UNIVERSITY OF SOUTHERN QUEENSLAND

Modelling Dispersion in Turbulent Boundary Layers Using Centre Manifold Technique

A dissertation submitted by

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For the award of the degree of

Doctor of Philosophy

2014

Dedication

To my family

Certification of Dissertation

I certify that the ideas, experimental work, results, analysis and conclusions reported in this dissertation are entirely my own effort, except where otherwise acknowledged. I also certify that the work is original and has not been previously submitted for any other award.

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ENDORSEMENT

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Acknowledgments

The research reported in this dissertation was carried out at the School of Agricultural, Faculty of Health, Engineering and Sciences, and the Computational Engineering and Science Research Centre (CESRC), USQ.

Firstly, I would like to thank my principal supervisor A/Prof. Dmitry Strunin for his invaluable guidance and useful suggestions. This dissertation would not have been completed without his continuous support.

I would like to thank my co-supervisor Prof. Thanh Tran-Cong for his instructive discussions during my PhD study.

I would like to thank Dr Duc Ngo-Cong for his cooperation with me especially regarding the experimental work.

Finally, I would like to thank the Iraqi government for giving me the opportunity to complete the PhD Study.

Abstract

The present research is concerned with modelling dispersion of contaminants and substances in turbulent boundary layers based on centre manifold approach. The results of this dissertation are as follows. (i) We derived accurate transport equations involving high-order spatial derivatives to describe the averaged turbulent transport of tracers and contaminants along turbulent channel flows. The central point of the model is the advection-diffusion equation supplemented by no-flux boundary conditions on the bottom and the surface. In all realisations of the flow analysed in this work, the layer has a universal velocity structure and well-defined turbulent diffusion coefficient. The turbulence is assumed well developed, which provides the mechanism of fast cross-flow mixing and thereby justifies the use of the centre manifold method (Mercer and Roberts, 1990). Using the originally two-dimensional transport equations, we derived the one-dimensional model in which the advection, diffusion, dispersion and higher-order coefficients are calculated in terms of the parameters controlling the flow. (ii) We formulated an analytical framework of the averaged transport of contaminants in turbulent boundary layers over smooth and rough substrates; (iii) An advection-diffusion equation is derived for the flow through urban canopies simulated by cubic arrays. (iv) We justified the centre manifold approach by the direct comparison of the numerical solutions of the averaged (1-D) and original (2-D) models using the one-dimensional integrated radial basis network (1D-IRBFN) method.

In the 1D-IRBFN method a Cartesian grid is used to discretise the spatial domain. The method uses the integration instead of conventional differentiation, which provides an effective way to implement derivative boundary conditions. The numerical solutions of the derived 1-D equations obtained by the centre manifolds are in a good agreement with those of the original 2-D advection-diffusion equation. In particular, the models yield practically the same value of the velocity of the point of maximum depth-averaged concentration along the channel.

We also compared the 1D-IRBFN solutions for the 1-D model with the solutions of the original 2-D model by successively adding higher-order derivatives into consideration such as the advection, diffusion and dispersion. A good convergence was observed. The numerical results confirm that the effect of longitudinal diffusion is negligible. We note that our work can be viewed not only as the confirmation of the centre manifold approach by the 1D-IRBFN method but also as a confirmation of the numerical method by the centre manifold theory.

In our analysis we considered two types of the velocity profile across the channel: the classical logarithmic profile and, according to an alternative and more recent model, power profile. The power profile is based on a different similarity hypothesis (Barenblatt, 2000) compared to the classical logarithmic theory. Arguably the power law better fits experimental measurements of the velocity distribution over the self-similar intermediate region adjacent to the viscous sublayer for a wide variety of boundary layer flows. We separately investigated the dispersion for both the logarithmic and power profile.

Further, we derived even higher-order partial differential equations governing the longitudinal dispersion. From numerical viewpoint, including higher-order spatial derivatives increases the accuracy of the averaged model derived by the centre manifold approach, especially for very large Reynolds numbers.

We also constructed an averaged model of shear dispersion in the turbulent flow above a canopy. The model contains as independent parameters the friction velocity, total thickness of the flow, height of the canopy and frontal area density of the canopy. The model is reduced by the centre manifold procedure to a universal one-dimensional model written in terms of the depth-average concentration of the tracer. The advection and diffusion coefficients, governing the transfer in the averaged model, are found in terms of the independent parameters. The used approach required lengthy derivations and produced quite cumbersome expressions.

However, we emphasize the following important aspect of all the derived one-dimensional centre manifold models: they reveal a hidden property of the transport process, namely the asymptotic one-dimensional law for the averaged concentration. This is a remarkable feature of the originally two-dimensional formulation. Whether such a law exists and what form it might have is not obvious beforehand. At the same time, from practical viewpoint, our results can be used to calculate the distance travelled by the contaminant spill, and the size of the spill; of course those can only serve as a tool for rough estimates.

Papers Resulting from the Research

1. Strunin, D. V. and Mohammed, F. J. (2012). Numerical Analysis of an Averaged Model of Turbulent Transport near a Roughness Layer, *Journal of Australian and New Zealand Industrial and Applied Mathematics*, **53**: C142-C154.

2. Mohammed, F. J., Ngo-Cong, D., Strunin, D. V., Mai-Duy, N. and Tran-Cong, T. (2014). Modelling Dispersion in Laminar and Turbulent Flows in an Open Channel Based on Centre Manifolds Using 1D-IRBFN Method, *Journal of Applied Mathematical Modelling* **38**: 3672-3691.

3. Mohammed, F. J., Strunin, D. V., Ngo-Cong, D. and Tran-Cong, T. (2014). Asymptotics of Averaged Turbulent Transfer in Canopy Flows, Journal of Engineering Mathematics, accepted.

4. Ngo-Cong, D., Mohammed, F. J., Strunin, D. V., Tran-Cong, T. and Mai-Duy, N. (2013). Higher-Order Transport Equations for Turbulent Channel Flows, *Journal of Australian and New Zealand Industrial and Applied Mathematics*, to appear.

Contents

Acronyms & Abbreviations

1D-IRBFN	One-dimensional Indirect/Integrated Radial Basis Function Network
FDM	Finite Different Method
1-D	One-Dimensional
2-D	Two-Dimensional
DRBFN	Direct Radial Basis Function Network
IRBFN	Indirect/Integrated Radial Basis Function Network
MQ	Multiquadric
PDE	Partial Differential Equation
RBF	Radial Basis Function
RBFN	Radial Basis Function Network
RHS	Right Hand Side

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