

Problem set 7

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The purpose of the section is to provide teachers and students with interesting problems. The topic of this issue is equations that are solvable/not solvable over integers. Such equations are called Diophantine equations in the honour of Diophantus who studied them many centuries ago. One of the famous equations that can be traced back to the Greeks (Theon of Smyrna) is an equation of Pell type.

It was Euler who controversially attributed the study of non-trivial solutions of the equation $x^2 - Dy = 1$, where D is a positive integer and not a perfect square, to John Pell whose 400 year anniversary since his birth took place on the 1 March 2011. The equation $x^2 - Dy = 1$ which appeared in Rahn (1659) was certainly written with Pell's help. Some historians assume it was written entirely by Pell (Scriba, 1974). Perhaps Euler knew what he was doing in naming the equation. The general solution to Pell's equation was first obtained by Lagrange who presented a number of papers on this topic to the Berlin Academy between 1768 and 1770.

One more famous equation is Archimedes' *problema bovinum* (cattle problem) posed as a challenge to Apollonius, which took until the twentieth century before a complete solution was found (Gelca, & Andreescu, 2007). Questions in the problem set below are less famous. However, a four of these were considered by Euler consideration over 200 years ago.

[Problems 1–4 from Euler]

1. Find all triples (x, y, z) of positive integers such that $x - y$, $x + y$, $x - z$, $x + z$, $y - z$, $y + z$ are perfect squares.
2. Prove that the equation $x^3 + y^3 + z^3 = t^2$ has infinite number of solutions over integers.
3. Prove that the equation $x^2 + y^3 + z^4 = t^2$ has infinite number of solutions over integers..
4. Solve in integers $x^2 + y^2 = z^n$ where $n > 1$ and x, y are relatively prime.
5. Solve in positive integers $x^2 + y^2 = 1997(x - y)$.

References

- Gelca, R. & Andreescu, T. (2007). *Putnam and beyond*. New York: Springer.
- Rahn, J. H. (1659). *Teutsche algebra* (in German). Zurich.
- Scriba, C. J. (1974). *John Pell's English edition of J. H. Rahn's Teutsche Algebra*. In R. S. Cohen et al. (Eds.), *For Dirk Struik: Scientific, historical and political essays in honor of Dirk J. Struik* (pp. 261–274). Reidel: Dordrecht, Holland.

Solutions to this set of problems for publication should be submitted to:
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Solutions to this set will be made available on the AAMT website (www.aamt.edu.au) after 1 October 2011.

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