ISSUES IN PROCESSING UNSTABLE TWISTED FIBRE ASSEMBLIES

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Abstract

During yarn formation by ring spinning, fibres are bent into approximately helical shapes and an unbalanced torque or twist-liveliness is created. Generally, the torque depends on yarn geometric factors such as the yarn twist and linear density and the fibre properties. In order to prevent the instability of yarn (e.g., snarling), twist-liveliness must be controlled. A practical solution to the problem of twist-liveliness is the formation of a two-fold yarn. This twisting or plying process produces a yarn structure where the energy of the system is determined by purely geometrical constraints of the plied structure and consequently when an energy minimum is reached the plied yarn obtained from the process is torsionally balanced and torque-free.

In the present paper, the instability of twisted textile yarns will be interpreted using the Topological Conservation Law (Fuller, 1971) which has been developed to study the dynamics of twisted rods by Van der Heijden et al. (2003). The present work considers the equilibrium configuration of a series of multi-ply twisted yarns (2, 4, 8, and 16 strands) of finite length. The influence of structural properties (the number of strands, the diameter and twist of each strand) on the parameters of a balanced multi-ply yarn (writhe and twist) using a topological invariant of the twisted yarn (link) is established and investigated using experimental results obtained at CSIRO.

Keywords: Topological conservation law, writhe, twist, link, multi-ply, balance twisted yarn, instability.

1. Introduction

For the last decade, one of the important subjects of the dynamics of twisted yarns is the source of their instability. In fact, issues of instability for twisted structures are well-known, such as snarling of yarn or the skewing of knitted fabrics. For example, if the ends of a freshly-spun yarn are brought together, the yarn will buckle locally and jump into a ply or form a snarl. By assuming helical shapes for the centre-line of strands in the ply, expressions of forces and moments in the ply can be obtained using the static equilibrium equations. Recently, based on the topological conservation law (TLC) (Fuller, 1971, 1978) for the dynamics of rods, Thompson et al. (2002) has employed the dynamics of a elastic fibre to model a single DNA molecule and analysed its super-coiled equilibrium ply configurations and Van der Heijden et al. (2003) have modeled the non-linear jump behaviour of twisted clamped rods. In the present work, the application of this law (TCL) to the instability of textile twisted yarns will be considered. The equilibrium configuration of a series of multi-ply twisted yarns of finite length and some influences of structural properties on a balanced multi-ply yarn (writhe and twist) are investigated and presented.

2. Model of a multi ply yarn

One way of representing the conformation of a strand within a multi-strand yarn is to regard the strand as lying on the surface of a cylinder. Figure 1 shows the case of a yarn of n strands of radius r whose centerlines are wound on a cylinder of R, where each strand is considered as an elastic circular filament. Hence, the configuration of a strand is specified by the position of a curve in space $r_i(s)$ where s is the arc length along the central axis of the yarn.

Let $\{\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3\}$ be a fixed rectangular Cartesian coordinate system, and $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ a moving coordinate system whose two axes coincide with the principal axes of the strand cross section ($\mathbf{x}_1$ and $\mathbf{x}_2$) and the third axis $\mathbf{x}_3$ coincides with the tangent to the strand $\left(\mathbf{x}_3 = \frac{d\mathbf{r}}{ds}\right)$ (see Fig.1). Let ψ, θ, ϕ be the angular twist in the strand, the helical angle inclination of the strand and the angular rotation of the fiber around the $\mathbf{x}_3$ axis, respectively.

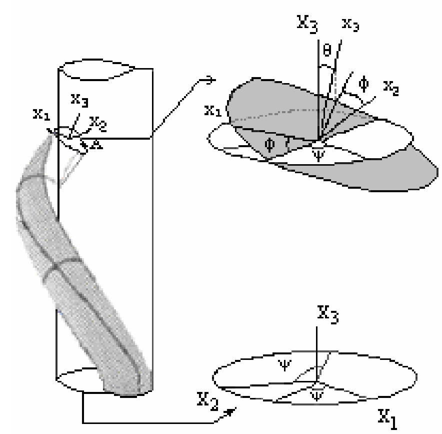

Fig. 1 Schema of a strand (r) wound on a cylinder (R): the fixed cartesian and moving coordinate frames.

For the sake of presentation, considering strands wound on a cylinder in the left handed side helix as shown in Fig.1. The centreline of a strand (i) can be expressed in fixed frame (Neukirch and Van der Heijden, 2002, Van der Heijden, 2001) as follows

$$\mathbf{r}_i(s) = \mathbf{K}_n^{i-1}\mathbf{r}_1 \qquad (1a)$$

With
$$\mathbf{K}_n = \begin{pmatrix} \cos\frac{2\pi}{n} & -\sin\frac{2\pi}{n} & 0 \\ \sin\frac{2\pi}{n} & \cos\frac{2\pi}{n} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_1 = \begin{pmatrix} -R\sin\psi \\ R\cos\psi \\ s\cos\theta \end{pmatrix} \qquad (1b)$$

Where
$$\psi'(s) = -\frac{\sin\theta}{R}; \quad \psi(0) = 0 \qquad (1c)$$

Hence, the position vector $\mathbf{r}_{A,i}$ of a point on the surface of a strand i can be determined by

$$\mathbf{r}_{A,i}(s,\phi) = \mathbf{r}_i(s) + r\sin\phi\,\mathbf{x}_1 + r\cos\phi\,\mathbf{x}_2 = \begin{pmatrix} -R\sin\psi + r\sin\phi\sin\psi + r\cos\phi\cos\theta\cos\psi \\ R\cos\psi - r\sin\phi\cos\psi + r\cos\phi\cos\theta\sin\psi \\ s\cos\theta - r\cos\phi\sin\theta \end{pmatrix} \qquad (2)$$

The geometrical model considered here is based on elastic rod mechanics in which the following assumption is introduced: the distance of the centreline of two adjacent strands is minimum and equal to the diameter of each strand. This is expressed as follows:

$$2 + m^2 \cos^2\theta - 2\cos\left(m\sin\theta - \frac{2\pi}{n}\right) = \frac{4}{\rho^2} \tag{3a}$$

$$m\cos^2\theta + \sin\theta\sin\left(m\sin\theta - \frac{2\pi}{n}\right) = 0 \tag{3b}$$

Where $m = \frac{s_{i+1} - s_i}{R}, \quad \rho = \frac{R}{r}$

Equations (1b,c) and $\mathbf{x}_3 = \frac{d\mathbf{r}}{ds}$ yields to $\mathbf{x}_3 = \begin{pmatrix} \sin\theta\cos\psi \\ \sin\theta\sin\psi \\ \cos\theta \end{pmatrix}$ (4)

Let u, whose components are u_1, u_2, u_3, be the curvatures and the twist of strand in the moving frame, the governing equations of the evolution of a strand (the centreline) are given by

$$\frac{d\mathbf{x}_i}{ds} = \mathbf{u} \times \mathbf{x}_i \qquad i = 1, 2, 3 \tag{5}$$

Because the centreline of each strand lies on a cylinder of R, a cylindrical coordinate frame $\{\mathbf{e}_r, \mathbf{e}_\psi, \mathbf{e}_z\}$ is introduced as follows

$$\begin{aligned} \mathbf{e}_r &= -\sin\psi\mathbf{X}_1 - \cos\psi\mathbf{X}_2; \\ \mathbf{e}_\psi &= -\cos\psi\mathbf{X}_1 + \sin\psi\mathbf{X}_2; \\ \mathbf{e}_z &= \mathbf{X}_3 \end{aligned} \tag{6a}$$

With $\frac{d\mathbf{e}_r}{ds} = \frac{d\psi}{ds}\mathbf{e}_\psi; \quad \frac{d\mathbf{e}_\psi}{ds} = -\frac{d\psi}{ds}\mathbf{e}_r; \quad \mathbf{x}_3 = -\sin\theta\mathbf{e}_\psi + \cos\theta\mathbf{e}_z$ (6b)

3. Governing equations of a multi-ply yarn

3.1 Link, twist and writhe in textile engineering (Kinematic equation)

For this presentation, consider a single Z twisted yarn of length L whose ends are glued together to form a closed loop or hank. Let the number of total end rotations of the yarn, before being formed into the hank, be specified as the link which is a topological invariant, i.e. it is unaffected by any deformations and is defined for as the following formula (Van der Heijden et al., 2003 and Fuller, 1971)

$$L_k = T_w + W_r \tag{7}$$

Where $T_w = \frac{1}{2\pi}\int_L u_3 ds$ is the total twist, and W_r (named the writhe) is a property of centre-line of the single yarn. It is produced from the out-of-plane deformation to form a snarl or plied yarn and equal to the signed crossing number averaged over planar projections from all possible directions (see Thompson et al. (2002), p.963). In a spinning process, the concept of the "twist" that is inserted into a single twisted yarn, is actually the 'link' per an unit length of yarn. If applying a rotation of Ω (rads) in a single yarn of length L, the spinning twist per length τ (positive for Z twist and negative for S twist) and twist angle are given by, respectively

$$\tau = \frac{\Omega}{L}, \quad \tan\theta = r\tau . \tag{8}$$

Since the twist is assumed constant along the yarn in textile engineering, the link is written as follows

$$L_k = \frac{\Omega}{2\pi} = \frac{L\tau}{2\pi} . \tag{9}$$

A similar explanation holds for a multi-ply assembly from more than one hank and the details can be seen in Neukirch and Van der Heijden (2002).

3.2. Kinetic balance Equations

Consider a multi-ply yarn of n strands whose physical and geometrical parameters are mentioned in previous sections. The governing equations of a strand for the force **F** and moment **M** are given by (Fraser and Stump, 1998a,b; Thompson et al., 2002; Stump and van der Heijden, 1999)

$$\frac{d\mathbf{F}}{ds} + \mathbf{p} = 0 \tag{10a}$$

$$\frac{d\mathbf{M}}{ds} + \frac{d\mathbf{r}}{ds} \times \mathbf{F} = 0 \tag{10b}$$

Where **p** is the pressure (force per unit length) on a strand in which the constitutive relations between the stresses and trains are assumed linear and given by

$$\mathbf{M} = B\mathbf{x}_3 \times \frac{d\mathbf{x}_3}{ds} + Cu_3\mathbf{x}_3 \tag{11}$$

and B and C are the two principal bending stiffness and torsional rigidity of strands, respectively. Let (F_r, F_ψ, F_z) be the components of F in the cylindrical coordinate, we have

$$\mathbf{F} \times \mathbf{d}_3 = \left(F_\psi \cos\theta + F_z \sin\theta\right)\mathbf{e}_r - F_r \cos\theta \mathbf{e}_\psi - F_r \sin\theta \mathbf{e}_z \tag{12}$$

From (12) and (11), the differential balance equation (10b) is rewritten as follows

$$-B\frac{d^2\theta}{ds^2} + B\left(\frac{d\psi}{ds}\right)^2 \sin\theta\cos\theta - Cu_3\frac{d\psi}{ds}\sin\theta = -F_\psi\cos\theta - F_z\sin\theta \tag{13}$$

$$F_r = -B\frac{d^2\psi}{ds^2}\sin\theta - 2B\frac{d\psi}{ds}\frac{d\theta}{ds}\cos\theta + Cu_3\frac{d\psi}{ds} \tag{14}$$

$$C\frac{du_3}{ds} = 0 \tag{15}$$

The derivatives of F in the cylindrical frame are given by

$$\frac{dF_r}{ds} = -\frac{F_\psi}{R}\sin\theta - p; \quad \frac{dF_\psi}{ds} = \frac{F_r}{R}\sin\theta; \quad \frac{dF_z}{ds} = 0 \tag{16}$$

With the assumption θ and u_3 are constant, equations (14), (16) and (11) show that the components of F are all constant and are calculated as follows

$$\begin{aligned} F_r &= 0 \\ F_Z &= \frac{F_o}{n} \\ F_\psi &= \frac{M_0}{nR} + \frac{B}{R^2}\sin^3\theta - \frac{Cu^3}{R}\cos\theta \end{aligned} \tag{17}$$

where F_o and M_o are the axial force and moment applied to the multi-ply yarn, respectively. From (17), the differential balance equation (13) is rewritten by

$$2n\sin^3\theta\cos\theta - n\frac{C}{B}\rho ru_3\cos 2\theta + \frac{\rho^2 r^2 F_0}{B}\sin\theta + \frac{\rho r M_0}{B}\cos\theta = 0 \tag{18}$$

With a given initial twist u_3, it can be expressed as follows (Antman, 1995, Love, 1904)

$$u_3 = \frac{d\phi}{ds} + \frac{d\psi}{ds}\cos\theta = \frac{\Omega}{L} - \frac{\sin 2\theta}{2R} = \tau - \frac{\sin 2\theta}{2R} \tag{19}$$

Substituting (19) into (18) yields

$$2n\sin^3\theta\cos\theta + \frac{C}{4B}n\sin 4\theta - \frac{Cr}{BL}n\tau\rho\cos 2\theta + \frac{\rho^2 r^2 F_0}{B}\sin\theta + \frac{\rho r M_0}{B}\cos\theta = 0 \tag{20}$$

3.3 The balance of a multi-ply yarn

When removing applied force and moment ($F_o = M_o = 0$), the multi-ply yarn reaches the balanced state and the equation (18) yields to

$$2n\sin^3\theta\cos\theta + \frac{1}{4}\gamma n\sin 4\theta - \tau\frac{r}{L}\gamma n\rho\cos 2\theta = 0 \tag{21}$$

where $\gamma = \frac{C}{B}$; Equation (21) gives the relationship between twist and writhe of the process of transformation single twisted yarn to n ply yarn. Hence, with given yarn structural parameters (yarn count, twist, bending stiffness, torsional stiffness and the number of strands of a multi-ply yarn), the balance state (θ) of the multi ply yarn is totally determined by (3) and (21).

4. Examples and discussions

In this work, the balance of multi ply yarns of 2, 4, 8, 16 strands will be considered and investigated. The analysis is carried out using a range of different yarn counts and initial twists in which the relationships between twist angle (link) of strands and helical angle of the multi-ply yarn (or the corresponding quantity) are determined. Although the torsional and bending stiffnesses of strands depend on the structural parameters of the yarn, the ratio of the torsional and bending stiffness (γ) and the Poisson's ratio (ν) are assumed constant at about 0.7 and 0.43, respectively, in the present work. Furthermore, the radius of wool worsted strands is calculated as a function of the yarn counts using the packing fraction of 0.63 by Booth, (1975).

<u>Plies from one hank</u>

For plied twisted yarns from one hank, i.e., two strands ($n = 2$, $R = r$ ($\rho = 1$)): this is a popular and simple case of multi-ply yarns. In the balance state, the kinematic and balance kinetic equations yields

$$\tan\psi_o = \tau_0 r = -\frac{1}{2}\tan 2\theta(1 + 2\nu \sin^2 \theta) \qquad (22)$$

where ψ_o is the twist angle of the two strands. The link and writhe can be determined from (7)-(8) and (22)

$$L_k = \frac{L}{2\pi r}\left(\nu \sin 2\beta - (1+\nu)\tan 2\beta\right) \qquad (23)$$

$$W_r = -\frac{L}{2\pi r}\sin 2\beta \qquad (24)$$

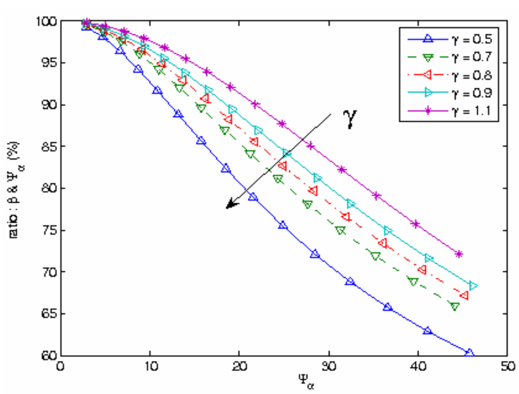

Fig. 2. Relationship between the twist angle of strand and helix twist angle of balance plied yarn using different γ's

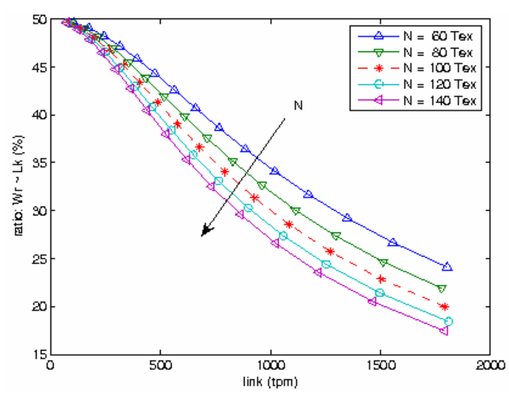

Fig. 3. Relationship between the twist angle of strand and helix twist angle of a balance plied yarn using different yarn counts

The relationship is shown between the initial twist in Figure 2 or the link of the single twisted strands in Figure 3 and structural stability of the plied yarn where the stability is given in terms of the ratio of twist angle of the strand to the helix angle of the plied yarn (Figure 2) or the ratio of writhe to link (Figure 3). It can be seen that this relationship depends clearly but not strongly on the geometrical property of strands (yarn count, packing fraction). The results also show good agreement between the present analysis using the TLC and kinetic balance and the experimental analysis (Fig.4). The difference between the theoretical values and the experimental results is always less than 8% for two strand plied yarns.

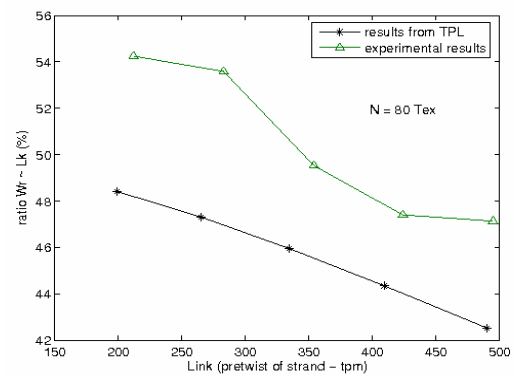

Fig. 4. The relationship between twist of strand and helix twist angle of a plied yarn: comparison between experimental results and the results from the present work using strands of 80 Tex.

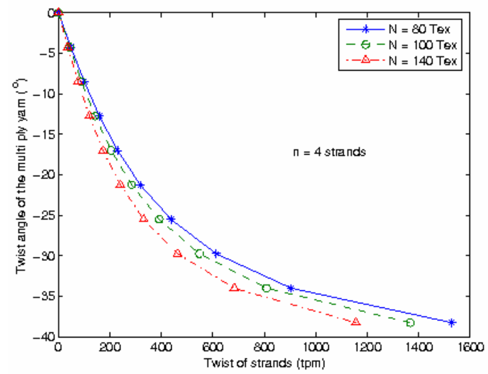

Fig. 5. The relationship between twist of strands and helix twist angle of a multi ply yarn for the different cases of used yarn counts : 80, Tex 100 Tex and 140 Tex using 4 strands.

Plies from more than two hanks

The influence of the number of strands and their yarn count on the structural stability of a multi-ply n-strand yarn is considered in the following. Figure 5 shows the theoretical relationship between twist angle of strands and the helical angle of the multi ply yarn of strands with respect to yarn count (80 Tex, 100 Tex, 140 Tex) at the balance situation. The relationship in Figure 6 shows the effect of an increased number of strands from 4 to 16.

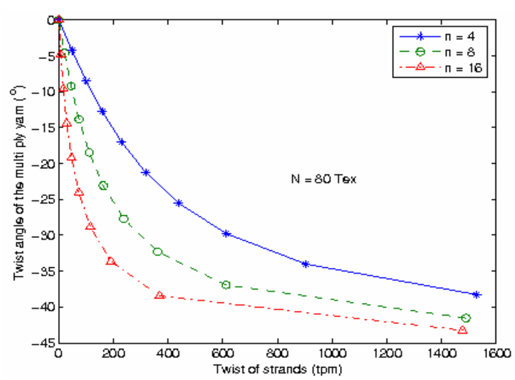

Fig. 6. Relationship between twist of strands and helix twist angle of plied yarn for the cases of 4, 8, 16 strands of 80 Tex

Due to the limitation of paper length, some other results which do not appear in the paper will be presented at the conference.

5. Conclusion

The Topological Conservation Law is used in combination with the governing equations to simulate the equilibrium configuration of a balanced multi-ply yarn. The influence of structural properties on the parameters of a balanced multi-ply yarn is also established and investigated using preliminary experimental results obtained at CSIRO. The relationship of the link and writhe of a given closed yarn structure may allow some multi-ply twisted yarns to be designed whose instability can be controlled.

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