



UNIVERSITY OF SOUTHERN
QUEENSLAND

INTEGRATED RADIAL BASIS FUNCTION
METHODS FOR NEWTONIAN AND
NON-NEWTONIAN FLUID FLOWS

A dissertation submitted by

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Dedication

To my family and to the memory of my father

Certification of Dissertation

I certify that the idea, experimental work, results and analyses, software and conclusions reported in this dissertation are entirely my own effort, except where otherwise acknowledged. I also certify that the work is original and has not been previously submitted for any other award.

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Date

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Notes to Readers

Attached to this thesis is the CD-ROM containing the following files:

1. thesis.pdf: An electronic version of this thesis
2. Movie_Cavity_Isotherms.wmv: An animation showing the evolution of temperature field (Chapter 3, Problem 2: Natural convection in a square slot)
3. Movie_Cavity_Streamlines.wmv: An animation showing the evolution of streamfunction field (Chapter 3, Problem 2: Natural convection in a square slot)
4. Movie_Annulus_Isotherms.wmv: An animation showing the evolution of temperature field (Chapter 3, Problem 3: Natural convection in a concentric annulus)
5. Movie_Annulus_Streamlines.wmv: An animation showing the evolution of streamfunction field (Chapter 3, Problem 3: Natural convection in a concentric annulus)

Abstract

In this PhD thesis, one-dimensional integrated radial basis function networks (1D-IRBFNs) are further developed for the simulation of viscous and viscoelastic flows in two dimensions. The thesis consists of two main parts.

In the first part, 1D-IRBFNs are incorporated into the Galerkin formulation to simulate viscous flows. The governing equations are taken in the streamfunction-vorticity formulation and in the streamfunction formulation. Boundary conditions are effectively imposed with the help of the integration constants. The proposed 1D-IRBFN-based Galerkin methods are validated through the numerical simulation of several benchmark test problems including free convection in a square slot and in a concentric annulus.

In the second part, 1D-IRBFNs are incorporated into the Galerkin and collocation formulations to simulate viscoelastic flows. The momentum and continuity equations are taken in the streamfunction-vorticity formulation and two types of fluid, namely Oldroyd-B and CEF models, are considered. Flows in a rectangular duct and in straight and corrugated tubes are simulated to validate the proposed 1D-IRBFN-based Galerkin/Collocation methods.

Main attractive features of the proposed methods include (i) easy implementation; (ii) avoidance of the reduction in convergence rate caused by differentiation; and (iii) effective treatment of derivative boundary conditions. Numerical results show that the proposed methods are stable, high-order accurate and converge well. This study further demonstrates the great potential of using RBFs in CFD.

Papers Resulting from the Research

Book Chapters

1. Ho-Minh, D., Mai-Duy, N. and Tran-Cong, T. (2010). “A Cartesian-grid integrated-RBF Galerkin technique”. In: B. Sarler and S.N. Atluri (eds.) *Recent Studies in Meshless and Other Novel Computational Methods*. Chapter VI, pp. 87–102. Tech Science Press (ISBN 978-0-9824205-4-6).

Journal Papers

1. Ho-Minh, D., Mai-Duy, N. and Tran-Cong, T. (2009). A Galerkin-RBF approach for the streamfunction-vorticity-temperature formulation of natural convection in 2D enclosed domains. *CMES: Computer Modeling in Engineering & Sciences*, vol. 44, no. 3, pp. 219–248.
2. Mai-Duy, N., Ho-Minh, D. and Tran-Cong, T. (2009). A Galerkin approach incorporating integrated radial basis function networks for the solution of 2D biharmonic equations. *International Journal of Computer Mathematics*, vol. 86, nos. 10-11, pp. 1746–1759.
3. Ho-Minh, D., Mai-Duy, N. and Tran-Cong, T. (2010). Simulation of viscous and viscoelastic flows using a RBF-Galerkin approach. Submitted

to *Australian Journal of Mechanical Engineering*.

4. Ho-Minh, D., Mai-Duy, N. and Tran-Cong, T. (2010). Galerkin/Collocation methods based on 1D-integrated-RBFNs for viscoelastic flows. *CMES: Computer Modeling in Engineering & Sciences*, vol.70, no.3, pp. 217-252.

Conference Papers

1. Ho-Minh, D., Mai Duy, N. and Tran Cong, T. (2007). Analysis of viscoelastic flow by a radial basis function networks method. In: P. Jacobs, T. McIntyre, M. Cleary, D. Buttsworth, D. Mee, R. Clements, R. Morgan and C. Lemckert (eds). *The 16th Australasian Fluid Mechanics Conference*, Gold Coast, QLD, Australia, 3-7 December 2007. *Proceedings of The 16th Australasian Fluid Mechanics Conference* (CD), pages 1321–1327. The University of Queensland (ISBN 978-1-864998-94-8).
2. Ho-Minh, D., Mai-Duy, N. and Tran-Cong, T. (2010). A Cartesian-grid integrated-RBF method for viscoelastic flows. In: N. Khalili, S. Valliappan, Q. Li and A. Russell (eds). *The 9th World Congress on Computational Mechanics and The 4th Asian Pacific Congress on Computational Mechanics (WCCM/APCOM 2010)*, Sydney, Australia, 19-23/Jul/2010. *IOP Conference Series: Materials Science and Engineering*, Vol. 10, Paper No. 012210, 9 pages. IOP Publishing (ISSN 1757-899X (Online) and ISSN 1757-8981 (Print)).
3. Ho-Minh, D., Le-Cao, K., Mai-Duy, N. and Tran-Cong, T. (2010). Simulation of fluid flows at high Reynolds/Rayleigh numbers using integrated radial basis function. In: G.D. Mallinson and J.E. Cater (eds). *The 17th Australasian Fluid Mechanics Conference*, Auckland, New Zealand, 5-9/Dec/2010. *Proceedings of 17th Australasian Fluid Mechanics Conference* (CD), Paper No 139, 4 pages. The University of Auckland (ISBN: 978-0-86869-129-9).

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Acronyms & Abbreviations

1D-IRBFN	One-dimensional Indirect/Integrated Radial Basis Function Network
BEM	Boundary Element Method
CEF	Criminale-Ericksen-Filbey model
CFD	Computational Fluid Dynamics
DAMs	Diffuse Approximation Methods
DOF	Degrees of Freedom
DRBEM	Dual Reciprocity Boundary Element Method
DRBFN	Direct Radial Basis Function Network
DSC	Discrete Singular Convolution
EEME	Explicitly Elliptic Momentum Equation
EFG	Element-free Galerkin
EVSS	Elastic Viscous Stress Splitting
FCC	Fourier-Chebyshev Collocation
FDM	Finite Difference Method
FEM	Finite Element Method
FVM	Finite Volume Method
GFEM	Galerkin Finite Element Method
IRBFN	Indirect/Integrated Radial Basis Function Network
LHS	Left Hand Side
LRPIM	Local Radial Point Interpolation Method

MLP	Multilayer Perceptron
MLPG	Meshless Local Petrov-Galerkin method
MPTT	Modified Phan-Thien–Tanner model
MQ	Multiquadric
MWR	Method of Weighted Residuals
NN	Neural Networks
ODE	Ordinary Differential Equation
PCFD	Pseudo-spectral Cylindrical Finite Difference method
PDE	Partial Differential Equation
PSFD	Pseudo-Spectral Finite Difference method
PTT	Phan-Thien - Tanner model
RBF	Radial Basis Function
RBFN	Radial Basis Function Network
RBFCM	mesh-free local RBF Collocation Method
RBF-DQM	RBF-based Differential Quadrature Method
RHS	Right Hand Side
RKPM	Reproducing Kernel Particle Method
SM	Spectral Method
SPH	Smooth Particle Hydrodynamics
UCM	Upper-convected Maxwell

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