BUSINESS CYCLE DYNAMICS WITH DURATION DEPENDENCE AND LEADING INDICATORS

By

Allan P. Layton

School of Economics and Finance

Queensland University of Technology

GPO Box 2434, Brisbane, Australia, 4001.

Email: a.layton@qut.edu.au

And

Daniel R. Smith

Faculty of Business Administration

Simon Fraser University

8888 University Drive, Burnaby, BC, V5A 1S6, Canada

Email: drsmith@sfu.ca

ABSTRACT

Durland and McCurdy (1994) investigated the issue of duration dependence in US business cycle phases using a Markov regime switching approach, introduced by Hamilton (1989) and extended to the case of variable transition parameters by Filardo (1994). In Durland and McCurdy's model duration alone was used as an explanatory variable of the transition probabilities. They found that recessions were duration dependent whilst expansions were not. In this paper, we explicitly incorporate the widely-accepted US business cycle phase change dates as determined by the NBER, and use a state-dependent multinomial Logit modelling framework. The model incorporates both duration and movements in two leading indexes - one designed to have a short lead (SLI) and the other designed to have a longer lead (LLI) - as potential explanatory variables. We find that doing so suggests that current duration is not only a significant determinant of transition out of recessions, but that there is some evidence that it is also weakly significant in the case of expansions. Furthermore, we find that SLI has more informational content for the termination of recessions whilst LLI does so for expansions.

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ABSTRACT

Durland and McCurdy (1994) investigated the issue of duration dependence in US business cycle phases using a Markov regime switching approach, introduced by Hamilton (1989) and extended to the case of variable transition parameters by Filardo (1994). In Durland and McCurdy's model duration alone was used as an explanatory variable of the transition probabilities. They found that recessions were duration dependent whilst expansions were not. In this paper, we explicitly incorporate the widely-accepted US business cycle phase change dates as determined by the NBER, and use a state-dependent multinomial Logit modelling framework. The model incorporates both duration and movements in two leading indexes - one designed to have a short lead (SLI) and the other designed to have a longer lead (LLI) - as potential explanatory variables. We find that doing so suggests that current duration is not only a significant determinant of transition out of recessions, but that there is some evidence that it is also weakly significant in the case of expansions. Furthermore, we find that SLI has more informational content for the termination of recessions whilst LLI does so for expansions.

Keywords: Business Cycle, Duration, Leading Indicators, Regime Shift, Multinomial Logit

1. INTRODUCTION

The question of whether business cycle phases are duration dependent has been of interest for many decades. One widely held view is that the older an expansion is, the more likely it is to end. There was much discussion along these lines in the US in the late 1990s as that expansion approached – and eventually passed – the longest previous US expansion ever recorded (since the 1850s). On the other hand, many economists have questioned whether there is any strong underlying rationale for this belief or whether it is simply the business cycle analogue of the view that ’nothing lasts forever’.

Whilst it is obvious that no business cycle phase has ever lasted forever – and is never likely to – the issue surrounding duration dependence is whether there exists statistical evidence that the probability of a phase change systematically increases with the length of the current phase. Furthermore, even if current phase duration does seem to be a determinant of the probability of phase termination, there may well be some underlying economic processes taking place systematically over time which increase the likelihood of the phase ending. Thus, if other factors are indeed important in determining phase shifts, then apparent duration dependence may simply reflect the influence of other more fundamental variables. In the same way therefore that a trend variable may or may not remain statistically significant after incorporating other explanatory variables, the same may well be true of duration once other explanatory variables are incorporated into the analysis. We believe this to be an important question and we therefore seek to address the issue here.

In our empirical analysis we use the business cycle phase chronology for the US determined by the National Bureau of Economic Research (NBER) dating panel. As we discuss below, this chronology represents a set of reference dates which is agreed upon by a group of recognized experts at the NBER, is widely used, and has been used to examine duration dependence in the business cycle using a range of different methodologies. However the business cycle is essentially a conceptual construct – and at that an ultimately unobservable construct. The best that can be done is to carefully conceptually define it and then use a range of datasets to triangulate on the most appropriate dates of the phase changes. There may well be alternative methodologies and resulting chronologies for the US to that of the NBER. However, we argue that the NBER methodology has the longest pedigree and the NBER chronology is the most widely accepted, cited and used set of phase change dates for the US business cycle. Given this, we elect to study duration dependence explicitly using the NBER chronology.

Our approach can be contrasted with the duration-dependent regime switching extensions of Hamilton (1989) which explicitly assume that the latent regime state is unobservable and must be inferred. In fact every Markov-switching model of US GDP we are aware of benchmarks the regime probabilities against the NBER chronology. Hamilton-type models are most useful in situations where there is no clear a priori knowledge of the phase change dates.[1] However, for the case of the US business cycle, to ignore the existence of such dates as the NBER chronology is to ignore very important relevant information.

Earlier research that uses the NBER chronology to explore duration dependence (i.e., Sichel (1991), Zuehlke (2003) and Diebold and Rudebusch (1990) discussed below) use either non-parametric methods or a hazard function which uses only the length of each phase. However, we model the state of the business cycle as a first-order Markov process and allow the transition probabilities to vary as a function of both leading indicators and phase durations. In particular, we directly model the probability of staying in a phase or changing phases as a multinomial Logit (and Probit) process.[2] We believe this modeling specification represents a more complete framework and will not only bring into further relief the strength of duration dependence in the US business cycle evident in the data, but will also provide for richer interpretations of the resulting estimated model.

[1] Knowing the phase states re-casts the econometrics completely. As suggested by an anonymous referee this allows hybrid models to be constructed where the known phase changes can be used to model the transition probabilities (assuming variable transition probabilities) separately from the inference in relation to the state-dependent parameters. Further details of some applications along these lines are available from the authors upon request.

[2] We used both types of models but, since the results of both were qualitatively the same, in the paper only the Logit results are reported. The Probit results are, of course, available upon request from the authors.

We also believe our approach to the issue of duration dependence nicely complements the Markov-regime switching approach of Durland and McCurdy (1994). Their approach – an extension of Hamilton's (1989) - models an observed cyclical variable (GDP in their case) whose distribution is influenced by unobserved latent states. The estimated conditional density of the observable cyclical variable allows the econometrician to infer something about both the unobserved state of the variable under study as well as its apparent duration dependence.[3] In contrast to this, we investigate duration dependence directly by relying on the phase changes, and hence business cycle durations, implied by the NBER-determined business cycle chronology.

Our approach is clearly only available when one has access to such a well respected business cycle chronology, and without it, one would apply the more computationally intensive approach of Durland and McCurdy (1994). Which approach yields better inference about business cycle duration depends ultimately on one's stance on whether the NBER chronology or the set of latent states implicit in US GDP is a better proxy for the US business cycle chronology. Interestingly, we find that the standard errors from our approach are tighter than the standard errors reported in approaches that do not use the NBER dates. This suggests that using the NBER chronology allows us to learn much about business cycle phase dynamics.

Of course, we do not take the position that the NBER dating committee is infallible nor that the dates are sacrosanct. Nonetheless, we believe a chronology determined by a committee of respected experts employing a consistent methodology and using a variety of macroeconomic indictors is likely to be superior to the noisy signal about the state of the business cycle inferred from a single cyclical variable such as GDP. We have also verified in a simulation exercise that the main conclusions about the importance of

[3] In an interesting paper Filardo and Gordon (1998) demonstrate that Markov switching models with transition probabilities which vary according to leading indicators produce time-varying conditional expected durations (see later). However, the ability of these models to capture the more traditional notion of duration dependence is clearly limited. We present a model in which the transition probability depends explicitly on the current phase duration as well as leading indicators, and demonstrate that it significantly improves the explanatory power of the model, produces significant variation in conditional expected durations, and leads to important insights into business cycle dynamics.

duration dependence in modelling phase changes is robust to minor variations in the chronology as determined by the dating committee. And so, given the widespread use of the NBER chronology, it seems sensible to us to directly use it to construct tests of US business cycle duration dependence?[4]

In the next section we discuss earlier related literature. We then present the modelling framework and how it relates to the Hamilton approach – of which it may be regarded as a special case variant. The estimation results follow, with concluding remarks presented in Section 5.

2. BACKGROUND LITERATURE IN RELATION TO PRESENT PAPER

In his test for duration dependency, Sichel (1991) used a hazard function approach in which a specific functional form for the hazard rate – the Weibull - was assumed, and the necessary parameters were estimated from the US business cycle chronology as determined by the NBER. Using phase lengths derived from the NBER chronology for post-WWII data Sichel found evidence supporting duration dependence in recessions but insignificant evidence for expansions. Zuehlke (2003) followed the approach of Sichel but used a longer dataset and a generalised Weibull for the hazard. The generalised Weibull allowed for the possibility that log hazard grew non-linearly with log duration rather than constraining it to grow linearly, as is the case with the Weibull. Using this modified approach, in addition to duration dependence in recessions, Zuehlke importantly also found evidence of duration dependence for the expansionary phase of the business cycle.

Diebold and Rudebusch (1990) also used the NBER dates but used a non-parametric methodology. They found against duration dependence for both expansions and recessions; however, they acknowledged that, although the evidence was statistically insignificant, the data available at the time more strongly favoured recession duration dependence. They also found some evidence of whole-cycle duration dependence.

[4] In this respect the approach is also similar to the earlier work of Neftci (1982, 1984).

With the immediate widespread popularity of Hamilton's Markov regime switching methodology, a number of papers subsequently tested the notion of duration dependence within that framework. This framework is described in more detail in the Modelling Framework section below but basically the approach amounts to allowing the transition parameters - representing the probability of transitioning out of particular phases – to vary over time in response to some underlying determinants. For example, Durland and McCurdy (1994) incorporated current phase duration as a potential explanatory variable for the transition probability parameters governing phase switches. They found that, within this framework, quarterly US GNP data suggested recessions were duration dependent but not so for expansions.

Finally, another, more distantly, related paper is that of Estrella and Mishkin (1998). Although these authors were not concerned with the duration dependency issue – and therefore they did not incorporate a duration variable as an explanatory variable - they also used the NBER dates. In their case it was to define a binary dependent variable representing recession. Unlike Estrella and Mishkin, we model phase changes and, importantly, relax their implied restriction that the model's coefficients be symmetric across the two phases. As potential drivers of the observed phase changes, we incorporate indexes of leading indicators as well as the current phase duration.

3. THE MODELLING FRAMEWORK

In many modelling situations it is sensible to allow for the possibility that the variable of interest may come from one of several different 'states', 'phases', or 'regimes' and that whatever is the data generating mechanism driving the observed variable it may differ across regimes. For instance, it is common to conceptualise the business cycle as consisting of two phases: expansion and recession.[5] It is widely accepted that there are a number of asymmetries across these two business cycle regimes. For instance, the

[5] Some researchers and analysts also sometimes allow for the possibility of a third "recovery" phase. See, for example, Sichel (1994) and Layton and Smith (2001).

average duration of expansions is much longer than for recessions, the variability of economic growth rates is different in each regime, and, to some extent, researchers have found that the dynamic properties of economic growth may differ across regimes.

A modelling approach which has gained great popularity for studying these asymmetries is the Markov regime-switching model of Hamilton (1989). It allows for shifts from one phase into another and, in its simplest form, it assumes constant transition probabilities with the distribution of the variable under study assumed to be normal with a different mean and variance across phases.[6] The probability of switching from one phase into the other is characterised by a discrete first-order Markov process where the probability of switching depends only on the most recent phase state.

Suppose the business cycle consists of two phases, summarized by the discrete random variable S_t=i (i =1,2) which takes two possible values respectively denoting expansion (1) and recession (2). The transition matrix describing the evolution of S_t is given by

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \tag{1}$$

where p_{11} denotes the probability of remaining in phase 1 from period *t*-1 to period *t*, and p_{22} is the probability of staying in phase 2 from period *t*-1 to period *t*. Because these are probabilities the off diagonal elements are simply: $p_{12} = 1 - p_{11}$, the probability of changing from phase 1 to phase 2; and $p_{21} = 1 - p_{22}$, the probability of changing from phase 2 to phase 1.

Let y_t denote the business cycle indicator whose distribution depends on the business cycle phase S_t. For simplicity we will assume that y_t is normally distributed conditional on the state, or $y_t | S_t = i \sim N(\mu_i, \sigma_i^2)$, which implies the conditional density of y_t is given by:

[6] It is also possible to allow for autoregressive dynamics which may be the same, or which may differ, across phases.

$$f(y_t \mid S_t = i; \theta) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left[-\frac{(y_t - \mu_i)^2}{2\sigma_i^2} \right], \tag{2}$$

with $\theta = (p_{11}\ p_{22}\ \mu_1\ \mu_2\ \sigma_1^2\ \sigma_2^2)'$ the relevant parameter vector to be estimated.

A natural extension of the simple model – and one which allows for some interesting causal hypothesis tests – is to allow the matrix *P* to be a time-varying function of some conditioning information variables. A more general version of (1) is:

$$P(S_t = 1 \mid S_{t-1} = 1, X_{t-1}) = p_{11t}, \qquad P(S_t = 2 \mid S_{t-1} = 1, X_{t-1}) = p_{12t} = 1 - p_{11t} \tag{3a}$$

$$P(S_t = 1 \mid S_{t-1} = 2, X_{t-1}) = p_{21t} = 1 - p_{22t}, \qquad P(S_t = 2 \mid S_{t-1} = 2, X_{t-1}) = p_{22t}$$

where

$$p_{iit} = (1 + \exp(-\gamma_i'\ X_{t-1}))^{-1} \text{ for } i = 1, 2; \tag{3b}$$

and

$$X_{t-1} = (1, x_{1,t-1}, x_{2,t-1}, \cdots, x_{k-1,t-1})'; \ \gamma_i = (\gamma_{i0}, \gamma_{i1}, \cdots, \gamma_{i,k-1})'$$

and where *k-1* is the number of determinants of the transition probabilities. The functional form, $(1+\exp\{-x\})^{-1}$, is the logistic and is one of several different specifications which could be used to ensure that the estimated transition probabilities lie between zero and one.

Durland and McCurdy (1994) test for business cycle duration dependence using a special case of the above “variable transition probability” model. In particular they used quarter-to-quarter GNP growth rates as their dependent variable (*y*) and used current business cycle phase duration to summarize the time-varying transition probabilities conditioning information (i.e., *X* in (3b) above consisted only of the variable, duration).

Estimation of this type of model is complicated because the phase change dates and hence phase durations are unobservable. Thus, at any given time, the precise value of the key explanatory variable is not known with certainty and it could take on any of an exponentially expanding range of possibilities as the sample period extends. To mitigate this problem Durland and McCurdy arbitrarily truncate the duration variable at a maximal value D^*. The probability of staying in phase *i* is simply assumed constant for durations above this upper threshold.

However, rather than needing to make this assumption, parameter estimation is considerably simplified when the phase change dates are known. Further, using these phase change dates may significantly increase the expected precision of the estimates of the various parameters of the model - including the duration parameters - since we avoid the noise involved in using an imperfect proxy variable to represent the business cycle chronology. In the current case, by using the available NBER dates as the US business cycle chronology, we effectively define the issue of phase duration dependence in terms of whatever apparent duration dependence is evident in these pre-defined phases.[7] This eliminates all uncertainty as to phase switches and defines exactly the value of the duration variable at each time period.[8]

Given this simplification, we retain a Markov-type process for phase changes, define transition probabilities conditional only on the phase last period, and model these transition probabilities as functions of a list of relevant explanatory variables, namely, current phase duration, and readings on some leading economic indicator indexes of interest. We use conditioning information available at time *t-1* to model the probability of staying in (and therefore of leaving) state *i* from period *t-1* to period *t.* As mentioned, the conditioning information consists of a constant, phase duration d_{t-1}, and a vector of other relevant explanatory variables Z_{t-1} (containing the leading indexes).[9] To ensure that the transition probabilities are well defined, we employed a LOGIT functional form. Specifically, the probability of staying in phase *i* (*i* =1,2) may be given as

[7] Of course, the added precision of the estimates is conditional upon the accuracy of the NBER dates as a representation of the US business cycle chronology. In cases of uncertain business cycle chronologies then Durland and McCurdy's approach has advantages. Later in the paper we provide some analysis of the impact of uncertainty in the NBER dates.

[8] This is not quite true as there will be some inevitable uncertainty surrounding the most recent observations subsequent to the most recent determination by the NBER of the last turning point but in advance of any further NBER turning point determination.

[9] Thus, from here, for convenience we split the vector X_{t-1} (in 3b) into our duration variable, d_{t-1}, and the vector, Z_{t-1}.

$$P(S_t = i \mid S_{t-1} = i, X_{t-1}) = (1 + \exp(-(\alpha_i + \delta_i d_{t-1} + \beta_i{}' Z_{t-1})))^{-1} \qquad (4)$$

where Z_{t-1} is a column vector of two selected leading economic indicator indexes (with β_i representing the two column vectors (one vector for each phase) of associated parameters), d_{t-1} is the duration of the current expansion or recession up to period t-1 (with associated parameters, δ_i) and defined as $d_{t-1} = \begin{cases} d_{t-2}+1 & \text{if } S_{t-1} = S_{t-2} \\ 1 & \text{if } S_{t-1} \neq S_{t-2} \end{cases}$.

Considering (4) at each point in time, t-1, only one of four possible outcomes can occur:

1. The economy can stay in expansion: $S_{t-1} = 1$ and $S_t = 1$.
2. The economy can transition from expansion to recession (a peak): $S_{t-1} = 1$ and $S_t = 2$.
3. The economy can transition from recession to expansion (a trough): $S_{t-1} = 2$ and $S_t = 1$.
4. The economy can stay in recession: $S_{t-1} = 2$ and $S_t = 2$.

To summarize these four outcomes and simplify the expression for the likelihood function, define the following four dummy variables h_t^A through h_t^D which we notionally collect into a four-element vector h_t :

$$\begin{aligned}
h_t^A &= \begin{cases} 1 & \text{if } S_t = 1 \text{ and } S_{t-1} = 1 \\ 0 & \text{otherwise} \end{cases} \\
h_t^B &= \begin{cases} 1 & \text{if } S_t = 2 \text{ and } S_{t-1} = 1 \\ 0 & \text{otherwise} \end{cases} \\
h_t^C &= \begin{cases} 1 & \text{if } S_t = 1 \text{ and } S_{t-1} = 2 \\ 0 & \text{otherwise} \end{cases} \\
h_t^D &= \begin{cases} 1 & \text{if } S_t = 2 \text{ and } S_{t-1} = 2 \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \qquad (5)$$

Thus, at each point in time, exactly one element of the vector h_t takes the value 1, while all the other three are zero. The log likelihood function may therefore be defined as

$$LL(h;\theta) = \sum_{t=1}^{T} h_t^A \log(\Phi(\alpha_1 + \delta_1 d_{t-1} + \beta_1 ' Z_{t-1})) + h_t^B \log(1 - \Phi(\alpha_1 + \delta_1 d_{t-1} + \beta_1 ' Z_{t-1})) + \\ h_t^C \log(1 - \Phi(\alpha_2 + \delta_2 d_{t-1} + \beta_2 ' Z_{t-1})) + h_t^D \log(\Phi(\alpha_2 + \delta_2 d_{t-1} + \beta_2 ' Z_{t-1})). \quad (6)$$

The highest possible likelihood obtains when the model assigns a probability of one to a phase shift at the NBER-determined turning points and assigns a probability of one to continuing in an expansion or recession at all other times—in other words the model 'gets it exactly right' at every date. In this case the log-likelihood will be zero. Whenever the model assigns a probability less than one to any observed phase "event", the log likelihood becomes more negative. Thus, in this case of observable phases, the calculated values of the log-likelihoods for the various models allows the use of the well-known likelihood ratio test to test directly the relative goodness-of-fit to the NBER business cycle chronology of the various alternative models under consideration.[10]

3. THE EMPIRICAL RESULTS

3.1 Data Issues

We use monthly data for the analysis spanning the period 1/1949 – 12/2002. Data on the four dummies, h_t^A, h_t^B, h_t^C, and h_t^D and the Duration variable are defined from the monthly NBER business cycle dates - as presented in Table 1.[11] The two other variables used in the analysis are the two US leading indexes compiled by the Economic Cycle

[10] Others who have used a regime switching modelling framework without explicitly using the NBER dates as dependent variable have used the quadratic probability score (QPS) to do this. The closer QPS is to zero the better the fit to the NBER dates. However, statistically comparing different QPS values for different models is problematic since its distributional properties are unknown. Layton and Smith (2005) discuss model evaluation in the context of the QPS.

[11] An important issue was recently raised by Pagan (2005). Pagan points out that a constructed phase state variable such as we construct using the NBER dates (or any other dating method for that matter) is very likely to be serially correlated. To correct for this serial correlation we calculate Newey-West (1987) standard errors with 6 lags which are robust to autocorrelation, heteroscedasticity and non-normality.

Research Institute (ECRI): the short leading index (SLI) and the long leading index (LLI).

The use of leading indicators – and the construction of leading indicator indexes - to try to anticipate imminent turning points in the business cycle has a long tradition. Certainly Geoffrey H Moore, founder of the Centre for International Business Cycle Research (CIBCR) in 1982 and, later, also founder of ECRI (after leaving CIBCR in 1995), would be one of the major pioneers, along with Victor Zarnowitz (brought into CIBCR as research director by Moore in 1994), Phil Klein and others of Moore's close associates. A few relevant references would be Zarnowitz and Moore (1982), Moore (1983, 1990) and Lahiri and Moore (1991).

Whilst no doubt very successful, characteristic of the early leading indicator (LI) work was that forecasting turning points using cyclical information in LIs was very judgmental without the construction of formal parametric models. Thus, the provision of actual estimated probabilities of imminent phase changes was not possible. Formally incorporating the very useful cyclical information in LIs into a parametric regime switching model of the type described herein allows the provision of such estimates (see, for example, Layton (1998)).

The individual components of the SLI and LLI indexes have been reported elsewhere (see Layton and Katsuura, 2001, Table 1, p409). Further information on the construction of the two indexes may be obtained by contacting ECRI directly at www.businesscycle.com . The splitting of leading indicators into those with a short lead and those with a longer lead is a little unusual. Interested readers may want to refer to Cullity and Moore ("Long-Leading and Short-Leading Indexes") in Moore (1990). One important difference between the two indexes is that LLI explicitly contains an interest rate measure.

For these two variables, the index data were first converted to month-to-month growth rates. Then, for each variable, a series was constructed consisting of a moving sum of the growth rates. For SLI this moving sum spanned the most recent six months and for LLI it spanned the most recent eight months. The use of a moving sum has been used successfully in previous research (see, for example, Layton (1998)) to capture the strength and persistence of any swing in the index. The different spans reflect the different expected leads of each index in relation to business cycle phase shifts. Graphs of the two resulting variables are provided in Figures 1 and 2. These were the data used in the estimation of the various models discussed below.

3.2 A Preliminary Model for Comparison with Durland and McCurdy (1994)

As mentioned in the previous section, in their test of US business cycle duration dependence, Durland and McCurdy (1994) used only duration as a potential explanatory variable in their variable transition parameter regime switching model. We therefore first estimate our regime switching multinomial Logit model with current Duration (up to period t-1) as the sole explanatory variable. The results are provided in Table 2 respectively in column 2 of the table.[12]

The first point to note derives from comparing column 2 with column 1 in the table. Column 1 represents the estimation results for the multinomial model assuming the switching probabilities for each phase are constant through time. Column 2 allows these switching probabilities potentially to depend on the duration of the current phase. A comparison of the value of the log likelihood (LL) for the two alternatives clearly statistically rejects that the switching probabilities are invariant with respect to Duration. The value of the likelihood ratio test statistic is 15.58. Thus the data quite strongly reject the null and, as was found by Durland and McCurdy, we conclude there is evidence supporting the notion that business cycles are duration dependent.

[12] It should be noted that the NBER chronology is predicated on the requirement that no phase be of shorter duration than 5 months. It is therefore necessary to allow for this in computing the likelihood of the estimated model. We did this by constraining the probability of a transition out of a phase to be zero within the first five months of the phase. We also estimated the parameters without this constraint and found no qualitatively significant or substantive changes to the results.

Furthermore, both estimated coefficients are negative which is consistent with the view that the probability of remaining in a particular business cycle phase decreases with the age of the phase. For recessions, the estimated parameter is -.3059 and, with a robust t-ratio of -4.31, is highly significant. The estimated parameter for expansions is -.0179, clearly much less negative than that for recessions. This implies expansion duration has a weaker estimated impact on the probability of an expansion terminating than in the case of recessions. All of this is also broadly consistent with Durland and McCurdy. However, of some interest here is that, contrary to Durland and McCurdy, the robust t-ratio for this coefficient is -1.72, implying the likelihood of this parameter being statistically significantly different from zero is much greater than what was found by Durland and McCurdy (with a t-ratio in their case of just -.86). This is most likely due, in part, to our direct use of the NBER business cycle chronology. We thereby avoid the noise generated by using some selected time series as an imprecise proxy from which the chronology is imperfectly inferred (quarterly GNP growth rates in the case of Durland and McCurdy).

3.3 Incorporating the Leading Indexes

Of course both the above analysis and that of Durland and McCurdy may be regarded as only partial in that the only explanatory variable included in the model is Duration. Suppose the actual determinants of observed phase durations were variations in some set of underlying economic fundamentals driving the business cycle. If these fundamental drivers were cyclically mean reverting but were omitted from the model and, in their place, observed duration was the only explanatory variable used, then it could appear that phase changes were duration dependent.

In this sub-section we report the results of incorporating the two leading indexes described in Section 3.1 into the models. Results are also reported in Table 2. There are a number of intermediate columns in the tables corresponding to various combinations of the explanatory variables. These are provided for the sake of completeness, however, the column of most interest is Column 8 which contains the estimation results arising from including all three explanatory variables in the model. A graphical indication of how the probability of expansion (recession) changes in accordance with changes in each of the three explanatory variables is provided in Figure 3.

First, the estimated model incorporating the two leading indexes is statistically superior (as measured by the difference in LLs) to the model with Duration alone. The converse is also true, viz., the inclusion of Duration in addition to the two leading indexes adds significantly to the statistical explanation of the business cycle phase change dates compared with a model with the leading indexes alone (refer to Column 5 in comparison to Column 8).

Second, all coefficients for which we had prior expectations as to their signs had the appropriate signs except for the coefficient of LLI in recessions (i.e., β_2^{LLI}) which should logically be negative. However, with a robust t-ratio of less than one, it is clearly statistically insignificant, and so the estimated sign is not really of concern.

Third, the expansion Duration parameter coefficient remains greater than its robust standard error but the t-ratio has reduced to -1.23. Its absolute magnitude has also reduced and is furthermore smaller relative to the recession Duration coefficient. The recession Duration coefficient is now larger in absolute magnitude and also continues to have a robust t-ratio of about three. All of this leads to the conclusion that phase duration is considerably more important in predicting the end of recession than it is for predicting the end of an expansion (refer also to Figure 3).

Fourth, the results for the leading indexes point to the conclusion that the long leading index is of no value in predicting the end of recessions once Duration is incorporated into the model but that the short leading index continues to have informational content. Furthermore, whilst both indexes seem to have predictive power as far as the termination of expansions is concerned, of the two indexes, the long leading index would appear to be the stronger explanatory variable. Its estimated coefficient is more than twice its SE while that of SLI is not and the actual estimated value of the coefficient of LLI is also quite a bit larger than that of SLI.[13]

In summary, the estimated results may be interpreted as suggesting that Duration and the SLI have significant informational content as far as predicting the probability of the imminent termination of a recession. However, once movements in the leading indexes are taken into account, Duration has little predictive information in predicting the probability of the imminent termination of an expansion. Of the two leading indexes, LLI seems to have the stronger predictive power in expansions.

In Figure 3, we provide the model-derived period-by-period probabilities of recession along with the true NBER-determined probabilities (taking values 0 or 1). As can be seen in the figure, the model incorporating Duration and the two leading indexes does very well in replicating the true probabilities.

3.4 Bootstrapping the Impact on the Findings of Probabilistic Variations in the Business Cycle Chronology

The official NBER chronology is certainly the most widely accepted set of business cycle dates in the US. In fact many commentators/analysts regard them as being the "true" US business cycle dates. However, as thorough as the NBER dating committee is, it is unlikely that the official dates perfectly coincide with the unobservable peaks and troughs in the US business cycle. To assess the degree to which errors in these dates may affect our results we conduct the following bootstrap experiment.[14]

We construct a new chronology with a new peak or trough $TP_i = TP_{nber} + d_i$ where d is an integer random variable $d \in \{-n, \ldots, n\}$ whose distribution is given by $P(d = 0) = p$ and $P(d = j) = P(d = -j) = (1-p)(1-q)^{j-1} q$, and when $q = 1 - .5^{1/n}$. In this setup the

[13] It should be noted in passing that SLI and LLI were both standardised by adjusting for their respective mean and standard deviation.

[14] We thank an anonymous referee for suggesting this experiment.

probabilities of dating errors of a given magnitude decline geometrically, and the variance of d is decreasing in both p and q.

To ensure that our artificial chronology is reasonable we impose the standard NBER constraints on minimum phase and cycle length. If a chronology does not meet these criteria it is rejected and we generate a new candidate series.[15] We estimate the model parameters on each of 1000 acceptable time series using p=0.6 and report the resulting mean value of each parameter estimate and its root mean square deviation in Table 3. These root mean square deviations represent the variation in the parameter estimates that can be attributed to uncertainty about the precise dates of business cycle peaks and troughs.[16]

The clearest conclusion which results from this experiment is that variation in the business cycle chronology would seem potentially to impact more substantively on the modeling results for recessions.[17] This is evident from the size of the root mean square deviations of the recession parameter estimates in relation to the original estimates in Table 2. This is not too surprising given the average length of recessions is so much shorter than that for expansions. The parameter estimates for recessions – for both duration and leading indicator variables – are likely therefore to be quite a bit more sensitive to shifts in the NBER recession dates by a few months.

Since US expansions are much longer than recessions, moving a trough or a peak by a month or two has a much more significant impact on recession durations without any major impact on most expansion phase durations. This then introduces greater variation into the recession duration parameter estimates than the expansion duration parameter estimates once we allow for the dates to shift randomly by a few months. Similarly, the

[15]We reject about 20 percent of the series. This is exclusively due to changes in the peak and trough shortening the already very short 1980 recession.
[16] Note that 0.6 assigns quite a bit of uncertainty to the NBER dates. A higher – and probably more realistic – probability of the official dates being the most appropriate would be expected to give even less variation to the parameter estimates.
[17] For instance, compare the very small root mean square errors for the expansion duration parameter estimate, δ_1, in Table 3 with those for the recession duration parameter estimate, δ_2.

estimated influence of the leading indicators in recessions could be expected to have greater variability for recessions than for expansions if trough and peak dates change randomly by a few months. Because expansion are so long, shifting the phase change dates by one or two months may not cause too much of a change to the overall apparent influence of leading indicators around peaks and troughs as far as expansions are concerned. However, for recessions, changing the dates by a few months can quite dramatically affect the implied behaviour of the indicators around the relevant dates. Assigning additional or fewer observations of the leading indicators from either subsequent or past expansions to a given recession can then have quite discernable effects on the variability in the parameter estimates.

3.5 Investigating the Properties of the Model-Generated Conditional Expected Phase Durations

Filardo and Gordon (1998) introduced the idea of investigating the remaining expected phase duration as a useful model diagnostic for models such as those used in this paper. In their analysis the expected remaining duration varied in response to variation in a leading indicator which was included to model the time varying transition probabilities in their model. In this sub-section we carry out a similar analysis for our models.

The expected remaining duration of the current phase of the business cycle at time t is defined as the expected number of periods the economy will remain in the current phase, or

$$D_t = \sum_{j=1}^{\infty} jP(S_{t+j} = 1 - s, S_{t+j-1} = s, \ldots \mid S_t = s, I_t),$$

which is calculated as

$$\begin{aligned} D_t &= E\left((1 - p_{ii,t}) + \sum_{j=2}^{\infty} j(1 - p_{ii,t+j}) \prod_{k=1}^{j-1} p_{ii,t+k} \mid I_t \right). \\ &= \lim_{n\to\infty} E\left(1 + \sum_{j=1}^{n-1} \prod_{k=1}^{j} p_{ii,t+k} - n \prod_{k=1}^{n} p_{ii,t+k} \mid I_t \right) \end{aligned} \qquad (7)$$

Note that the transition probabilities in equation (7) are random variables and unknown at time *t*. Because the transition probabilities are time-varying we must integrate over possible future value of the leading indicators. [18]

Filardo and Gordon (1998) considered the problem in a Bayesian setting. They propose an algorithm that produces draws from the posterior distribution of the parameter vector, with the conditional remaining duration being computed by calculating forecasts of the leading indicator. More specifically, they propose to use the forecast values of the leading indicator to compute the transition probabilities up to some critical value, and then use the transition probability implied by the unconditional mean $\bar{p}$ to calculate the conditional duration thereafter ($\bar{p}/(1-\bar{p})$). However, their approach ignores Jensen's inequality and uses $p(E(z_{t+i}))$ rather than $E(p(z_{t+i}))$, and because $p(.)$ is concave when transition probabilities are greater than 0.5, as they generally are, this will tend to underestimate the expected remaining duration (since for concave *g*, Jensen's inequality implies that $E(g(z_{t+i})) \le g(E(z_{t+i}))$).

We calculate the expectations used to calculate conditional duration in equation (7) by Monte Carlo methods. Initially we fit a time-series model to the leading indicators. We then simulate *N* different sample paths of the leading indictor *M* periods into the future using this econometric model conditioning on the current value of the leading indicator: $\tilde{Z}_{t+i}^{(n)}$ for $i = 1,\ldots,M$ and $j = 1,\ldots,N$. The current duration d_{t-1} increases deterministically. The conditional expected remaining duration is then calculated as

$$D_t = \frac{1}{M}\sum_{m=1}^{M}\left((1-p_{ii,t}^{(n)}) + \sum_{j=2}^{N} j(1-p_{ii,t+j}^{(n)})\prod_{k=1}^{j-1} p_{ii,t+k}^{(n)}\right) \quad (8)$$

where $p_{ii,t+i}^{(n)} = \Phi(\alpha_i + \delta_i(d_t + j - 1) + \beta_i'\tilde{Z}_{t+i}^{(n)})$ (or

$p_{ii,t+i}^{(n)} = (1+\exp(-\{\alpha_i + \delta_i(d_t + j - 1) + \beta_i'\tilde{Z}_{t+i}^{(n)}\}))^{-1}$).

[18] As an aside, since Durland and McCurdy (1994) imposed an arbitrary upper limit on duration dependence of D^*, once that duration is reached, expected remaining duration becomes constant at $p_{ii,D*}/(1-p_{ii,D*})$ for their model.

In this experiment we use 1200 observations and, for the models that we consider in this paper, we find that the terms in the summation converge. This is relatively unsurprising given that the longest expansions are no more than around 10 years in length (ie.,120 monthly observations).

We estimate VAR models for the dynamics of SLI and LLI. The Bayesian information criteria suggested a VAR(2) adequately captured the dynamics of these two leading indicators

$Y_t = B_1' Y_{t-1} + B_2' Y_{t-2} + u_t$

where $Y_t' = (SLI_t, LLI_t)$ (after demeaning) and $E(u_t u_t') = \Sigma$,[19] and to calculate the conditional expected remaining duration we need the state variables $I_t = (d_t, Y_t, Y_{t-1})'$.

In Figure 5 the conditional expected remaining duration for each phase is plotted against time for Model 1 (constant transition probability model), Model 2 (the model with duration alone), Model 5 (the model with both the SLI and the LLI), and Model 8 (the model with duration and the two leading indicators). Of most interest is the pattern evident for expansions which we also compare with Filardo and Gordon's (1998) results (refer to their Figure 2).

In their analysis, using a model incorporating their leading indicator only, the variation in expected remaining duration is rather small for expansions and shows little tendency to decline as the expansion gets longer. From Figure 5, we find a similar result for Model 5 which does not include current phase duration as a driving variable of the transition probabilities. This can be contrasted with models 2 and 8 which either include duration alone (Model 2) or Duration with SLI and LLI (Model 8) and we find that, as an expansion lengthens, the remaining expected duration declines dramatically. Interestingly, Model 5 which uses only SLI and LLI to account for time-varying transition probabilities does quite a poor job of picking the end of the long expansions in the 1960's and 1990's, whereas Models 2 and 8 do a much better job.

[19] In the interests of space, the parameter estimates are not presented here. They are, however, available from the authors upon request.

We can also usefully contrast the dynamics implied by Models 2 and 8. Using Model 8 there are periods early in an expansion phase – and when the leading indicators were growing strongly – when the expected remaining duration was above that implied by Model 2. Since Model 2 omitted the important information in the leading indicators' pattern of growth, it was forecasting an expansion with a shorter remaining expected duration than Model 8 which incorporated not only the current expansion duration but also the added information from the leading indicators. This is a demonstration of the advantage of incorporating both current duration as well as the leading indicators in the model. Model 8, with the combination of leading indicators and duration, also results in a very interesting drop in expected remaining duration just before the onset of the 2001 recession. This model clearly performs the best in picking this recession.

4. CONCLUSIONS

In this paper we have revisited the issue of phase duration dependence in the US business cycle. We believe the approach we have employed entails some interesting variations on other earlier papers on the topic. Rather than use some macroeconomic variable (like GNP) to infer (imperfectly) the US business cycle chronology, we use the widely accepted NBER chronology. If one is prepared to accept this chronology as representing the US business cycle chronology – as many analysts and commentators seem to – then its use avoids the issue of measurement error imprecision in the modeling analysis. We have also incorporated other potentially relevant explanatory variables into our extended models to account for other possible determinants of the probability of a business cycle phase terminating beyond its duration. Finally, we have allowed our models' parameters to vary across the two different business cycle phases.

Results include the following. When only duration is included as the explanatory variable in the phase switching model our results not only support the finding that recessions appear to be strongly duration dependent but also provide some evidence in favour of duration dependence in expansions. This supports the most recent finding by Zuehlke

(2003) but contrasts with other earlier work. In our case we believe this is due to the explicit use of the NBER chronology.

Perhaps the most important additional results embodied in the paper relate to the inclusion of other variables into the model. Once this is done, recessions continue to appear to be quite strongly duration dependent, but the evidence for duration dependence in expansions becomes considerably weaker. Furthermore, the selected leading indicators introduced into the model also appear to have important informational content in predicting the probability of imminent business cycle phase shifts beyond that contained in duration alone. This is the case for both expansion and recession phases.

More specifically, whilst duration continues to be an important and statistically significant determinant of the termination of recessions, the short leading index also has statistically significant informational content. On the other hand, it is the long leading index which exhibits strong predictive power in anticipating the end of expansions and its inclusion substantially mitigates any informational content that phase duration previously contained in explaining the transition probability out of expansions. One interpretation of this is that the transition probabilities for expansionary phases of the business cycle are in fact driven by changes in the economic fundamentals of the economy, as captured by the leading index, rather than by phase duration. An alternative interpretation of the lack of significance of duration in expansions may be structured on the notion that changes in the leading index around turning points of the business cycle are a result of the aging of the business cycle itself; i.e., it is some characteristic of lengthening duration that itself causes consequential changes in the leading index.[20]

Another point is also worthy of mention here. In investigating the issue of duration, the key question is whether phase durations cluster around some average duration such that the longer the duration of the business cycle phase the greater is its probability of terminating. Thus, one can argue that, irrespective of whether the underlying cause of an observed phase switch is actually the result of some changing economic fundamentals,

[20] We thank an anonymous referee for suggesting this interpretation.

the key issue is simply whether or not the evidence on phase switches supports the existence of a systematic relationship between the probability of experiencing a phase switch and phase duration itself.[21] From this viewpoint, the insignificance of the duration variable when considered with the leading index does not necessarily nullify the previous evidence found regarding positive duration dependence in expansions. On this argument, a reasonable conservative conclusion then to reach from the model incorporating both duration and the leading indexes is simply that duration is evidently considerably less significant for expansions than it is for recessions.

[21] An analogy would be whether a trend existed in data on a variable. It may well be that there is a set of economic fundamentals which give rise to the trend but the key issue is simply whether the data have a trend.

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TABLES AND GRAPHS

Table 1: NBER Chronology - http://www.nber.org/cycles/.

Trough	**Peak**	**Recession Duration Peak to Trough**	**Expansion Duration Trough to Peak**
October 1949	July 1953		45 months
May 1954	August 1957	10 months	39 months
April 1958	April 1960	8 months	24 months
February 1961	December 1969	10 months	106 months
November 1970	November 1973	11 months	36 months
March 1975	January 1980	16 months	58 months
July 1980	July 1981	6 months	12 months
November 1982	July 1990	16 months	92 months
March 1991	March 2001	8 months	120 months
November 2001		8 months	

Table 2 Parameter Estimates of Various Markov Regime Switching Multinomial Logit Models.

	1.	2.	3.	4.	5.	6.	7.	8.
α_1	4.0850	4.9666	4.2619	4.5649	4.6248	5.0121	5.3454	5.3240
	(0.3361)	(0.6748)	(0.4021)	(0.4719)	(0.5044)	(0.7283)	(0.8030)	(0.8040)
	[0.3361]	[0.7010]	[0.3550]	[0.4253]	[0.4286]	[0.7718]	[0.8676]	[0.8425]
δ_1		-0.0179				-0.0143	-0.0142	-0.0127
		(0.0100)				(0.0100)	(0.0101)	(0.0099)
		[0.0104]				[0.0103]	[0.0104]	[0.0099]
β_1^{SLI}			1.2491		0.7516	1.2806		0.7571
			(0.4219)		(0.4877)	(0.4552)		(0.5250)
			[0.2952]		[0.4051]	[0.3029]		[0.4439]
β_1^{LLI}				1.2362	1.0097		1.2868	1.0452
				(0.3588)	(0.4100)		(0.3892)	(0.4419)
				[0.3067]	[0.4042]		[0.3542]	[0.4690]
α_2	2.2304	4.6753	1.4287	1.9999	1.5458	4.2159	4.3469	4.7891
	(0.3328)	(0.9725)	(0.4817)	(0.3444)	(0.5091)	(1.0848)	(1.0504)	(1.2922)
	[0.3327]	[0.7268]	[0.5277]	[0.3467]	[0.5186]	[0.8726]	[0.7652]	[1.2055]
δ_2		-0.3059				-0.3353	-0.2759	-0.4234
		(0.0948)				(0.1057)	(0.1029)	(0.1481)
		[0.0710]				[0.0965]	[0.0742]	[0.1379]
β_2^{SLI}			-0.8062		-0.4988	-0.6355		-0.9418
			(0.4312)		(0.4599)	(0.3368)		(0.4785)
			[0.5760]		[0.5710]	[0.2732]		[0.4109]
β_2^{LLI}				-0.7032	-0.5541		-0.2644	0.4488
				(0.2978)	(0.3208)		(0.2960)	(0.4948)
				[0.2458]	[0.2591]		[0.2672]	[0.5109]
LL	-78.660	-70.870	-72.307	-69.582	-67.817	-64.930	-64.931	-61.762

Note: The probability of staying in regime *i* (i=1 denotes expansion, i=2 denotes recession) is given by

$$P(S_t = i \mid S_{t-1} = i, \psi_{t-1}) = (1 + \exp\{-(\alpha_i + \delta_i d_{t-1} + \beta_i ' Z_{t-1})\})^{-1}$$

where Z_{t-1} is a vector of leading indicators, d_{t-1} is the duration of the current expansion or recession and defined as $d_t = \begin{cases} d_{t-1} + 1 & \text{if } S_t = S_{t-1} \\ 1 & \text{if } S_t \neq S_{t-1} \end{cases}$. Parameter estimates are reported for a range of models with asymptotic standard errors in parenthesis and Newey-West robust standard errors with 6 lags in square brackets.

Table 3: Means and root mean squared error of parameter estimates across simulations

	1	2	3	4	5	6	7	8
α_1	3.7581	4.3774	3.9298	4.2613	4.3220	4.4446	4.6796	4.7891
	(0.0109)	(0.0529)	(0.1008)	(0.1051)	(0.1472)	(0.1424)	(0.1506)	(0.2198)
δ_1		-0.0157				-0.0123	-0.0099	-0.0106
		(0.0012)				(0.0017)	(0.0020)	(0.0025)
β_1^{SLI}			1.0554		0.6948	1.0590		0.7292
			(0.1526)		(0.2003)	(0.1664)		(0.2150)
β_1^{LLI}				1.0039	0.7903		1.0070	0.7908
				(0.1012)	(0.1282)		(0.1126)	(0.1404)
α_2	1.9260	4.3502	1.1540	1.7933	1.3489	3.9022	4.0576	4.7857
	(0.0672)	(0.8573)	(0.3658)	(0.1303)	(0.4095)	(1.1964)	(1.0527)	(1.9150)
δ_2		-0.2516				-0.2763	-0.2283	-0.3940
		(0.0896)				(0.1274)	(0.1087)	(0.2261)
β_2^{SLI}			-0.7598		-0.4857	-0.6022		-0.9610
			(0.4615)		(0.5149)	(0.4277)		(0.6271)
β_2^{LLI}				-0.5401	-0.3651		-0.2365	0.4875
				(0.1864)	(0.2175)		(0.2588)	(0.5804)

Note: We report the average coefficient across 1000 simulated business cycle chronologies and in parenthesis below it the root mean squared error of the estimated coefficients across the simulations.

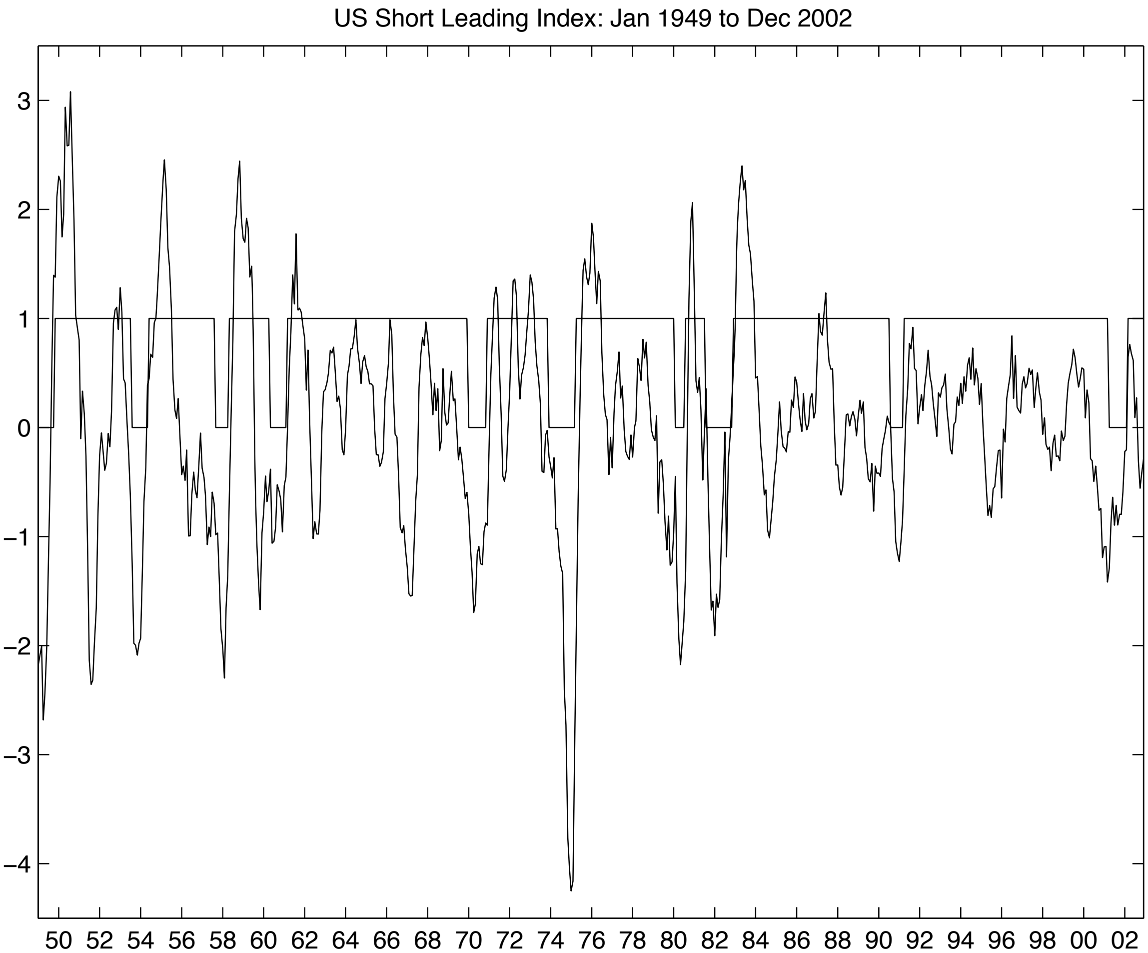

Figure 1: Plot of Short Leading Index with NBER Chronology.

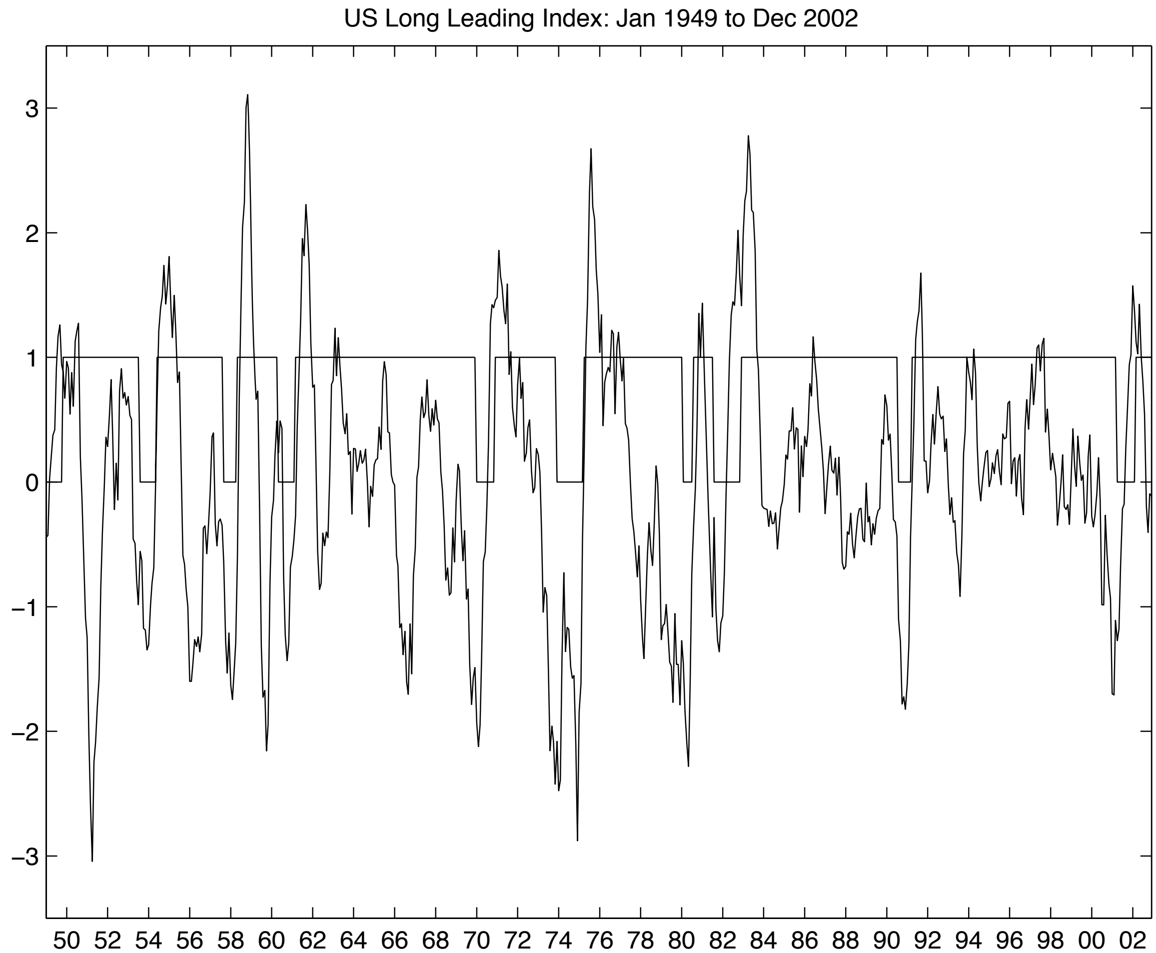

Figure 2. Plot of Long Leading Index with NBER Chronology.

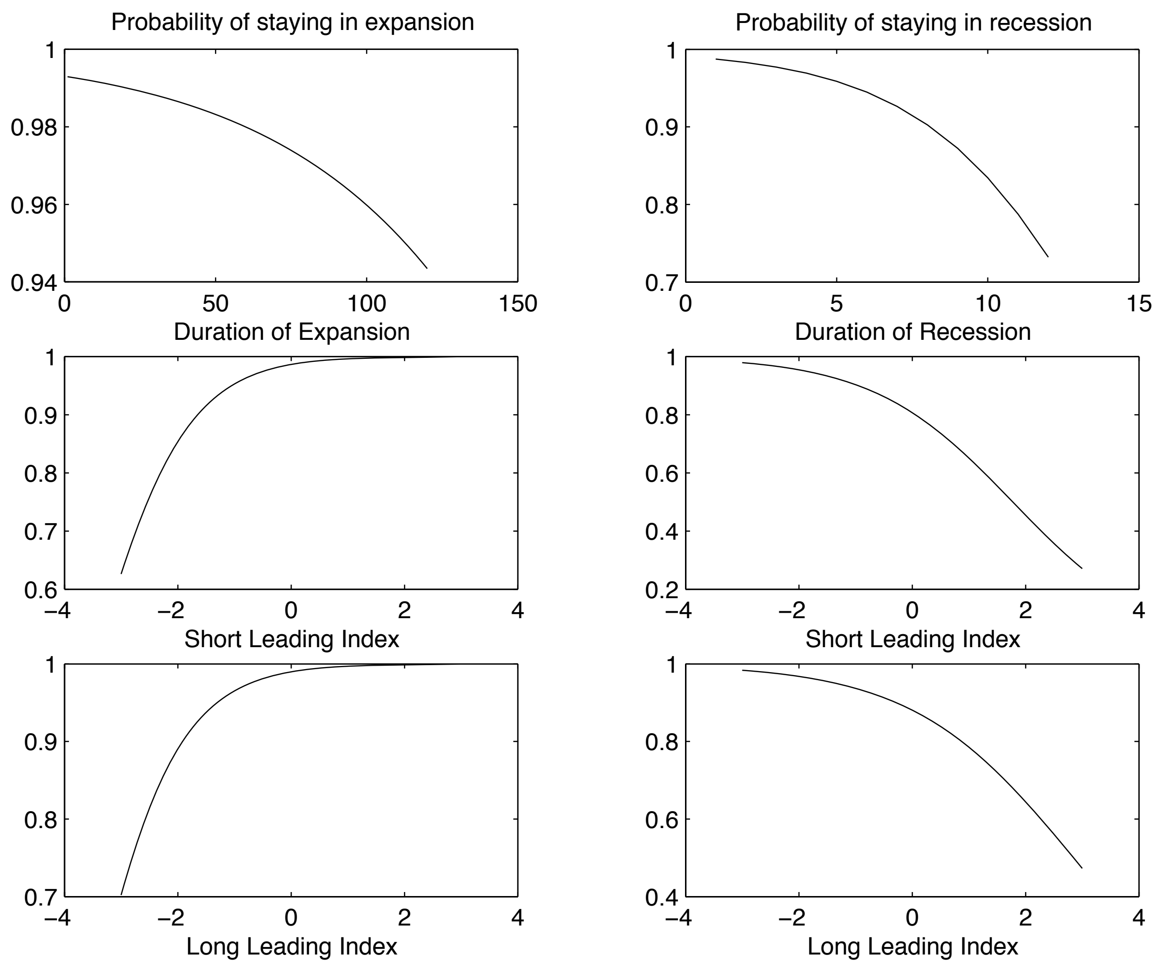

Figure 3. Plot of Transition Probabilities for Recessionary and Expansionary States with only one explanatory parameter.

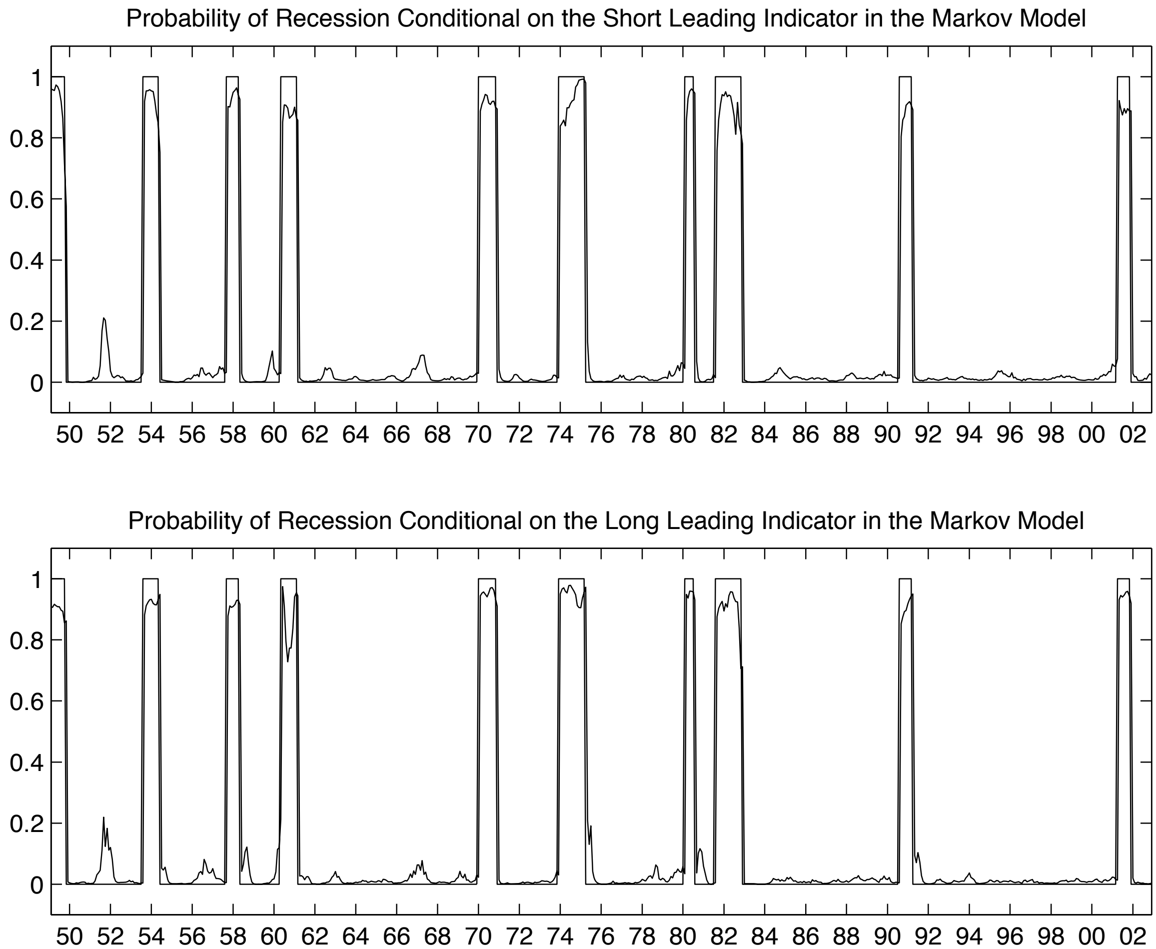

Figure 4a. Plot of Recession Probabilities in the Multinomial LOGIT Markov Model $P(S_t = 2 \mid S_{t-1}, \Psi_{t-1})$.

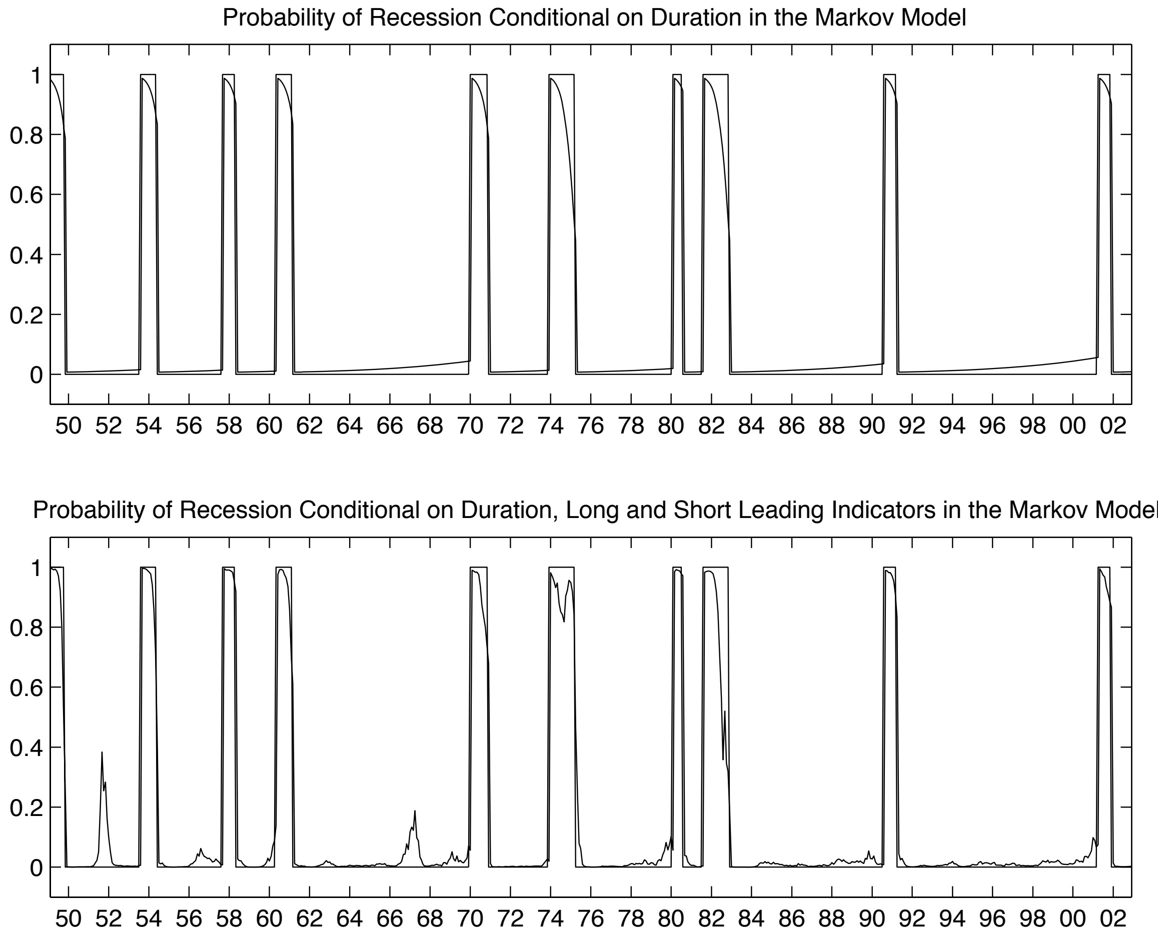

Figure 4b. Plot of Recession Probabilities in the Multinomial LOGIT Markov Model $P(S_t = 2 \mid S_{t-1}, \Psi_{t-1})$.

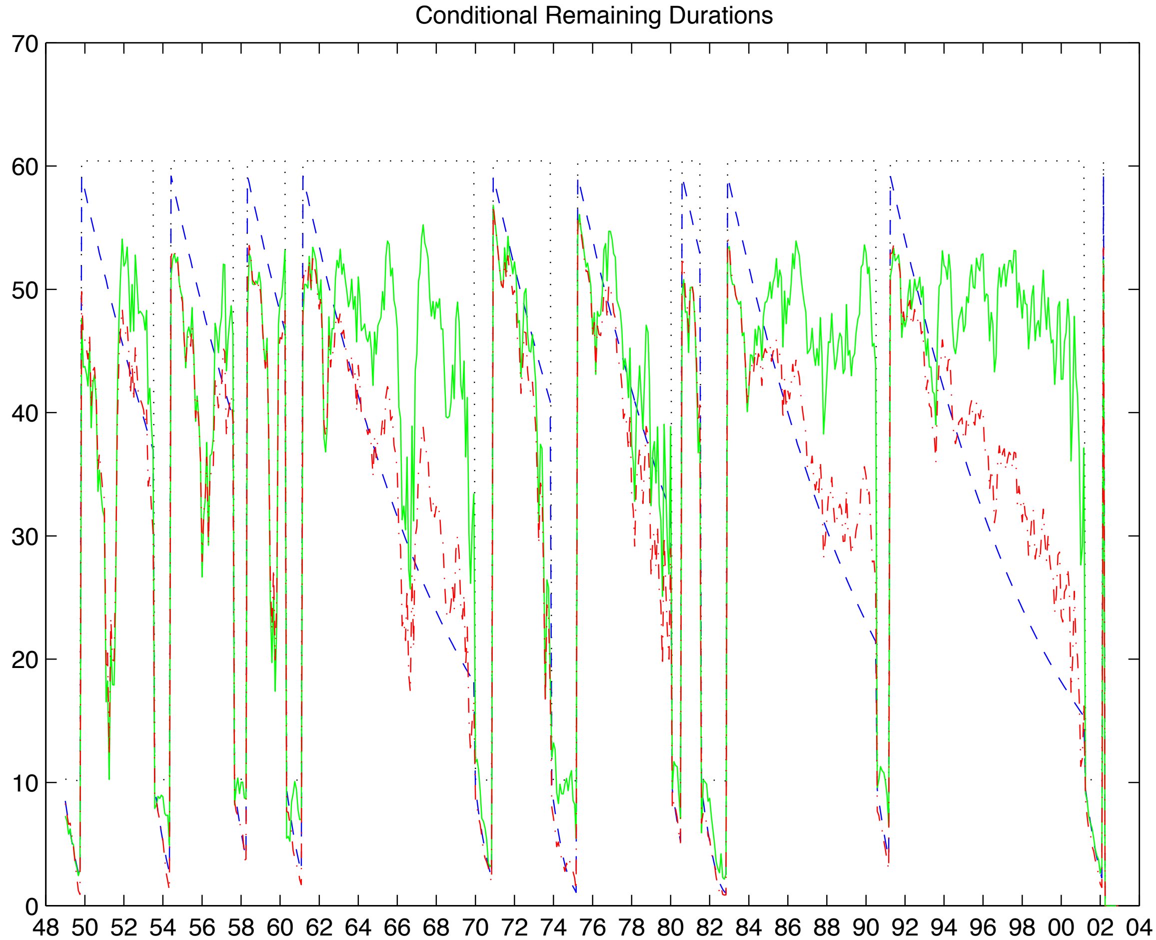

Figure 5. Plot of conditional remaining duration.

Notes: The four plots are for the following models: 1) Model 1, constant transition probabilities (dotted line); 2) Model 2, only duration (dashed line); Model 5, only leading indicators (solid line); and Model 8, both leading indicators and duration (dashed and dotted line).