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MATHEMATICS, COMPUTERS, AND UMBILICAL CORDS

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Abstract. This report describes an investigation of the attitudes, achievement and behaviour of students in a technology-enriched first-year Australian tertiary Algebra & Calculus course. The study indicated that the technology choices and competencies that students demonstrated under test conditions and in a computer workshop were not always consistent with their general technology attitudes and behaviour over the semester. Mathematics confidence showed consistently positive relationships with achievement on a range of course work. Computer confidence and technology attitudes did not, despite the technology intervention. Of particular interest were students whose computer confidence and enthusiasm did not translate into achievement. Factors that failed to explain students' technology choices and demonstrated levels of competency were computer confidence, prior use of technology for mathematics, attitudes towards such use, completion of weekly exercises, and tutorial/laboratory attendance. The study offered further compelling evidence of the low relationship between mathematics confidence and computer confidence. Students' views indicated clearly that they valued hand exercises. Curriculum development and assessment profiles need to accommodate their learning needs and preferences, and the rates at which these evolve. Questions arise about the reasons for students' technology choices and behaviour: beliefs and preferences developed through prior experiences may be like umbilical cords when students assimilate new cultures of learning and practice.

1. Introduction

Tertiary educators seek ways to enliven the learning of mathematics by exploiting powerful scientific software in new technology-rich learning environments provided within undergraduate programs. See the Laboratory in [**11**]), for example. A growing number of students are comfortable using computers, many adopt technology learning platforms very readily, and it seems reasonable to assume that the enthusiasm of those who enjoy using technology might be channelled positively for the learning of mathematics. Interacting with mathematics and computers at the same time makes new cognitive and affective demands on students, however (see [**8**]), and these demands may differ substantially from one technology program to another. Artigue [**1**] reflects on some aspects of the dialectics between technical and conceptual work when learning mathematics in a CAS environment, for example.

While the direction of influence is not clear, there is evidence that attitudes in mathematics learning are related to mathematical achievement. Tartre and Fennema [**10**] describe *mathematics confidence* as the affective variable most consistently related to achievement. If mathematics confidence relates to achievement,

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questions arise about the relationship between computer confidence and achievement in learning environments in which students use computers regularly and are required to interact with mathematics and computers almost simultaneously.

To gain deeper understanding, the authors and others have embarked on studies of the interacting effects of affective, cognitive and behavioural factors within technology-rich programs. Early findings in this body of research provide compelling evidence that computer confidence and mathematics confidence are very different constructs. Consistently very low correlations between these confidences in a range of different technology programmes suggest that these factors are almost orthogonal: see [**6**], [**4**] and [**7**]. These studies also reveal that students' attitudes to the use of technology for learning mathematics correlate far more strongly with computer confidence than with mathematics confidence.

These findings raised questions about students with different levels of mathematics and computer confidence in technology-enriched programs. Students with mixed confidences (high mathematics confidence/low computer confidence, and vice versa) are of particular interest: how do they behave and achieve?

A 2001 study [**3**] of attitudes and achievement in a technology-enriched mathematics course, confirmed the attitude findings above and revealed no evidence that any of the computer attitudes investigated correlated with achievement. On the other hand, mathematics attitudes (both confidence and motivation) correlated moderately with achievement (up to $\rho = 0.65$).

2. Background

The setting for the 2 studies reported here was a first-year tertiary Algebra & Calculus course in an Australian university in 2001 and 2003. The course exposed students to basic use of scientific software in carefully supported stages. The role of technology as a valued aid for graphing and computation in the course was unambiguous. The lecturers demonstrated use of MATLAB frequently in lectures and included computer-based tasks in the weekly exercises and assignments. Laptop computers and graphics calculators were permitted in one of two exit examinations. In 2003, students were also given advance notice that the midsemester test would be held in a computer laboratory and would include hand and computer tasks of the type practised weekly.

The weekly tutorial/laboratory exercises were designed to develop hand-skills and use technology for computation, visualisation and exploration. Students spent the first hour of the weekly small-group tutorial doing hand tasks in a classroom before spending a second hour in a computer laboratory with their tutor using MATLAB (or their own graphics calculators, where appropriate) to complete the tasks. Technology was exploited for concept development and applications. Typical exercises included comparing graphs and properties of functions, finding gradients and derivatives, optimising, finding and refining Riemann sums, investigating lines and planes in three dimensions, row-reducing matrices, finding determinants and inverses, solving systems of linear equations, and a wide range of applications.

A pilot study investigated the behaviour of 29 early undergraduate mathematics students under non-assessment conditions in their weekly computer laboratory in 2001, doing workshop tasks typical of those covered earlier in their course. The data suggested low relationships between computer confidence and levels of success on a range of tasks, despite full access to the use of a computer. Pertinent aspects of this Workshop study are summarised below; for more details, see [**2**].

To investigate the robustness of these small-group findings, and to explore relationships between attitudes and achievement under different conditions, data were captured for the full class of students working under assessment conditions in 2003. This paper reports some of the findings of these two studies, and reflects on the implications.

3. The 2001 Pilot Workshop Study

Data were captured from a focus group of 29 students from the class of 176. The sample was selected from those who had attended all the weekly labs over the semester. These students worked voluntarily on 8 mathematics tasks in their weekly computer laboratory. Some tasks needed the aid of technology; some could be done easily by hand if their mathematical properties were recognised; either hand or technology methods were appropriate for other tasks. Students could also choose to use technology to confirm hand methods, or vice versa.

Students' levels of mathematics confidence, computer confidence, and attitudes to technology in the learning of mathematics were captured on entry to the course using the USQ MATH/TECH scales (see 4.2 below). Each student's work was rated between 0 and 8 on three *performance indicators* listed below. For more on the study and criteria used, see [**2**].

- *Score* s: overall level of success on the 8 tasks, based on a rating of 1 per task.
- *Approach* a: how appropriate the 8 choices of hand or technology methods were, based on a rating of 1 per task.
- *Competency* c: the level of technology competency demonstrated, based on the type of task. Simple row-reduction requiring single data entry and use of one command was rated 1, for example; graphing or calculating a Riemann sum required additional technology skills and were rated 3, based on a count of 1 per stage.

Technology competency levels c were generally quite good, and it was not surprising that overall score s on the Workshop tasks related well to appropriate choice of approach a. Mathematics confidence related positively with achievement on the Workshop tasks and on the assessment tasks of the course generally (around $\rho = 0.5$); and many students performed predictably given their espoused attitudes.

However, computer confidence related poorly to technology competency level c and to choice of approach a. Some students' Workshop technology indicators (a and c were inconsistent with their computer confidence and attitudes, and with their commitment to attending the computer lab sessions over the semester.

Figure 1 shows the students' $s(a, c)$ performance indicators as labels on a plot of the confidence levels they espoused at that time, near the end of the course. Note the spread of technology competencies c (third in each label) in the dotted section, in particular. Note the much closer relationship between achievement s and approach a, than between s and c. Missing data are a result of near coincidence of points.

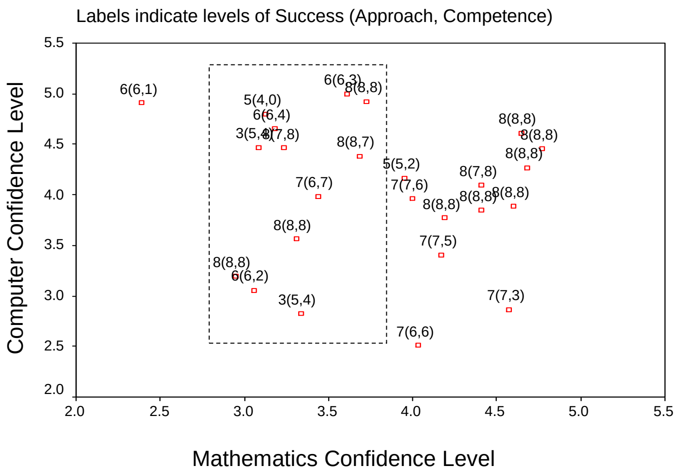

FIGURE 1. 2001 Workshop Data (N=29).

4. The 2003 Midsemester Test Study

To investigate relationships between attitudes, technology behaviour, and achievement under different conditions, and especially under test conditions, data were captured for a class of 119 students (initially) in 2003. The questions addressed here are as follows:

(1) What are the relationships between confidences, attitudes, and achievement in this technology-enriched mathematics course?
(2) Are these findings consistent with those of the 2001 pilot Workshop study of students working under non-assessment conditions ?
(3) What other factors may affect the nature and level of students' use of technology when doing mathematics in such a course?
(4) What are the implications for teaching and curriculum design?

4.1. Factors and data capture: The attitudinal factors under focus were computer confidence, mathematics confidence, and attitudes to the use of technology in the learning of mathematics. Achievement was measured by performance on a one-hour midsemester test in a computer lab, and on a 2-hour pen-and-paper exit examination. Technology indicators identified for analysis were frequency of use, range of use, and competency. Other factors included prior use of technology for doing mathematics, mathematics proficiency on entry, performance on other course tasks, levels of completion of the weekly tasks over the semester, attendance at the weekly small group tutorial/laboratory sessions, gender, and degree program.

- Mathematics confidence, computer confidence, and attitudes to technology in the learning of mathematics were captured on entry to the course using the USQ MATH/TECH scales. Tested over a number of years, these scales demonstrate high internal consistency and test/re-test reliability and are in use in universities in several countries (for example, see [**12**], and [**5**]). For further information, see [**4**] or contact the first author.

- Data on achievement and performance were captured in the mid-semester test, exit examination, two assignments, and the weekly exercises.
- Mathematics proficiency on entry was established from the grade achieved prior to entry. Circumstances did not permit a diagnostic test on entry.
- Students' behaviour and attitudes were observed by the first author (one of the lecturers) in two-hour weekly tutorial/lab sessions over the full semester.
- General views on the use of technology for learning mathematics, and on the approaches used in this course in particular, were captured in a mid-semester questionnaire which included open-ended questions. Interview data is not yet available for the 2003 class. Focus interviews from earlier years are used for reference in this report, including a representative sample of 15 students with different confidence levels in the 2001 class.

4.2. Analysis and findings from the 2003 Data: Most students took advantage of support for the use of technology and engaged willingly in the voluntary weekly computer laboratory tasks, as in prior years. Almost all demonstrated competent basic use of technology on the compulsory assignment tasks.

Attitudes (measured on entry) and achievement on the midsemester test and an exit examination, confirmed findings in previous studies:

- Mathematics confidence correlated weakly with computer confidence as expected from prior work (Pearson's $\rho = 0.22, n = 119$).
- Mathematics-technology attitudes correlated more strongly with computer confidence than with mathematics confidence ($\rho = 0.44$ against 0.24).
- Computer confidence showed no relationship with achievement on the midsemester test and on an exit examination, as in earlier studies.
- Mathematics confidence measured on entry to the course showed some positive correlation with achievement on the midsemester test and exit examination (around $\rho = 0.30$). Relationships of up to $\rho = 0.65$ have been found with mathematics confidence measured later in the semester: see [**3**].

Students' use of technology in the one-hour mid-semester test was investigated. The test tasks could all be done by hand, given enough time, but students were not aware of that in advance. Basic use of technology for calculation and row-reduction could yield some savings in time and there was opportunity for more sophisticated use. There were 6 opportunities to use technology: to calculate the values of a function, row-reduce 2 systems of linear equations, and plot graphs in two and three-dimensions (a polynomial, and a line or planes in 3-D).

Each student's work was rated on each of the following *technology indicators*:

- *frequency of use* of technology (0 to 6): a count of opportunities taken;
- *competency level* when using technology (a total between 0 and 10): or example, row reduction required only the entry of a matrix and use of 1 command, and was assigned a competency level of 1: plotting the 2-D graphs included basic syntax skills and was assigned a competency of 2, etc.
- *range of types of use* of technology (0 to 9): a rating on a hierarchy appropriate for the tasks. A reasonable attempt to use a simple type, like row-reduction was assigned a rating of 1. Higher ratings were assigned for use of more types and increasingly complex use.

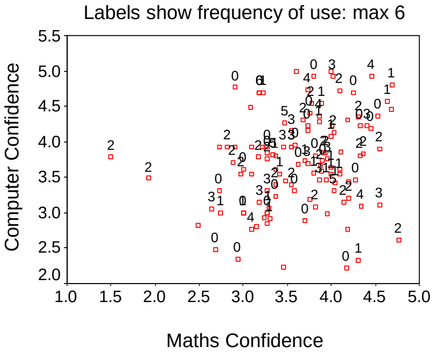

Figure 2a: Scatterplot of confidences showing frequency of use of technology.

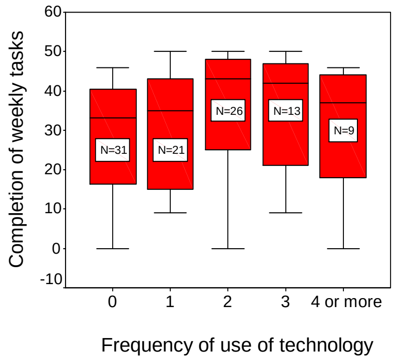

Figure 2b: Weekly effort versus frequency on the test.

FIGURE 2. 2003 Midsemester Test Data (N=100).

Viewed together, the three indicators provide a profile of the student's use of technology. For example, note that *frequency of use* alone does not distinguish between repeated use of the same process and different types of use.

The data revealed some low levels of use of the computer under test conditions and low relationships between computer attitudes and the technology indicators under test conditions. Figure 2a shows each student's frequency of use of technology in the test as a labels on their position on a scatterplot of computer confidence against mathematics confidence. Note the variation in the label indicating *frequency of technology use* as one moves vertically through a range of computer confidences: for example, higher points showing 0 frequency denote students with high computer confidence who chose *not* to use technology under test conditions. Considerable variation in *competency level* and *range of types of use* were also detected across the spectrum of computer attitudes.

The data also revealed no significant relationships between the technology indicators on the mid-semester test and the factors listed below:

- the degree to which they engaged in homework practice tasks of a similar nature over the semester (this related well to overall achievement in the course, however);
- their level of prior experience using technology for doing mathematics;
- their level of attendance at the weekly tutorial/laboratory sessions;
- gender; and degree program.

Figure 2b shows boxplots of the spread of levels of completion of the weekly tutorial/lab tasks (maximum 50) for five sets of students: grouped according to their frequency of use of technology in the test. While 22 students took strong opportunity to use technology in the test (ratings of 3 or above on frequency with good range of use and competency), 26 demonstrated only some substantial use, 21 used it for only one type of process (row-reduction or 2-D graphing) and 31 did not use it at all.

The boxplot on the left of Figure 2b shows the wide range of levels of completion (maximum 50) of the voluntary weekly tasks among those 31 students who chose not to use technology for the test tasks. The upper part indicates those students who demonstrated high levels of completion of the weekly tutorial/lab exercises but abandoned use of technology under test conditions. The boxplot on the right of Figure 2b shows a similar spread of levels of weekly task completion among the 9 students who used technology at least 4 times out of the maximum of 6 opportunities.

5. Discussion

The role of technology as a valued aid for calculation and graphing was unambiguous in the course, and appropriate use of technology was permitted in the assessment work. Quite high levels of computer use were evident on the Workshop tasks which were not done under assessment conditions. Lower levels of computer use were observed for the large group under test conditions, and 31 students chose not to use the computer at all, despite quite high levels of participation in the weekly tutorial/laboratory tasks.

Analysis of the 2003 data confirmed earlier findings on mathematics and computer confidence and attitudes in prior research carried out by the authors and others: see 4.2. Earlier findings on the relationships between attitudes and achievement were confirmed: computer confidence showed no relationship at all with achievement on typical tasks in this technology-enriched mathematics course, despite its role in concept develpment and problem solving. Mathematics confidence showed some positive correlation with achievement.

Other findings for the large-group data also resonated quite strongly with trends in the small-group pilot study. This is of particular interest because the data were captured under quite different conditions, and analysed quite differently. In summary, computer confidence and attitudes did not relate well to achievement levels, or to levels of use of technology, on typical assessment tasks in a technology-enriched mathematics classroom.

Investigation of prior experience of using technology for doing mathematics, level of attendance at the weekly tutorial/laboratory sessions, gender, and degree program also failed to reveal any clear relationships with levels of technology use and competencies under test conditions; nor did the degree to which they engaged in homework practice tasks similar to those in the test over the semester. The latter related well to overall achievement in the course, however.

Amongst other findings, these studies identified students who opted for hand approaches when under pressure of some kind, despite demonstrating willingness to use technology as an aid generally over the semester. The time needed to build the skills necessary for effective use of technology, especially under conditions of assessment, needs careful consideration.

Interviews in 2001 and earlier, and open ended questions in 2003 (see 4.1), yielded clear strong statements from students about their beliefs and habits: students express the "need" to do basic things "by hand" to "understand". Students value hand exercises, and at times using technology challenges their sense of ownership and control of learning. Their views resonate with typical responses echoed in [**11**] and "tales of resistance" in [**8**]. When invited to elaborate, some refer to feelings of satisfaction and confidence that arise from handwork: "I feel better if I can do it by hand" and "I know I can do it". Some allude to feelings of discomfort with the black-box nature of computer software. Others express feelings of insecurity under timed test conditions in a laboratory, citing that it is hard to cope with both hand-work and computer simultaneously: it "takes too much time" and "it makes things worse". Defending hand methods, the vocal responses of a few echo habits from the past like verbal umbilical chords: a simple repetitive "that's how I learn", or "that's how I always work".

6. Summary and Conclusion

This paper reports trends found in a technology-enriched early undergraduate mathematics program in which students did weekly tasks that used computer software for concept development, graphing and computation. Almost all students demonstrated competent use of technology on compulsory assignment tasks and many took the opportunity to use technology under conditions of choice. Relatively high levels of competence gave some students an advantage over others under assessment conditions, suggesting evidence of *the rich getting richer*.

Despite the opportunities afforded by technology, there were low relationships between students' technology attitudes and their achievement on a broad range of course tasks. Positive student attitudes to using technology and engagement in weekly practice did not always yield demonstrable early outcomes. In particular, there were clear signs of mismatch between some students' technology attitudes and their technology choices and behaviour under the pressure of a midsemester test. Much higher levels of technology use were evident under non-assessment workshop conditions, but there were some similar signs of mismatch between attitudes and demonstrated competencies.

Mathematics confidence emerged as the affective factor that related most reliably with achievement in this learning environment, even though technology played a substantial role. It yielded low to moderate relationships with achievement on a wide range of course work. Computer confidence and technology attitudes did not. Foreshadowed by earlier studies ([**3**], [**2**]), these findings emerged here from data captured from students working under different conditions. These findings need investigation in other technology contexts.

Earlier findings on attitudes were confirmed by this study (see 4.2), providing compelling evidence that computer and mathematics occupy different dimensions in education contexts. Curriculum development and profiles for assessment need to accommodate students' learning needs and preferences and the rates at which they evolve. Schuck [**9**] described students' mathematical beliefs and attitudes that are established over many years as "chains" that bind them to the past. Perhaps the deeply intertwined learning beliefs and preferences that students develop through prior experiences are more like umbilical cords: students grow and flourish when they are ready for new experiences, but may flounder if those ties are cut too soon.

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