



University of Southern Queensland
Faculty of Engineering & Surveying

PHD DISSERTATION

**WIRELESS NETWORK CODING FOR
MULTI-HOP RELAY CHANNELS**

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Abstract

Future wireless communication systems are required to meet growing demands for high spectral efficiency, low energy consumption and high mobility. The advent of wireless network coding (WNC) has offered a new opportunity to improve network throughput and transmission reliability by exploiting interference in intermediate relays. Combined with network coding and self-information cancelation, WNC for two-way relay channels (TWRCs) has come to the forefront.

This dissertation focuses on exploiting WNC in multi-hop two-way relay channels (MH-TRCs). Particularly, a multi-hop wireless network coding (MH-WNC) scheme is designed for the generalized L -node K -message MH-TRC. Theoretical studies on the network throughput and performance bounds achieved by the MH-WNC scheme with different relaying strategies (i.e., amplify-and-forward (AF) and compute-and-forward (CPF)) are carried out. Furthermore, by introducing different numbers of transmission time intervals into the MH-WNC, a multiple-time-interval (Multi-TI) MH-WNC is proposed to determine an optimal MH-WNC which can achieve the best outage performance for all-scale MH-TRCs. Finally, this study extends the research on WNC one step forward from two-user networks to multi-user networks. An extended CPF joint with a dominated solution for maximizing the overall computation rate is proposed for the multi-way relay channel (mRC) in the last chapter.

The contributions of this dissertation are multifold. First, the proposed MH-WNC scheme with fixed two transmission time intervals can achieve a significantly improved network throughput compared to the non-network coding (Non-NC) scheme in the generalized L -node K -message MH-TRC. Theoretical results are derived for both multi-hop analog network coding (MH-ANC) and multi-hop compute-and-forward (MH-CPF). Moreover, both theoretical and numerical re-

sults demonstrate that the two MH-WNC schemes can be applied to different scale MH-TRCs to achieve a better outage performance compared to the conventional Non-NC scheme (i.e., MH-ANC for the non-regenerative MH-TRC with a small number of nodes, and MH-CPF for the regenerative MH-TRC with a large number of nodes.). Furthermore, a Multi-TI MH-WNC scheme is generalized with a special binary-tree model and characteristic matrix. The determined optimal MH-WNC scheme is able to provide the best outage performance and outperform the Non-NC scheme in all scale MH-TRCs. Last but not least, this dissertation provides a preliminary investigation of WNC in mRCs. The proposed dominated solution for maximizing the overall computation rate can ensure that all the nodes in the mRC successfully recover their required messages. Moreover, the extended CPF strategy is proven superior to Non-NC in the mRC with a small number of users.

Certification of Dissertation

I certify that the ideas, designs and experimental work, results, analyses and conclusions set out in this dissertation are entirely my own effort, except where otherwise indicated and acknowledged.

I further certify that the work is original and has not been previously submitted for assessment in any other course or institution, except where specifically stated.

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List of publications

The following publications were produced during the period of candidature:

[1] Gengkun Wang, Wei Xiang, Jinhong Yuan, and Tao Huang, "Outage performance of Analog Network Coding in Generalized Two-Way Multi-Hop Networks," in *Proc. IEEE Wireless Communications and Networking Conference*, Cancun, Mexico, Mar. 2011, pp. 759-764.

This paper was awarded the Best Paper Award by IEEE WCNC 2011, which is one of world's three most prestigious conferences in communication technologies.

[2] Gengkun Wang, Wei Xiang, Jinhong Yuan, and Tao Huang, "Outage Analysis of Non-Regenerative Analog Network Coding for Two-Way Multi-Hop Networks," *IEEE Commun. Lett.*, vol. 15, pp. 662 - 664, May 2011.

The work in the paper is presented in Chapter 4.

[3] Gengkun Wang, Wei Xiang, and Jinhong Yuan, "Multi-Hop Compute-and-Forward for Generalized Two-Way Relay Channels," accepted by *Trans. Emerging Tel. Tech. (formerly known as European Transactions on Telecommunications (ETT))*.

The work in the paper is presented in Chapter 5.

[4] Gengkun Wang, Wei Xiang, and Jinhong Yuan, "Outage Performance for Compute-and-Forward in Generalized Multi-Way Relay Channels," *IEEE Commun. Lett.*, vol. 16, pp. 2099 - 2102, Dec 2012.

The work in the paper is presented in Chapter 7.

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Acronyms & Abbreviations

AF	Amplify-and-forward
ANC	Analog network coding
AWGN	Additive white Gaussian noise
BER	Bit-error-rate
BPSK	Binary phase-shift keying
BS	Base station
BSC	Binary symmetric channel
CDMA	Code division multiple access
CDNC	Channel decoding network coding
CF	Compress-and-forward
CPF	Compute-and-forward
CSI	Channel state information
DF	Decode-and-forward
EF	Estimate-and-forward
FDMA	Frequency division multiple access
FFNC	Finite field network coding
GF	Galois field
LDPC	Low-density parity-check
LLL	Lenstra-Lenstra-Lovsz
LNC	Linear network coding
LTE	Long-term evolution
MAC	Multiple-access channel
MH-ANC	Multi-hop analog network coding
MH-CPF	Multi-hop compute-and-forward

MH-TRC	Multi-hop two-way relay channel
MH-WNC	Multi-hop wireless network coding
MIMO	Multi-input multi-output
ML	Maximum likelihood
mRC	Multi-way relay channel
MS	Mobile station
NC	Network coding
Non-NC	Non-network coding
P2P	Peer-to-peer
PNC	Physical-layer network coding
QPSK	Quadrature phase-shift keying
RA	Repeat accumulate
RLNC	Random linear network coding
SNR	Signal-noise-ratio
SVP	Shortest vector problem
TDMA	Time division multiple access
TI	Transmission time interval
TWRC	Two-way relay channel
WNC	Wireless network coding
XOR	Exclusive or