

University of Southern Queensland



**Steady and Unsteady Free Surface Flow
Past a Two-Dimensional Stern**

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Doctor of Philosophy

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Abstract

The research examines an influence of a platform shape on free surface waves generated behind a semi-infinite two-dimensional platform moving with a constant speed on a water surface of a finite depth h . The fluid is assumed to be inviscid, incompressible and irrotational; the surface tension effect is neglected. The aim of the research is to find analytically and numerically such platform shape which minimizes generated waves and reduces wave drag exerting on a moving platform when the Froude number is relatively small, $F < 1$. It is shown that for certain platform shapes, generated waves can be minimised or even eliminated, at least, within the framework of a linearized theory.

Linearized hydrodynamic equations for a fluid of finite depth are solved analytically by means of the Fourier transform and Wiener–Hopf technique, as well as numerically with the help of boundary integral technique. A weakly nonlinear solution is also obtained for shallow-water approximation within the framework of the forced Korteweg–de Vries (KdV) equation.

The problem is investigated for steady motion of a platform having a different stern shape. Then the analysis is performed for unsteady motion of a platform having a flat shape. The linearized problem for a water of finite depth is solved by means of the Laplace transform and Wiener–Hopf technique. The linear problem is formulated by assuming that at the initial instant of time the free surface is slightly perturbed due to the platform submerging onto the depth $d \ll h$ beneath the free surface. It is shown that the unsteady solution approaches the steady state solution as $t \rightarrow \infty$. The dependence of maximum wave perturbation on the fluid depth is found numerically.

In the last Chapter 6 the analysis is extended to steady motion of a flat platform at the interface between two fluids of different density. It is assumed that the lower layer has a finite depth h , whereas the upper layer is infinite. Results obtained for internal waves on the sharp density interface depend on the density ratio $a = \rho_1/\rho_2$ and in the limit $a \rightarrow 0$ they coincide with the results obtained for surface waves.

The results of this research can help in understanding of the physics of wave generation past a bluff body (e.g., wide blunt ships) and shed some light on solving an engineering problem of ship building of an optimal shape.

Statement of Original Authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher educational institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

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Keywords

Boundary integral technique, Wiener–Hopf technique, weakly nonlinear theory, free surface flow, steady flow, unsteady flow, Fourier transform, Laplace transform, conformal mapping, Korteweg–de Vries (KdV) equation.

List of Publications

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