PROPORTIONAL REASONING: A CASE STUDY HIGHLIGHTING ITS SIGNIFICANCE IN MATHEMATICS CURRICULUM

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ABSTRACT

This paper outlines a study that was undertaken with a typical thirteen year old boy. This boy was experiencing difficulty with his work on percentages, basic trigonometry and simple equation solving. Our intervention with the boy led us to suspect that there was a connection between his understanding (or lack thereof) of these concepts and his capacity to think and reason proportionally. As part of our investigation we explored several questions including "How necessary is it that a child be able to think and reason proportionally?" and "How important is this skill for the successful understanding of other mathematical concepts?" The paper documents our experiences, describing the many strategies that we used to help this boy improve his capacity to think and reason proportionally. It reports his progress over a period of several weeks, and analyses his learning and his attitude/s to what he was learning. Most importantly this study serves to highlight the important interrelationship between proportional reasoning and other math topics such as trigonometry, percentages and equation solving.

INTRODUCTION

There is a body of evidence suggesting that proportional reasoning is not afforded a high enough priority in the typical middle school mathematics classroom. Some researchers (Ilany, Keret & Ben-Chaim, 2004; Lo & Watanabe, 1997) highlight the importance of proportional reasoning in understanding concepts such a trigonometry, gradient, percentages and algebra. Despite such emphases the typical middle school mathematics textbook generally limits teaching of such to one chapter, and often less that this. Lamon (1995) shares this view, adding that symbols are then introduced into proportion problems before sufficient groundwork has been laid for students to understand the role of symbols in proportional reasoning problems. Also, since the topic of ratio usually leads into proportion, the situation is further exacerbated by misconceptions in ratio that can be traced back to misconceptions in decimal and fractional concepts (Pearn & Stephens, 2004; Lo & Watanabe, 1997).

We would suggest that there is a strong argument in favour of adopting a more integrated approach to the teaching of proportional reasoning. It has been suggested that this needs to be commenced at an early stage, as early as the teaching of whole number and fractions. It has been further suggested that developing proportional reasoning skills is akin to number sense or data sense, which is developed by a wide variety of experiences over time (Norton, 2005). To emphasis this, Norton used his audience at PME to pose the problem "It costs 12 cents to buy 20 straws. How much will 8 straws cost?" and it soon became evident that there were a number of methods for solving this problem, with probably the least of these being the traditional drill and practice technique of cross-multiplication.

THE CASE STUDY

Tim (not his real name) is thirteen years old and in grade nine at school. He first came to our attention through the recommendation of his teacher. We found him to be reasonably motivated yet frustrated by his failure to understand concepts that he was being taught. At the time his class was about one week into the topic of trigonometry and so we decided to focus our interview questions in this area:

Interviewer: Tim – you have been studying the expression "tan" – can you explain what it means?
Tim: Umm …… you can use it to find the side of the triangle that you don't know.
Interviewer: Yeah – that's correct but can you tell us what tan is? Why can it be used to find the missing side of a triangle. What is it about tan that makes this possible (hoping that he might mention the relationship between similar triangles that we know had been covered in his class)?
Tim: Umm …..

At this point we used the computer software Geometer's Sketchpad to construct a right-angled triangle. The software was then used to record measurements for two of the triangle's sides and the single included angle as indicated in figure 1a. Also recorded was a value for the ratio of the two sides ie 0.67. Tim noted with a degree of interest that when he clicked on the side opposite the marked angle and resized the triangle, that the ratio did not change (figure 1b). We let him play with this for awhile and then asked him if there was a situation when the ratio was not the same. He immediately responded that this happened only when he changed the angle and he demonstrated this (figure 1c). We met Tim again the following week and picked up where we left off:

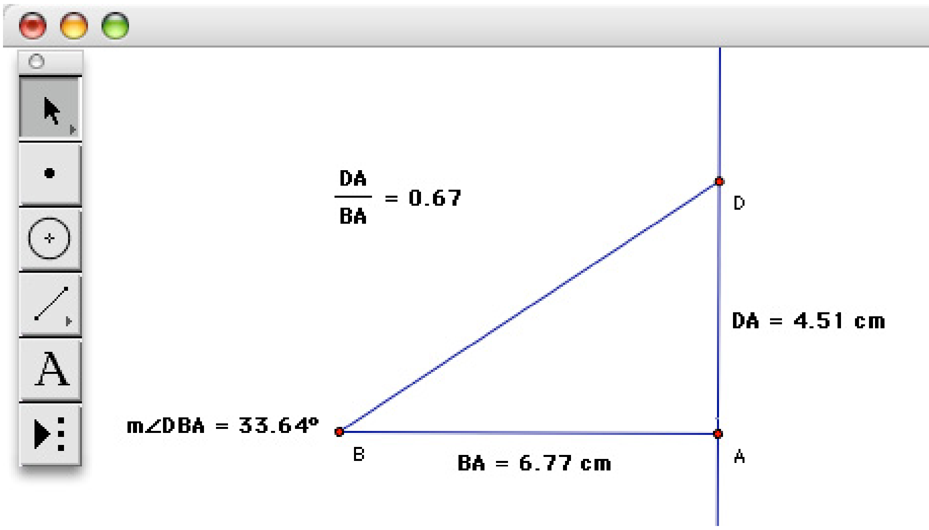

Figure 1a

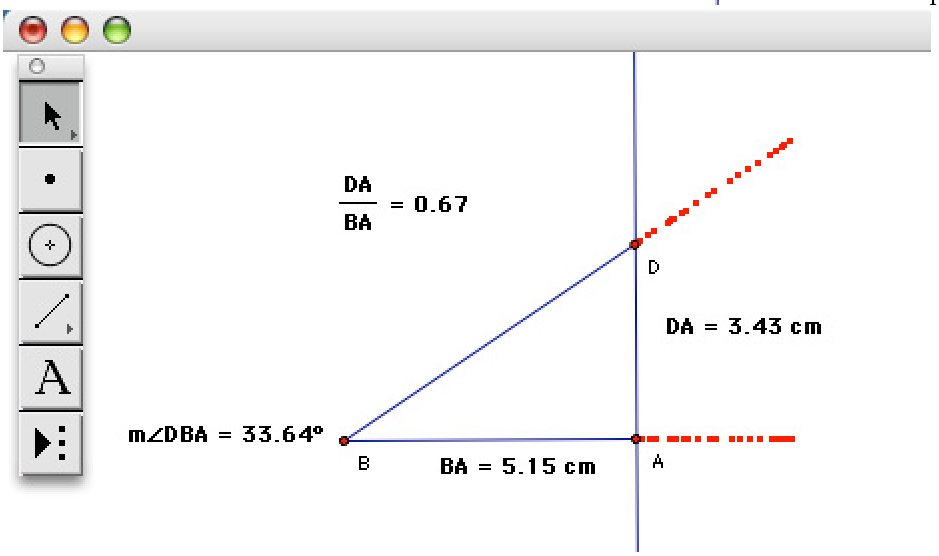

Figure 1b

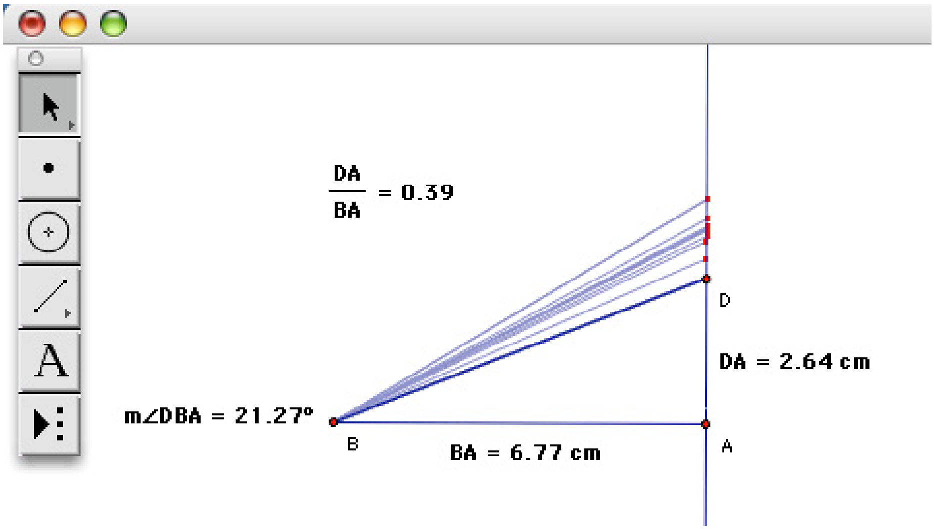

Figure 1c

Interviewer:	What about if you always kept the angle at 33.64 degrees?
Tim:	No – the ratio won't change ….. umm …. Yeah it's going to stay the same.
Interviewer:	(Hands Tim a calculator) if I now asked you to put 33.64 degrees and press tan ….. what do you think you would get for an answer on the calculator.
Tim:	Would it be that ratio???? (in an enquiring tone)
Interviewer:	What do you think?
Tim:	Yeah – I reckon it will be that ratio – yeah it will be (as he inputs the values into the calculator).
Interviewer:	How did you know this so quickly?
Tim:	Well …… if the angle always stays at 33.64 degrees and the triangle just gets bigger or smaller ….. that ratio is going to be the same and we learnt in class how to find the tan of an angle.

We then bring up on the computer Tim's earlier illustration (figure 1b. We use a stickit to cover up the measurement of one of the two sides, in this case, the base measurement of 5.15 .

Interviewer:	Tim – we have covered up one of the sides but note that we kept the angle the same. Do you reckon you could work out what the measurement is of this side that we covered up?
Tim:	Use the tan of 33.64.
Interviewer:	Ok, how?
Tim:	(obtains tan with calculator and writes) 3.43/x = 0.67
Interviewer:	Great – now find x and you have your missing side.
Tim:	0.67 divided by 3.43 …. Isn't it?
Interviewer:	You could be right ….. why do you say this?
Tim:	Well ….. it's 3.43 divided by x here … so x has to go to the other side ….. this is when I get mixed up …. When I have to solve an equation.
Interviewer:	Ok – lets make it a little easier. Lets say that we had an angle where the tan was 0.6 exactly rather than 0.67. What is 0.6 as a fraction?
Tim:	6 tenths.
Interviewer:	ok – now rewrite what you had before but with this fraction value instead.
Tim:	(writes) 3.43/x = 6/10
Interviewer:	3.43 parts out of how many parts is roughly the same as 6 parts out of 10 parts. Read this backwards ……. It might make it a little easier.
Tim:	Umm ….. 6 parts out of 10 parts is 3.43 parts out of how many parts. Umm ….. it would be about half wouldn't it?

Interviewer:	I think I know what you mean but try to explain it to me.
Tim:	Half of 6 is about 3 …. so for 3.43 it would be about half of 10 ……. Pause
Interviewer:	Keep going …. I don't mind a rough answer.
Tim:	Well …… a half of 10 ….. 5
Interviewer:	Ok – so roughly 5 wouldn't you agree. So the missing side is roughly 5 or exactly 5.15 (removing the stickit to illustrate answer ie figure 1b).
Interviewer:	So 3.43/5.15 = 6/10 ….. does this make sense? Would you like to use your calculator to confirm this? (Tim uses calculator and appears happy with his result)

This experience opened our minds to the following question. Was it the trigonometry content or was it Tim's proportional reasoning skills that prevented him from understanding this area of mathematics? We asked Tim's opinion. He explained that only now did he understand "why the ratio stayed the same when the angle didn't change". When we asked him why, he said that he could "see it better when the computer was doing the calculations". We asked if there was anything else that he did not understand from before. He explained that even though he understood the steps for finding the missing value, x, he seemed to "get it wrong a lot of the time".

Tim appeared to be saying that he had problems with trigonometry initially because he did not understand the rationale that underpins the tangent ratio. Our observation was that he was able to formulate a "proportionality equation" which when solved would yield the missing side. He did this by following a drill and practice procedure. When it came to solving the equation he again relied on a rote memorised procedure for doing so – albeit a procedure that he was not able to adapt to cater to all situations. For all intensive purposes then, Tim was able to follow the problem through to it's conclusion and yet here he was telling us that he did not understand what he was doing until now.

Tim's experience in this particular instance does highlight one thing. In Tim's case or in the case of children like Tim, there is a need to understand a concept at a conceptual level before he/she is asked to move on to more routine and algorithmic procedures. The other question that we ask is how much of Tim's difficulties could be attributed to his proportional reasoning skills (or lack thereof)? We decided to explore this area further. We used an excerpt from Tim's interview when he stated that it was equations like 3.43/x = 6/10 that confused him, and explained that we were going to try and see how he was thinking about such equations. (Note that at this point, we would want to caution the reader that there is a difference, conceptually at least, between a ratio and a fraction despite the fact that in many texts they are treated as being the same. What we mean by this, using an example, is that the fraction 2/5 is conceptualized as 2 parts out of the 5 parts that make up the whole. On the other hand, the ratio 2:5 may be conceptualised as the comparison of 2 parts with 5 parts, thereby leading one to conceptualise the situation as 7 parts making up the whole. We decided, nevertheless, to proceed as follows and manage this contradiction should it prove necessary.)

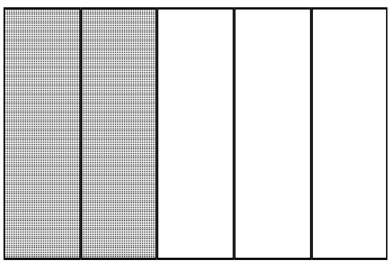

Figure 2

We used a typical paper folding exercise (figure 2) that is sometimes used for illustrating the equivalence of fractions. Tim is given this A4 sheet of paper which is representative of the common fraction 2/5. We explain that the sheet of paper illustrates the fraction two fifths because two out of the five equal parts have been shaded in. We ask Tim to fold it in such a way that what he sees now is the representation in figure 3 following.

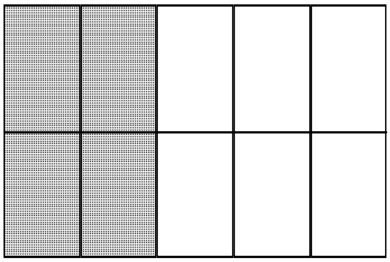

Figure 3

Tim is able to tell us that what he sees now is the fraction 4/10 and he does so with an air of confidence. With a little direction Tim is able to readily accept the conclusion that 2/5 = 4/10. The dialogue continues:

Interviewer:	If I told you that we had the equation 2/4 = x/10 then what would be the value of x?
Tim:	4
Interviewer:	How do you know?
Tim:	From what we just did – the paper.
Interviewer:	Ok …… what if I gave you 4/10 = x/20 …. Would you know how to find x?
Tim:	8 ….. I think you'd just double the four.
Interviewer:	Ok ….. that was fast …. Well done …… show me why …. Use the sheet of paper.

We expected Tim to simply fold the sheet again along the horizontal plane just once more so that the representation would show 8/10. Interestingly Tim tried to do several folds along the vertical plane. We let him proceed. After a few minutes, it was evident that he was not getting anywhere, so we gave him a copy of the same sheet of paper that we used earlier in figure 2 and asked him to make the first fold to illustrate that 2/5 = 4/10. At this point, he made the connection and folded the sheet along the correct plane, proudly explaining why 4/10 = 8/20, going even further by attempting (with difficulty) to make one more fold in an attempt to show us why 8/20 = 16/40. We should mention that we very nearly corrected Tim before he had the chance to make that first "incorrect" fold. We would now argue that it is a good thing that we resisted the temptation to do so. Our view is that it was this process of self-correction that allowed speedier understanding as well as a sense of ownership of that understanding on Tim's part.

We decided to stay on the topic of equivalent fractions a little longer and so we posed the following problem on cricket wicket taking averages. "Which bowler has the higher wicket taking average? Bowler A who takes 7 wickets in 10 overs or bowler B who takes 9 wickets in 12 overs?"

We asked Tim to consider what he thought Bowler A's average might have been after 5 overs to which Tim responded "half of that wouldn't it be …… 3 and a half". Assuming that bowler A kept up the same average, we asked for the average after 20 overs to which Tim again correctly gave the answer as "14". We asked Tim to write this as a proportion to which he eagerly wrote "7/10 = 14/20". We asked Tim to use an A4 sheet of paper to illustrate that these two fractions were in fact equal. After a little thought he divided the sheet into 10 vertical strips and then shaded in 7 of these, correctly completing the task by folding the sheet along the horizontal plane (figure 4):

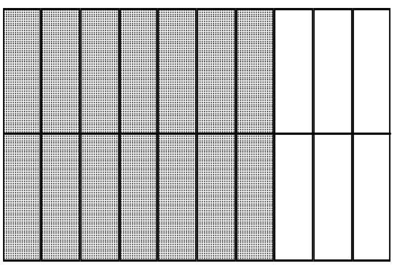

Figure 4

At this juncture we decided to help Tim extend this kind of thinking with the help of a spreadsheet. We were aware that Tim had done some work in Excel and had some basic spreadsheeting skills. We gave him the following (figure 5) spreadsheet containing only (what you see in bold is all that Tim was given) and asked Tim to use it to illustrate what he had done earlier, this time for both bowlers. He needed a little prompting but this was more to do with the nature of the task than the doubling and halving aspect of this proportional reasoning task. He completed this task to our satisfaction. We asked Tim who he thought had the better average to which he responded bowler B because 4.5 wickets out of 6 overs is better that 3.5 wickets out of 5 overs. We nearly asked Tim why he thought this, but then guessing (rightly or wrongly) that he probably did not have a justifiable reason, we asked him what he thought bowler B's average would have been after 5 overs rather than 6. Tim answer was "oh …. About 4 … I don't know".

	A	B	C	D	E	F	G
1	**A**	3.5	**7**	14	28	52	
2		**5**	**10**	**20**	40	80	
3							
4	**B**	**4.5**	**9**	**18**	36	72	
5		6	**12**	24	48	96	
6							

Figure 5

We then varied the spreadsheet and presented Tim with the following (figure 6). Again, all Tim could see were the bolded numbers while the remaining numbers were the answers that we were hoping Tim would be able to compute with minimal assistance. We left the top halves of columns G and H blank. Tim filled in the missing values in columns F and G without any assistance. He thought about column D, left it for a minute and looked at column C, returned to column D, asked us for confirmation whether all that was required for him to get the answer was to divide 7 by 10 and 9 by 12. When we confirmed this he appeared a little bemused that the problem should be that straightforward. He then moved to column C and filled in these figures with confidence. He then moved onto column B, thought for a bit, and asked for help. We asked if he was to add what was in column C and D, whether this would give him the right answer. He seemed to accept this. He finally moved onto column H and after two failed attempts, decided for himself that adding together the values in C4 and G4 would give him the required answer.

	A	B	C	D	E	F	G	H
1	**A**	2.1	1.4	0.7	**7**	35	-	-
2		**3**	**2**	**1**	**10**	**50**	-	-
3								
4	**B**	2.5	1.5	0.75	**9**	18	36	37.5
5		**3**	**2**	**1**	**12**	**24**	**48**	**50**

Figure 6

We noted that now we could be truly convinced that the better average was 9 out of 12 and we asked him why we were sure of this. He responded that 37.5 out of 50 was more wickets than 35 out of 50. We asked him to study this table carefully and then we asked him if it was necessary to use an average out of 50 for each bowler in order to decide who had the better average? It was at this point, we believe, that Tim underwent a light-bulb type of experience, exclaiming " just make them the same ….. as long as it is the same number of runs for each bowler that …. Yeah ….. 2.5 out of 3 is better than 2.1 out of 3….. 1.5 is better than 1.4 ….. 0.75 is better than 0.7 ….. oh ….. so all you have to do is just divide …. Yeah ….. divide the wickets by the overs …. Wickets per over".

When we re-visited the above transcripts after the interview we came to the conclusion that Tim was in fact working with concepts that stretched well beyond simple proportion. There were elements of ratio, rate, equation-solving and maybe even the idea of using percentages as a standard by which comparisons may be made between ratios.

At this point in the paper we would like to remind readers that when we were first introduced to Tim we were told that he did not understand the work that he had been taught on percentages. We decided that at our next meeting with Tim we would use his understanding so far as a bridge for developing understanding in percentages. We commenced the next session by referring Tim to the bowling averages problem and asking "35 out of 50 is the same as what out of 100" and "37.5 out of 50 is the same as what out of 100". Tim simply doubled these figures to arrive at the respective answers. We asked him what the connections were between what he had just done and the topic of percentages. He confidently explained that since 35 out of 50 was the same as 70 out of 100, this meant that it was 70%. We then asked Tim whether this is how he did it before. Tim considered our question for a moment and then responded "…. actually …. no – that's not the way I did it in class. We learnt to do it like 35 divided by 50 and then times this by 100" and he proceeded to demonstrate this on his calculator obtaining 70 as his answer. We then asked him which method he preferred to which he responded that he preferred our method because he understood why it worked. He went on to explain that even though he remembered to "divide and then times by 100" he would often get confused with the wording of some questions. We asked him if our method would work for all questions and we asked him to try our method on a bowling average of 4 wickets in 15. Tim used a doubling procedure to obtain 8/30, 16/60 and then added these two to get 24/90 and after a pause, appeared happy to hand back the problem to us. We then asked him that given an average of 4 wickets in 15 overs, whether he would be able to work out how many wickets this was in 1 over. Tim picked up his calculator saying "4 divided by 15 …… 0.2667 ….. 0.2667 wickets per over" to which we responded "and what is this per 100 overs?" to which Tim responded replied "0.2667 times 100 …… 26.67 …… ahh ….. I get it …… that's why you divide and then times by 100". Our observation of this is that it was another light-bulb experience for Tim.

We went on to explain to Tim that the percentage problem that he had just solved could in fact have been written as 4/15 = x/100. We summed up what he had learnt so far by saying that the reason for first dividing the 4 by 15 was so that we are able to establish that 4 parts compared to 15 parts is in same ratio as 0.2667 parts compared to 1 part, which we then multiply by 100 in order to establish that this then is in the same ratio as 26.67 parts to 100 parts. Tim seemed pleased with himself which we took to mean that he was experiencing a sense of self-satisfaction with his comprehension of the work. We further explained that 26.67 parts out of 100 parts is commonly referred to as 26.67 percent (%) and it comes from the Latin derivation per centum meaning per hundred.

The last area identified as problematic for Tim had to do with "equation solving". This is a topic that is not easy to teach, especially if you want to do it well. It takes some highly skilled teaching to have children develop the required facility in this skill. There are numerous activities highlighting concepts such as backtracking, balancing, flowcharts and function-machines that should be covered over a period of years. At this point in time we did not feel it wise to deviate too far from the main theme of our investigation, proportional reasoning, in order to undertake such a diagnostic task. At the same time, however, we did not want to lose a valuable opportunity to link at least a little bit of equation-solving with the concept of proportional reasoning.

We gave Tim the proportion 2/4 = 4/8 and got him to write the cross-product for this ie 2x8 = 4x4, noting that they were equal. We got Tim to volunteer several proportionality expressions including fractional ones such as 1.5/2 = 3/4 and illustrate to us that the two cross-products were equal. We then gave Tim 4/2 = 2/1, and Tim applied the process to this proportion easily. We then gave Tim 15/5 = 3 and asked Tim if it was possible to apply the procedure to this. Tim explained as he wrote " er ….. 5x3 = 15x …. Er … there is no number here …… ah ….. would I just put times 1?". We confirmed that he was correct and asked how he came up with this answer. Tim explained, " there is no denominator for 15 so I just put 15 over 1".

At this point we returned to the equation 4/15 = x/100, reminding Tim that he had previously worked out the answer to this percentage question as being x = 26.67. Tim proceeded as follows:

Tim writes and explains : (writes) 4/15 = x/100
4 x100 = 15 x X
400 = 15X
….. Tim pauses

Interviewer:	what is 15 times 10?
Tim:	150
Interviewer:	ok …. So what is 15 times 20 then?
Tim:	double that …. 300
Interviewer:	what is 15 times 30?
Tim:	add those …… 450
Interviewer:	ok …… so roughly 15 times what is 400?
Tim:	I reckon …. About 25 … no 26
Interviewer:	that was pretty good ….. how did you know 26
Tim:	oh …. Cos … I said 25 and then I remembered the answer was 26.6 something. …… Tim appears to be grinning with pride
Interviewer:	Would you like to see a quicker way of getting an accurate answer?
Tim:	yes
Interviewer:	Think of a small number less that 10 and multiply it by 2 and tell me what you got
Tim:	14
Interviewer:	your number is 7 ….. how did I get that so quickly?
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Interviewer:	your number is 18 ….. how did I know this?
Tim:	you multiplied it by 2
Interviewer:	do you understand what we are doing here? Getting back to our original equation …. after I multiplied my number by 15 I got 400 so before I multiplied it by 15, I must have had one fifteenth the amount which is 400 divided by 15.
Tim:	yes.

Next we gave Tim the following problem: "30 is what percent of 50?" Tim wrote the equation 30/50 = x/100 and used this to record the cross-products 30 x100 = 50 x X. He then comfortably solved this equation to obtain X = 60%. We jokingly suggested to Tim that he must have been lying when he said he had trouble with percentages. Tim responded that he understood questions like this but when they were asked a different way he "get's all confused". Our suspicion was that Tim might have been having difficulty distinguishing the part/whole and the numerator/denominator aspects of the percentage equation.

In light of this we decided to illustrate the IS/OF strategy for obtaining the correct equation. Although not widely found in the typical math text book, it is a strategy that we personally have found useful in the past. Every percentage problem has <u>three</u> pieces of information of which <u>one</u> (X) is missing. The two numerators are read from left to right followed by the two denominators to read somewhat like "something is something percent of something".

$$\frac{\ldots\ IS}{OF\ \ldots} = \frac{\ldots\ \%}{100}$$

The student is encouraged to fill in the easiest bits of information from the question into the template above. When seen this way, it should become apparent that every percentage question can be grouped into one of the following three types.

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Type 1
"30 is what percent of 50?" would be entered into the template and hence read as "30 is X % of 50".

$$\frac{\mathit{30}\ \text{IS}}{\text{OF}\ \mathit{50}} = \frac{X\ \%}{100}$$

Type 2
"46 is 20% of what number?" This question would be entered as follows and hence read as "46 is 20% of X".

$$\frac{\mathit{46}\ \text{IS}}{\text{OF}\ X} = \frac{20\ \%}{100}$$

Type 3
"What is 15% of 125??" This question would be entered as follows and hence read as "X is 15% of 125".

$$\frac{X\ \text{IS}}{\text{OF}\ \mathit{125}} = \frac{15\ \%}{100}$$

We got Tim working on a collection of percentage questions that contained a mix of the above three types of questions. In almost every case he was able to determine the proportionality equation associated with the question and then use the cross product and his understanding of equation solving so far to arrive at an answer. He got incorrect answers for only two questions and in both cases these were due to computational errors.

Satisfied that Tim could analyse percentage problems and solve the resulting equations we decided to return to trigonometry. We told Tim to research the internet and tell us when we see him next what a "clinometer" is used for. When we saw Tim again he proudly explained how it was used by surveyors to measure angles of elevation and angles of inclination and he was able to clearly explain what these terms meant. We then presented him with the scenario of having to measure the height of the flagpole by constructing a triangle and by using the tan of an angle. We helped Tim by drawing the flagpole and asked if he could use a clinometer to find any angles that he would need if he was standing a set distance from the base of the flagpole. Tim explained his thinking in the following way:

Tim:	I'd measure out a distance, eg 10 metres from the base of the flagpole and get the angle from the ground to the top of the flagpole.
Interviewer:	Good.
Tim:	say the angle is 30 degrees and the height will be X
Interviewer:	Keep going
Tim:	tan 30 = X/10
Tim:	0.57 = X/10
Tim:	now I have to solve this ….. umm …. Cross-product …. Could I change the 0.57 to 57/100
Interviewer:	yes.
Tim:	ok so …. 100X = 57x10 = 570
Interviewer:	Keep going
Tim:	X = 57 ….. yeah the flagpole is 57 metres high. Must be a really tall flagpole.
Interviewer:	Make sense?
Tim:	Yeah … that's interesting.
Interviewer:	could we have made that 0.57/1 instead of 57/100
Tim:	um …. I don't know
Interviewer:	Well – try it and find out.
Tim:	0.57/1 = X/10
Tim:	1X = 0.57x10 = 57
Tim:	oh yeeah ….. the answer is the same …. It's quicker that way cos it's the same isn't it ….. yeah ….. 0.57/1 is the same as 57/100 ….. I get it …. Yeah I get it now

Conclusion

This boy came to us experiencing difficulties with percentages, basic trigonometry and equation solving, and at first glance there did not appear to be a common reason for this. It was fortunate for us, however, that early in the study we began to suspect a connection between the boy's understanding (or lack thereof) of these concepts and his ability to think and reason proportionally and it was partly this knowledge that led us, initially at least, to make the decisions that we made. It would be fair to say that this study did contain an element of action research in the sense that each new discovery resulted in us asking new questions and taking new pathways. Along the way we gained a fascinating insight into not just the challenges faced by the teacher who attempts to teach children how to think and reason proportionally, but also the critical place that proportional reasoning holds or should hold in the middle-school mathematics curriculum. Most importantly this study gives us a vivid illustration of the distinctive interconnectedness between proportional reasoning and other math topics such as trigonometry, percentages and equation solving.

The study also suggests certain implications for the profession. Proportional reasoning needs to be woven into a variety of mathematical topics in such a way that there will be ongoing connections made between not just the topics themselves but also within the skill of proportional reasoning itself. Proportional reasoning should certainly be taught in tandem with topics such as trigonometry, percentages and equation-solving. It would probably be of benefit to integrate it into other topics such as fractional understanding in the number strand and when learning about rate, similarity, gradient and even algebra.

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