more risky prospect dominates by FSD.

Table 6 presents the (approximate) intersection points of the cumulative distributions for each pair of attack methods. A value of 0 identifies those cases where the cumulative distributions do not intersect (i.e. where there are clear FSD outcomes). The other values are the intersection points of the cumulative distributions. For example, $a_{15} = 4$ in Table 6 indicates that the cumulative distributions for 'armed attacks' and 'bombing' intersect at 4 fatalities. This result can be interpreted in conjunction with Figure 4 (a). For example, beyond an outcome of 4 fatalities the cumulative distribution for 'bombing' lies to the right of the cumulative distribution for 'armed attacks'. With reference to Braido and Ferreira (2006), the intersection points in Table 6 may be activated by the application of a convex schedule of incentives. Although such a schedule is conceivable and might be applied deliberately by a terrorist group or inadvertently, incentives might be less important in shaping the terrorist's decision to select the more risky attack method than we might think. If the terrorist context, especially the outcomes of recent attacks, has reshaped terrorists' reference points, they may already be inclined to select a more risky attack method.

Table 6

RAND-MIPT: Cumulative Distribution Intersection Points

	1	**2**	**3**	**4**	**5**	**6**	**7**
1	0	0	2.5	3	4	3	0
2		0	6	0.5	0	0.5	1.5
3			0	2.5	2.5	2.5	0
4				0	2.5	2.5	1.5
5					0	3	1
6						0	1.5
7							0

V. Conclusions

In this paper, we explore the circumstances under which a terrorist decision-maker might be expected to choose the more risky and—generally potentially more damaging—of two attack methods. We use two related approaches: (1) cumulative prospect theory (CPT); and (2) stochastic dominance. The two approaches are linked. Any preference ordering obtained from CPT must not—and will not—violate first-order stochastic dominance (FSD). It can be shown that there are points within both frameworks that are associated with the decision-maker switching to the more risky alternative in a given pair of risky prospects. We call these points 'trigger points'. Within CPT these trigger points are associated with reference points. When the decision-maker's reference point is high enough, the more risky prospect is accorded the higher prospect value. Within stochastic dominance these trigger points are associated with the intersection of the cumulative distributions for each prospect. Convex incentives schedules applied at the intersection of the cumulative distributions will activate the trigger point. Trigger points emerge without violating FSD. In the case of CPT, trigger points emerge for prospects where no clear dominance exists to be violated. In the case of stochastic dominance, second-order dominance (SSD) has embedded within it the property that past the point of intersection of two cumulative distributions, the prospect dominated by SSD dominates its alternative by FSD[6].

We analyse two sets of data: (1) RAND-MIPT data for the period 1968 to 2007; and (2) GTD data for the period 2000 to 2008. We treat the attack methods encompassed within these databases as risky prospects expected to inflict fatalities x_i with probability p_i, where $(p_1 + p_2 + \cdots + p_n = 1)$. Applying stochastic dominance to pairs of risky attack methods reveals that the RAND data contains few instances of clear FSD or SSD. This contrasts markedly with the GTD where 'armed assault' dominates each other attack method by FSD and 'bombing' dominates each other attack method (excluding 'armed assault') by FSD. This is reflected in the plots of the cumulative distributions. For a wide class of risk-averse decision-makers, stochastic dominance implies that 'armed assault' will be the most common choice from any pair of attack methods that can be formed from the GTD. Conversely, stochastic dominance reveals somewhat less about the RAND data. Far from being uninteresting, the absence of clear FSD results and the many intersections that characterise the cumulative distributions for the RAND attack methods provide scope for the existence of 'trigger points'.

When CPT is applied to pairs of attack methods formed from the RAND data, the decision-maker with reference point 1 (expected fatality)[7] invariably accords the highest prospect value to the less risky alternative. For example, at reference point 1 'assassination' is accorded a prospect value of 1.172. This would make it the preferred attack method from any pair for a decision-maker characterised by this reference point. Like 'assassination', 'kidnapping' also has a very low standard deviation of outcomes. It is accorded the second highest prospect value for a decision-maker with reference point 1. Although both

[6] Its cumulative distribution lies to the right of its alternative past the point at which the two cumulative distributions intersect.

[7] It is implicitly assumed, of course, that the parameter values determined by Tversky and Kahneman (1992) apply in this context. Clarifying this point is a matter for future research.

attack methods are associated with a low expected number of fatalities—1.04 for 'assassination' and 0.39 for 'kidnapping'—the outcomes are relatively certain. This ordering is quite sensitive to the decision-maker's reference point. As the reference point increases, a lower prospect value is accorded to each attack method but the prospect values accorded to the less risky alternatives decline faster than the prospect values accorded to their more risky alternatives (Phillips and Pohl 2014). In most cases, the trigger point for each pair of attack methods is located at reference points 2, 3 or 4. For example, 'armed attacks' is accorded a higher prospect value than 'bombing' by decision-makers with a reference point of 1 but is accorded a lower prospect value by decision-makers with reference points of 2 or more. The same is true of 'armed attacks' and 'hostage-taking' and 'bombing' and 'hijacking'.

In each case, the CPT trigger points lie below the intersection points of the cumulative distributions. Approximately, a decision-maker with a reference point that corresponds to the point at which the two cumulative distributions begin to converge will accord a higher prospect value to the more risky of the two prospects. For example, the cumulative distributions for 'bombing' and 'armed attacks' intersect at approximately 3.5 fatalities where 'bombing' is accorded a higher prospect value by decision-makers with a reference point of 2 or more. We explained that convex incentives applied at the intersection of two cumulative distributions will compel the decision-maker to choose the more risky alternative from a pair of risky prospects. In a context where there is reason to believe that terrorists may have formed relatively high reference points, incentives are likely to be a less important driver of risk-taking behaviour and *vice versa*. If it is plausible to consider that the fatalities inflicted by the most recent terrorist attack—or perhaps the most devastating previous attack—form the basis of the reference point from which a terrorist planning a subsequent attack assesses alternative attack methods, the results contained in this paper are operationally relevant from a law enforcement perspective. In particular, attacks that inflict high fatalities may prompt terrorists to choose high risk attack methods which have a higher number of expected fatalities.

References

Anderton, C.H. and Carter, J.R., 2005, On Rational Choice Theory and the Study of Terrorism, *Defence and Peace Economics*, pp.275-282.

Atkinson, S.E., Sandler, T. and Tschirhart, J., 1987, Terrorism in a Bargaining Framework, *Journal of Law and Economics*, 30, pp.1-21.

Barros, C.P., Proenca, I., Faria, J.R. and Gil-Alana, L.A. 2007, Are U.S. Citizens at Risk of Terrorism in Europe? Defence and Peace Economics, 18, pp.495-507.

Bawa, V.S., 1982, Stochastic Dominance: A Research Bibliography, *Management Science*, 28, pp.698-712.

Becker, G.S., 1968, Crime and Punishment: An Economic Alternative, *Journal of Political Economy*, 78, pp.169-217.

Bleichrodt, H. and Pinto, J., 2000, A Parameter-Free Elicitation of the Probability Weighting Function in Medical Decision Analysis, *Management Science*, 46, pp.1485-1496.

Braido, L.H.B. and Ferreira, D. 2006, Options Can Induce Risk Taking for Arbitrary Preferences, *Economic Theory*, 27, pp.513-522.

Brandt, P.T. and Sandler, T., 2010, What do Transnational Terrorists Target? Has it Changed? Are We Safer? *Journal of Conflict Resolution*, 54, pp.214-236.

Butler, C.K. 2007, Prospect Theory and Coercive Bargaining, *Journal of Conflict Resolution*, 51, pp.227-250.

Camerer, C. and Ho, T., 1994, Violations of the Betweeness Axiom and Nonlinearity in Probability, *Journal of Risk and Uncertainty*, 8, pp.167-196.

Enders, W. and Sandler, T., 2002, Patterns of Transnational Terrorism, 1970-1999: Alternative Time-Series Estimates, *International Studies Quarterly*, 46, pp.145-165.
Enders, W. and Sandler, T., 2005, Transnational Terrorism 1968-2000:Thresholds, Persistence and Forecasts, *Southern Economic Journal*, 71, pp.467-482.
Ehrlich, I., 1973, Participation in Illegitimate Activities: A Theoretical and Empirical Investigation, *Journal of Political Economy*, 81, pp.521-565.
Fishburn, P.C.,1964, *Decision and Value Theory*, Wiley, New York, New York.
Frey, B.S. and Luechinger, S. 2003, How to Fight Terrorism: Alternatives to Deterrence, *Defence and Peace Economics*, 14, pp.237-249.
Gonzalez, R. and Wu, G., 1999, On the Shape of the Probability Weighting Function, *Cognitive Psychology*, 38, pp.129-166.
Hadar, J. and Russell, W.R., 1969, Rules for Ordering Uncertain Prospects, *American Economic Review*, 59, pp.25-34.
Hanoch, G. and Levy, H., 1969, The Efficiency Analysis of Choices Involving Risk, *Review of Economic Studies*, 36, pp.335-346.
Hart, S., 2011, Comparing Risks by Acceptance and Rejection, *Journal of Political Economy*, 119, pp.617-638.
Intriligator, M.D., 2010, The Economics of Terrorism, *Economic Inquiry*, 48, pp.1-13.
Kahneman, D. and Tversky, A. 1979, Prospect Theory: An Analysis of Decision Under Risk, *Econometrica*, 47, pp.263-292.
Landes, W.M. 1978, An Economic Study of U.S. Aircraft Hijacking: 1961 to 1976, *Journal of Law and Economics*, 21, pp.1-31.
Lapan, H.E. and Sandler, T., 1988, To Bargain or Not Bargain: That is the Question, *American Economic Review Papers and Proceedings of the One-Hundredth Annual Meeting of the American Economic Association*, 78, pp.16-21.
Levy, H., 1992, Stochastic Dominance and Expected Utility: Survey and Analysis, *Management Science*, 38, pp.555-593.
Levy, H. and Wiener, Z., 2013, Prospect Theory and Utility Theory: Temporary Versus Permanent Attitude Towards Risk, *Journal of Economics and Business*, 68, pp.1-23.
Meyer, J., 1977a, Further Application of Stochastic Dominance to Mutual Fund Performance, *Journal of Financial and Quantitative Analysis*, 12, pp.235-242.
Meyer, J., 1977b, Choice Among Distributions, *Journal of Economic Theory*, 14, pp.326-336.
Meyer, J., 1979, Second Degree Stochastic Dominance with Respect to a Function, *International Economic Review*, 18, pp.477-487.
Pesendorfer, W., 2006, Behavioural Economics Comes of Age: A Review Essay on Advances in Behavioural Economics, *Journal of Economic Literature*, 44, pp.712-721.
Phillips, P.J., 2009, Applying Portfolio Theory to the Analysis of Terrorism: Computing the Set of Attack Method Combinations From Which the Rational Terrorist Group will Choose in Order to Maximise Injuries and Fatalities, *Defence and Peace Economics*, 20, pp.193-213.
Phillips, P.J., 2011, Lone Wolf Terrorism, *Peace Economics, Peace Science and Public Policy*, 17, Issue 1, Article 1.
Phillips, P.J., 2013, How to Fight Terrorism: The Relevance of Qualitative Economics, *Applied Economics Quarterly*, 59, pp.197-207.
Phillips, P.J. and Pohl, G., 2014, Prospect Theory and Terrorist Choice, *Journal of Applied Economics*, 17, pp.139-160.
Rosendorff, B.P. and Sandler, T., 2004, Too Much of a Good Thing? The Proactive Response Dilemma, *Journal of Conflict Resolution*, 48, pp.657-671.
Rothschild, M. and Stiglitz, J.E., 1970, Increasing Risk: I. A Definition, *Journal of Economic Theory*, 2, pp.225-243.
Rothschild, M. and Stiglitz, J.E., 1971, Increasing Risk: II. Its Economic Consequences, *Journal of Economic Theory*, 3, pp.66-84.
Sandler, T. 2011, New Frontiers of Terrorism Research: An Introduction, *Journal of Peace Research*, 48, pp.279-286.
Sandler, T. and Scott, J.L., 1987, Terrorist Success in Hostage-Taking Incidents: An Empirical Study, *Journal of Conflict Resolution*, 31, pp.35-53.
Sandler, T. and Enders, W., 2004, An Economic Perspective on Transnational Terrorism, *European Journal of Political Economy,* 20, pp.301-316.
Sandler, T., Tschirhart, J.T. and Cauley, J. 1983, A Theoretical Analysis of Transnational Terrorism, *American Political Science Review*, 77, pp.36-54.

Santifort, C., Sandler, T. and Brandt, P.T. 2013, Terrorist Attack and Target Diversity: Changepoints and Their Drivers, *Journal of Peace Research*, 50, pp.75-90.

Schneider, F., Brück, T. and Meierrieks, D., 2014, The Economics of Counterterrorism: A Survey, *Journal of Economic Surveys*, In Press.

Siqueira, K. and T. Sandler. 2006. Terrorists Versus the Government: Strategic Interaction, Support, Sponsorship. Journal of Conflict Resolution 50: 878-898.

Tversky, A. and Kahneman, D., 1986, Rational Choice and the Framing of Decisions, Journal of Business, 59, pp.251-277.

Tversky, A. and Kahneman, D. 1992, Advances in Prospect Theory: Cumulative Representation of Uncertainty, *Journal of Risk and Uncertainty*, 5, pp.297-323.

Tversky, A. and Fox, C., 1995, Weighing Risk and Uncertainty, *Psychological Review*, 102, pp.269-283.

Wu, G. and Gonzalez, R., 1996, Curvature of the Probability Weighting Function, *Management Science*, 42, pp.1676-1690.