UNIVERSITY OF SOUTHERN QUEENSLAND

C^2-ELEMENT RADIAL BASIS FUNCTION METHODS FOR SOME CONTINUUM MECHANICS PROBLEMS

A dissertation submitted by

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For the award of the degree of

Doctor of Philosophy

February 2013

Dedication

To my parents

Thanh Vo-Kim and Dung An-Bang

and

The woman in my life

Thach Huynh

Certification of Dissertation

I certify that the ideas, experimental work, results and analyses, software and conclusions reported in this dissertation are entirely my own effort, except where otherwise acknowledged. I also certify that the work is original and has not been previously submitted for any other award.

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Acknowledgments

I would like to acknowledge my supervisors Professor Nam Mai-Duy, Professor Thanh Tran-Cong and Dr Canh-Dung Tran for their effective guidance, constant support and encouragement throughout. I deeply appreciate their knowledge and patience to answer any question I have asked and correct my manuscripts. They are indeed wonderful supervisors.

In addition, I would like to thank A/Prof. Armando A. Apan, Mrs Juanita Ryan (Faculty of Engineering and Surveying), Ms Katrina Hall (Office of Research and Higher Degrees), Mr Martin Geach (P9 operational officer) for their kind support; Dr Andrew Wandel and Dr Kazem Ghabraie for offering me teaching assistant positions.

I am grateful to numerous friends and people, whom I have not mentioned by name, for their invaluable help during the course of my study.

My candidature was at all possible owing to a Postgraduate Scholarship from the University of Southern Queensland and Scholarship supplements from Faculty of Engineering and Surveying and Computational Science and Engineering Research Centre. These financial supports are gratefully acknowledged.

Finally, I am indebted to my parents, my brother and my wife for their unconditional support, love and encouragement.

Abstract

This work attempts to contribute further knowledge to high-order approximation and associated advanced techniques/methods for the numerical solution of differential equations in the discipline of computational science and engineering. Of particular interest is the numerical simulation of heat conduction, highly non-linear flows and multiscale problems. The distinguishing feature in this study is the development of novel local compact 2-node integrated radial basis function elements (IRBFEs) and their incorporation into the subregion/point collocation formulations based on Cartesian grids. As a result, a new class of C^2-continuous methods are devised, representing a significant improvement on the usual C^0-continuous methods. Incorporation of the new C^2-continuous methods into the development of a high-order multiscale computational framework provides advantageous features compared to other multiscale frameworks available in the literature, including (i) high rates of convergence and levels of accuracy; and (ii) converged C^2-continuous solutions of two-dimensional multiscale elliptic problems.

Firstly, a new control-volume (CV) discretisation method, based on Cartesian grid and IRBFEs, for solving PDEs is proposed. Unlike the standard CV method (Patankar 1980), the flux values at CV faces are presently estimated with high-order IRBF approximations on 2-node elements and the solution is C^2-continuous across the interface between two adjacent elements. Only two RBF centres (a smallest RBF set) associated with the two nodes of the element are used to construct the approximations locally leading to a very sparse and banded system matrix. Moreover, a wide range of RBF-widths can be used to effectively control the solution accuracy. Secondly, the proposed 2-node IRBFEs are incorporated into the subregion and point collocation frameworks for the discretisation of the streamfunction-vorticity formulation governing the fluid flows. Several high-order upwind schemes based on 2-node IRBFEs are developed for highly non-linear flows. Thirdly, the ADI procedure (Peaceman and Rachford 1955, Douglas and Gunn 1964) is applied to enhance the efficiency of the proposed methods. Especially novel C^2-continuous compact schemes based on 2-node IRBFEs are devised and combined with the ADI procedure to yield optimal tridiagonal system matrices on each and every grid line. Such tridiagonal matrices can be solved effectively and efficiently with the Thomas algorithm (Fletcher 1991, Pozrikidis 1997). Finally, the proposed C^2-continuous CV method is employed in a multiscale basis function approach to develop a high-order multiscale CV method for the solution of multiscale elliptic problems.

Accuracy, stability and efficiency of the proposed methods are verified with extensive numerical results.

Papers Resulting from the Research

Journal Papers

1. **D.-A. An-Vo**, N. Mai-Duy, T. Tran-Cong (2010). Simulation of Newtonian-fluid flows with C^2-continuous two-node integrated-RBF elements, *SL: Structural Longevity*, 4(1):39 – 45.

2. **D.-A. An-Vo**, N. Mai-Duy, T. Tran-Cong (2011). A C^2-continuous control-volume technique based on Cartesian grids and two-node integrated-RBF elements for second-order elliptic problems. *CMES: Computer Modeling in Engineering and Sciences*, 72(4):299 – 335.

3. **D.-A. An-Vo**, N. Mai-Duy, T. Tran-Cong (2011). High-order upwind methods based on C^2-continuous two-node integrated-RBF elements for viscous flows. *CMES: Computer Modeling in Engineering and Sciences*, 80(2):141 – 177.

4. C.-D. Tran, **D.-A. An-Vo**, N. Mai-Duy, T. Tran-Cong (2011). An integrated RBFN based macro-micro multi-scale method for computation of visco-elastic fluid flows. *CMES: Computer Modeling in Engineering and Sciences*, 82(2):137 – 162.

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method based on C^2-continuous two-node integrated-RBF elements for viscous flows. *Applied Mathematical Modelling*, 37:5184 – 5203.

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1. **D.-A. An-Vo**, D. Ngo-Cong, B.H.P Le, N. Mai-Duy, T. Tran-Cong (2010). Local integrated radial-basis-function discretisation schemes. *The ICCES Special Symposium on Meshless & Other Novel Computational Methods (ICCES-MM'10)*, 17-21/Aug/2010, Busan, Korea Republic. ICCES Journal, Tech Science Press (ISSN 1933-2815).

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3. **D.-A. An-Vo**, C.-D. Tran, N. Mai-Duy, T. Tran-Cong (2011). IRBFN-based multiscale solution of a model 1D elliptic equation. In *Boundary Element and Other Mesh Reduction Methods XXXIII*, 241 – 251. ISSN 1743-355X (invited).

4. C.-D. Tran, T. Tran-Cong, **D.-A. An-Vo** (2011). A macro-micro multiscale method based on RBFNs control volume scheme for the Non-Newtonian

fluid flows. *The* 1^{st} *International Conference on Computational Science and Engineering*, 19-21/Dec/2011, HCM city, Vietnam (invited).

5. **D.-A. An-Vo**, N. Mai-Duy, C.-D. Tran, T. Tran-Cong (2012). Modeling strain localisation in a segmented bar by a C^2-continuous two-node integrated-RBF element formulation. In *Boundary Element and Other Mesh Reduction Methods XXXIV*, 3 – 13. ISSN 1743-3533 (invited).

6. **D.-A. An-Vo**, C.-D. Tran, N. Mai-Duy, T. Tran-Cong (2012). RBF computation of multiscale elliptic problems. *ICCES-MM'12*, 2-6/Sep/2012, Bubva, Montenegro (keynote lecture).

Contents

Acronyms & Abbreviations

1D-IRBF	One-dimensional Integrated Radial Basis Function
2D-IRBF	Two-dimensional Integrated Radial Basis Function
ADI	Alternating Direction Implicit
BEM	Boundary Element Method
C2NIRBFM	Compact 2-node Integrated Radial Basis Function Method
CD	Central Difference
CFD	Computational Fluid Dynamics
CM	Collocation Method
CM	Convergence Measure
CPU	Central Processing Unit
CV	Control Volume
CVM	Control Volume Method
DGM	Discontinuous Galerkin Method
DRBF	Differentiated Radial Basis Function
ETCM	Explicit Treatment of Convection Method
FD	Finite Difference
FDM	Finite Difference Method
FE	Finite Element
FEM	Finite Element Method
FSS	Fine Scale Solver
FVM	Finite Volume Method

HMM	Heterogeneous Multiscale Method
HOC	High-Order Compact
IRBF	Integrated Radial Basis Function
IRBFE	Integrated Radial Basis Function Element
LCR	Local Convergence Rate
LHS	Left Hand Side
LR	Line Relaxation
MD	Multidomain
MFEM	Multiscale Finite Element Method
MFVM	Multiscale Finite Volume Method
MHM	Mathematical Homogenisation Method
MQ	Multiquadric
N-S	Navier-Stokes
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
PR	Peaceman and Rachford
RBF	Radial Basis Function
RHS	Right Hand Side
SVD	Singular Value Decomposition
UW	Upwind

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