PERFORMANCE-BASED OPTIMIZATION OF STRUT-AND-TIE MODELS IN REINFORCED CONCRETE BEAM-COLUMN CONNECTIONS

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ABSTRACT: This paper presents the performance-based optimization of strut-and-tie models in reinforced concrete beam-column connections. The performance-based optimization (PBO) technique is employed to investigate optimal strut-and-tie models in reinforced concrete beam-column connections. Developing strut-and-tie models in reinforced concrete beam-column connections is treated as a topology optimization problem. The optimal strut-and-tie model in a concrete beam-column connection is generated by gradually removing inefficient finite elements from the connection in an optimization process. Optimal strut-and-tie models for the design and detailing of opening knee joints, exterior beam-column connections and interior beam-column connections are presented.

KEYWORDS: beam-column connection; performance-based optimization; reinforced concrete; strut-and-tie model; topology optimization.

1. INTRODUCTION

Strut-and-tie models are particularly suitable for designing the disturbed regions (D-regions) of a concrete structure where the strain distribution is significantly nonlinear, such as at point loads, corbels, deep beams, and openings as shown by Marti [1] and Schlaich et al. [2]. Standard truss models or sectional methods can be used to design the B-regions of a concrete structure where the strain distribution is linear. Strut-and-tie modeling has been proved to be a rational, unified, and safe approach for the design and detailing of structural concrete under combined load effects [3, 4].

Traditionally, strut-and-tie models in structural concrete are developed using the elastic stress distribution and load path methods in practice. These conventional methods involve a trial-and-error process based on the designer's intuition and past experience. The strut-and-tie model obtained by such methods is not unique and varies with the designer's experience. As a result, topology optimization and computer graphics have been used to develop strut-and-tie models in concrete structures [5, 6]. To overcome the limitations of conventional methods, the performance-based optimization (PBO) technique has been developed by Liang [7] and Liang et al. [8-10] for automatically generating optimal strut-and-tie models for the design and detailing of reinforced and prestressed concrete structures.

This paper extends the PBO method for topology optimization of continuum structures with mean compliance constraints to the strut-and-tie modeling of reinforced concrete beam-column connections. The basic theory of the PBO method is described. The PBO technology is employed to investigate the optimal strut-and-tie models for the design and detailing of opening knee joints, exterior beam-column connections and interior beam-column connections.

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2. PERFORMANCE-BASED OPTIMIZATION

2.1 OPTIMIZATION PROBLEM

In reality, some parts of a structural concrete member are not as effective in carrying loads as other parts. By eliminating underutilized portions from a structural concrete member, the actual load path in the member can be found. The PBO method has the capacity to identify the underutilized portions of a structure and gradually remove them from the structure to maximize its performance. Therefore, the strut-and-tie modeling of structural concrete can be transformed to a topology optimization problem of continuum structures. In nature, loads are transmitted by the principle of minimum strain energy. The strategy is to find a strut-and-tie system as stiff as possible at minimum weight. The optimization problem can be stated as

$$\text{minimize} \sum_{e=1}^{n} w_e(t) \tag{1}$$

$$\text{subject to } C \leq C^* \tag{2}$$

Where w_e is the weight of the eth element, t is the thickness of all elements, C is the mean compliance or strain energy of the structure under applied loads and C^* is the limit of the C and n is the total number of elements in the structure. It is noted that the mean compliance of a structure under applied loads is usually used as a revised measure of its overall stiffness.

2.2 ELEMENT REMOVAL CRITERIA

Design sensitivity analyses indicate that the strain energy density of an element, which is defined as the strain energy of an element per unit weight can be used to evaluate the contribution of the element to the overall stiffness performance of a structure modeled by finite elements [7]. The element strain energy density can be calculated by

$$\gamma_e = \left| \tfrac{1}{2} \{u_e\}^T [k_e] \{u_e\} \right| / w_e \tag{3}$$

Where $\{u_e\}$ is the displacement vector of the eth element and $[k_e]$ is the stiffness matrix of eth element. The element removal criteria state that elements with the lowest strain energy densities should gradually be removed from the continuum design domain to achieve the performance objective as presented by Liang and Steven [11]. The element removal criteria lead to the stiffest load-carrying system in a continuum structure.

2.3 PERFORMANCE-BASED OPTIMALITY CRITERIA

By gradually eliminating elements with the lowest strain energy densities from a structural concrete member, its performance in terms of the efficiency of material usage and overall stiffness can be improved. The performance of strut-and-tie systems in an optimization process has to be evaluated in order to identify the optimum. Performance indices (PI) that characterize the structural responses and the weight of a structural member in an optimization process have been developed for quantifying the performance of structural topologies by Liang et al. [8-11]. The performance-based optimality criteria for structures with mean compliance constraints can be stated as [11]

$$\text{maximize } PI = \frac{C_0 W_0}{C_i W_i} \tag{4}$$

where C_0 and W_0 are the strain energy and weight of the initial concrete member without element removal, respectively and C_i and W_i are the strain energy and weight of the current system at the *i*th iteration, respectively. The optimal topology obtained represents the most efficient load transfer mechanism in the structural concrete member. As a result, optimal topologies generated by the PBO technique can be treated as optimal strut-and-tie models for the design and detailing of structural concrete members. The physical meaning of the performance-based optimality criteria is that the optimal strut-and-tie model transfers loads in a way such that the product of its associated strain energy and material consumption are a minimum.

2.4 OPTIMIZATION PROCEDURE

The basic concepts of the innovative and advanced PBO technique are that an optimal design can be generated by repeating the process of the finite element analysis (FEA), performance evaluation and element removal until the performance index is maximized. The PBO algorithm has been developed to automate this process. The user only needs to set up a simple finite element model for a structural concrete member. The main steps of the performance optimization procedure for strut-and-tie models are given as follows:

(1) Model the 2D concrete member with plane stress elements.
(2) Perform FEA on the concrete member.
(3) Evaluate the performance of the resulting system using the performance index.
(4) Calculate the strain energy densities of elements.
(5) Remove a small number of elements with the lowest strain energy densities from the member.
(6) Repeat step (2) to (5) until the performance index is maximized.
(7) Select the optimal strut-and-tie model.

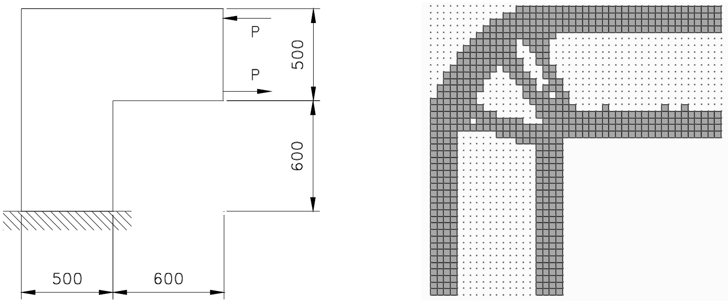
The PBO program developed can be used to generate optimal strut-and-tie models for the design and detailing of both reinforced and prestressed concrete structures. The prestressing forces can be treated as external loads in the finite element models.

3. EXAMPLES

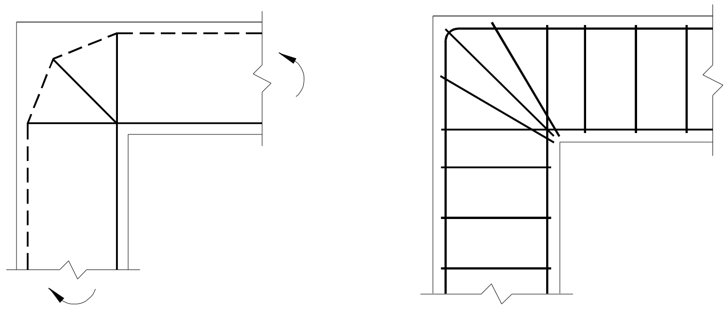
3.1 OPENING KNEE JOINTS

Opening knee joints are beam-column connections under positive moments in portal frames. The opening knee joint is treated as a D-region shown in Figure 1(a). In the finite element model, the positive moment was modeled by a couple as depicted in Figure 1(a). The thickness of the connection was 350 mm. The compressive cylinder strength of concrete was 32 MPa and Poisson's ratio was 0.15. The connection was discritized using 25 × 25 mm plane stress finite elements. The PBO technique was used to optimize this opening knee joint. The element removal ratio of 2 per cent was used in the performance optimization process. The optimal topology obtained is presented in Figure 1(b). The discrete strut-and-tie model idealized from the optimal topology is depicted in Figure 1(c), where the solid bold lines represent tension ties and the dashed lines represent struts. The figure shows that diagonal tensile force developed within the corner of the joint. The strut-and-tie model clearly indicates the layout of tension ties and therefore the steel reinforcement. Figure 1(d) depicts the layout of steel reinforcement in the opening knee joint. Three stirrups are fan out from the re-entrant corner to control possible diagonal cracking in the joint. U-bars should be used as main tensile steel reinforcement in beam and column of the connection to achieve effective anchorage.

3.2 EXTERIOR BEAM-COLUMN CONNECTIONS



(a) Structural model (b) Optimal topology



(c) Strut-and-tie model (d) Reinforcement

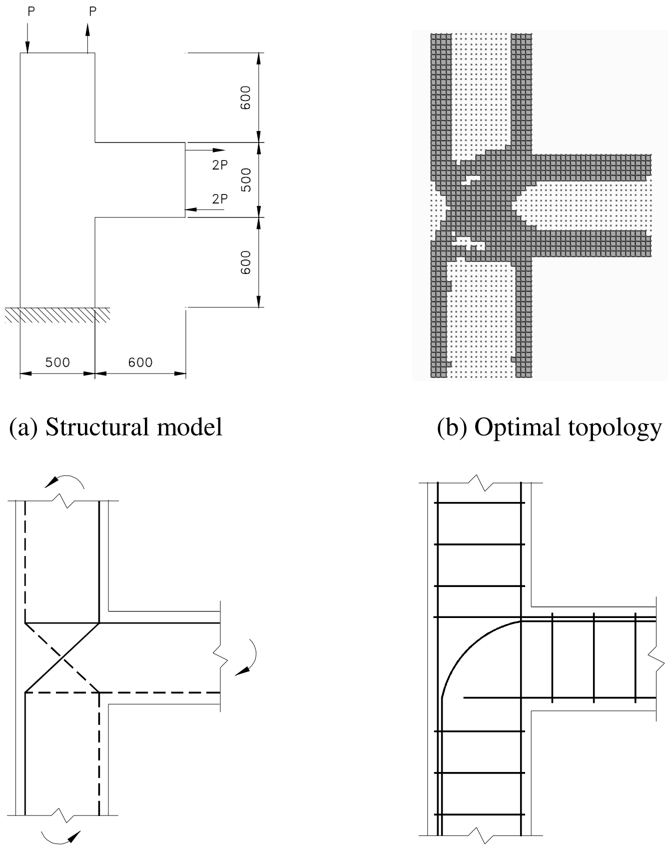
Figure 1. Opening knee joint

An exterior beam-column connection is a region where a beam joins an exterior continuous column. In an exterior beam-column connection, the bending moment acting on the beam is transferred to the upper and lower columns. Under applied loads, one corner of the connection is closing and the other is opening. The exterior beam-column connection is a D-region, which can be simulated using the structural model depicted in Figure 2(a). The beam bending moment was modeled using a couple forces of 2*P*. It was assumed that the stiffness of the upper and lower columns was the same, half of the bending moment was transferred to the upper column and another half was transferred to the lower column. The bending moment in the upper column was simulated using a couple forces of *P*. The bottom of the lower column was fixed. The thickness of the connection was specified as 350 mm. The compressive concrete strength of 32 MPa and Poisson's ratio of 0.15 were assumed for the concrete material. The connection was discretized using 25 × 25 mm four-node plane stress elements. The element removal ratio of 2 per cent was used in the optimization process.

The optimal topology generated by the PBO technique is presented in Figure 2(b). The discrete strut-and-tie model can be readily obtained from the optimal continuum strut-and-tie model and is shown in Figure 2(c). The strut-and-tie model indicates that tensile forces developed in the upper part of the beam, the right side of the upper column, the diagonal of the connection, and the left side of the lower column. These tensile forces in these parts can induce cracking of concrete. To control crack, the exterior beam-column connection is reinforced like that depicted in Figure 2(d). It is suggested that continuous and curved reinforcing bars should be used to resist the diagonal tensile forces in exterior beam-column connections as shown in Figure 2(d).

3.3 INTERIOR BEAM-COLUMN CONNECTIONS

The interior beam-column connections consisting of a continuous beam supported on a column is considered here. Under horizontal loads, one of the beams is subjected to positive moment and the



(a) Structural model (b) Optimal topology

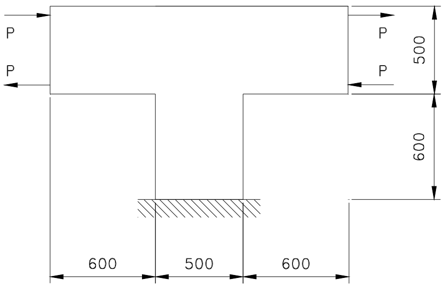
(c) Strut-and-tie model (d) Reinforcement

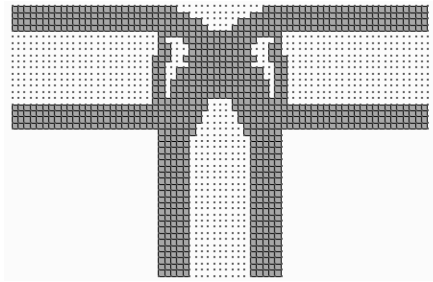
Figure 2. Exterior beam-column connection

other is under negative bending. The structural model depicted in Figure 3(a) was used to simulate interior beam-column connections in the finite element analysis. Beam bending moments were represented using a couple forces as shown in Figure 3(a). The connection was discretized into finite element mesh using four-node plane stress elements. The compressive strength of concrete of 32 MPa and Poisson's ratio $\nu = 0.15$ were specified in the finite element analysis. The element removal ratio of 2 per cent was used in the optimization process. The optimal topology generated by the PBO method is presented in Figure 3(b). The discrete strut-and-tie system in the connection is depicted in Figure 3(c). It appears from the strut-and-tie model that diagonal tensile forces developed in the connection. Figure 3(d) shows that the beam-column connection is reinforced with continuous and curved steel bars to resist diagonal tensile forces induced by two horizontal load cases.

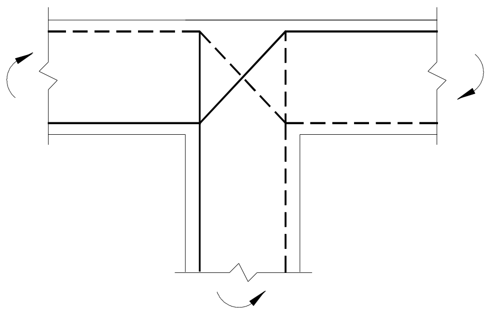
4. CONCLUSIONS

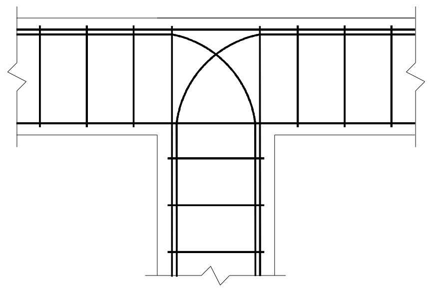
The performance-based optimization of strut-and-tie models in reinforced concrete beam-column connections has been presented in this paper. The basic theory underlying the PBO technology has been described. By visualizing the beam-column connections as D-regions, the PBO technology can be used to automatically generate optimal strut-and-tie models for the design and detailing of reinforced concrete beam-column connections. Optimal strut-and-tie models in opening knee joints, exterior beam-column connections and interior beam-column connections were presented. The reinforcement detailing of beam-column connections based on the layouts of strut-and-tie models produced were also presented. It is demonstrated that the PBO technology is an effective tool for the design and detailing of structural concrete particularly the D-regions such as beam-column connections.





(a) Structural model (b) Optimal topology





(c) Strut-and-tie model (d) Reinforcement

Figure 3. Interior beam-column connection

5. REFERENCES

1. Marti, P., "Basic tools of reinforced concrete beam design." *ACI Structural Journal,* Vol. 82, No. 1, 1985, pp. 46-56.
2. Schlaich, J., Schäfer, K. and Jennewein, M., "Toward a consistent design of structural concrete." *PCI Journal*, Vol. 32, No. 3, 1987, pp. 74-150.
3. Ramirez, J.A. and Breen, J.E., "Evaluation of modified truss model approach for beams in shear." *ACI Structural Journal,* Vol. 88, No. 5, 1991, pp. 562-571.
4. ASCE-ACI Committee 445 on Shear and Torsion, "Recent approaches to shear design of structural concrete." *J. Struct. Eng.*, ASCE, Vo. 124, No. 12, 1998, pp. 1375-1417.
5. Ramm, E., Beltzinger, K.-U. and Maute, K., "Structural optimization." *Current and Emerging Technologies of Shell and Spatial Structures, Proc. IASS Colloquium,* Madrid, 1997, pp. 201-216.
6. Alkshegeir, A. and Ramirez, J., "Computer graphics in detailing strut-and-tie models." *J. Computing in Civil Engineering*, ASCE, Vol. 6, No. 2, 1992, pp. 220-232.
7. Liang, Q.Q. *Performance-Based Optimization of Structures: Theory and Applications*. Spon Press, London, 2005.
8. Liang, Q.Q., Xie, Y.M. and Steven, G.P., "Topology optimization of strut-and-tie models in reinforced concrete structures using an evolutionary procedure." *ACI Structural Journal*, Vol. 97, No. 2, 2000, pp. 322-330.
9. Liang, Q.Q., Xie, Y.M. and Steven, G.P., "Generating optimal strut-and-tie models in prestressed concrete beams by performance-based optimization." *ACI Structural Journal*, Vol. 98, No. 2, 2001, pp. 226-232.
10. Liang, Q.Q., Uy, B. and Steven, G.P., "Performance-based optimization for strut-tie modeling of structural concrete." *Journal of Structural Engineering*, ASCE, Vol. 128, No. 6, 2002, pp. 815-823.
11. Liang, Q.Q. and Steven, G.P., "A performance-based optimization method for topology design of continuum structures with mean compliance constraints." *Computer Methods in Applied Mechanics and Engineering,* Vol. 191, No. 13-14, 2002, pp. 1471-1489.