Three Dimensional Lower Bound Solutions for the Stability of Plate Anchors in Sand

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Abstract

Soil anchors are commonly used as foundation systems for structures that require uplift or lateral resistance. These types of structures include transmission towers, sheet pile walls and buried pipelines. Although anchors are typically complex in shape (e.g. drag or helical anchors), many previous analyses idealise the anchor as a continuous strip under plane strain conditions. This assumption provides numerical advantages and the problem can solved in two dimensions. In contrast to recent numerical studies, this paper applies three dimensional numerical limit analysis and axi-symetrical displacement finite element analysis to evaluate the effect of anchor shape on the pullout capacity of horizontal anchors in sand. The anchor is idealised as either square or circular in shape. Results are presented in the familiar form of breakout factors based on various anchor shapes and embedment depths, and are also compared with existing numerical and empirical solutions.

Contents

Nomenclature

A	anchor area
B	anchor width
D	anchor diameter
L	anchor length
H	anchor embedment depth
γ	the soil unit weight
ψ'	the soil dilation angle
ϕ'	the soil friction angle
c'	the soil cohesion
N_γ	the anchor break-out factor
H/B	anchor embedment ratio
H/D	anchor embedment ratio
L/B	anchor aspect ratio
q_u	the ultimate anchor pullout capacity
S_F	the dimensionless anchor shape factor

1. Introduction

1.1. Background and Objectives

Soil anchors can be square, circular or rectangular in shape and are commonly used as foundation systems for structures requiring uplift resistance, such as transmission towers, or for structures requiring lateral resistance, such as sheet pile walls. More recently anchors have been used to provide a simple and economical mooring system for offshore floating oil and gas facilities. As the range of applications for anchors expands to include the support of more elaborate and substantially larger structures, a greater understanding of their behaviour is required.

The theory of soil uplift resistance may also be used to solve a number of geotechnical problems where primary uplift resistance of a structure is not provided by the addition of soil anchors. For example, structures such as submerged pipelines or buried foundations, although not supported by anchors, may be modelled effectively as soil anchors.

The objective of the present paper is to quantify the effect of anchor shape upon the ultimate pullout capacity. To do this, lower bound solutions for the ultimate capacity of horizontal square, and circular anchors in sand are determined. In addition, axi-symetrical displacement finite element analyses are also undertaken. The results are then compared to a previous study of strip anchors in sand (Merifield 2001), along with the available empirical and numerical results presented in the literature.

The general layout of the problem to be analysed is shown in Figure 1

The ultimate anchor pullout capacity in cohesionless soil is usually expressed as a function of the soil unit weight γ and embedment depth H in the following form

$$q_u = \gamma H N_\gamma \tag{1}$$

where N_γ is referred to as the anchor break-out factor.

1.2. Previous studies

To provide a satisfactory background to subsequent discussions, a summary of research into plate anchor behaviour is presented. A comprehensive overview on the topic of anchors is given by Das (1990).

One of the earliest applications of soil anchors was in supporting transmission towers. This application was responsible for the driving force behind a lot of the initial research into anchor behaviour (Balla 1961). Initially these towers were supported by large deadweight concrete blocks where the required uplift capacity was achieved solely due to the self weight of the concrete. This simple design came at considerable cost and, as a result, research was undertaken in order to find a more economical design solution. The result was what is known as belled piers or mushroom foundations. As the range of applications for anchors expanded to include the support of more elaborate and substantially larger structures, a more concerted research effort has meant soil anchors today have evolved to the point where they now provide an economical and competitive alternative to these mass foundations.

It will become clear that the majority of past research has been experimentally based and, as a result, current design practices are largely based on empiricism. In contrast, very few thorough numerical analyses have been performed to determine the ultimate pullout loads of anchors. Of the numerical studies that have been presented in the literature, few can be considered as rigorous.

Numerous investigators have performed model tests in an attempt to develop semi-empirical relationships that can be used to estimate the capacity of anchors in cohesionless soil. This is evidenced by the large number of studies shown in Table 1. However, for the sake of brevity, discussions will be limited to those investigations that have made the most significant contribution to anchor uplift theory.

The works prior to 1970 have not been presented in Table 1 and Table 2. This includes the field and/or model testing of horizontal circular anchors or belled piles by Mors (1959), Giffels *et al* (1960), Balla (1961), Turner (1962), Ireland (1963), Sutherland (1965), Mariupolskii (1965), Kananyan (1966), Baker and Konder (1966), and Adams and Hayes (1967). A number of these studies were primarily concerned with testing foundations for transmission towers (Mors (1959), Balla (1961), Turner (1962), Ireland (1963)).

In the majority of earlier studies, a failure mechanism was assumed and the uplift capacity was then determined by considering the equilibrium of the soil mass above the anchor and contained by the assumed failure surface. Based on the underlying assumptions, these methods of analysis are commonly referred to as the "Soil cone" method (Mors (1959)) and the "Friction cylinder" method (Downs & Chieurzzi (1966)).

Table 1 Laboratory model tests on horizontal anchors in cohesionless soil.

Author	Type of Testing	Anchor shape	Anchor size	Friction angles	Anchor Roughness	*H/B or H/D*
Hanna & Carr (1971)	Chamber	CIRC	38mm	37°	?	4-112
Hanna *et al* (1971)	Chamber & Field	CIRC	38mm & 150mm	37°	?	4-112
Das & Seeley (1975a)	Chamber	SQ RECT	51mm *L/B*=1-5	31°	?	1-5
Rowe (1978)	Chamber	SQR RECT	51mm	32°	16.7°	1-8
Andreadis *et al* (1981)	Chamber	CIRC	50mm - 150mm	37°, 42.5°	?	1-14
Ovesen (1981)	Centrifuge & field	CIRC SQR	20mm	29.5° - 37.7°	?	1-3.39
Murray & Geddes (1987)	Chamber	CIRC RECT *L/B*=1-10	50.8mm	44° Dense 36° Med	11 smooth 42 rough	1-10
Saeedy (1987)	Chamber	CIRC	37.8-75.6 mm	42	?	5-10
Frydman & Shamam (1989)	Field Chamber (Summary)	STRIP RECT	19mm 200mm	30° Loose 45° Dense	?	2.5-9.35
Dickin (1988)	Centrifuge Chamber	SQR RECT *L/B*=1-8	25mm 50mm	38°-41°* Loose 48°-51°* Dense	?	1-8
Tagaya *et al* (1988)	Centrifuge	CIRC RECT	15mm	42°	?	3-7.02
Murray & Geddes (1989)	Chamber	SQR RECT *L/B*=1-10	50.8mm	43.6° Dense 36° Med dense	10.6°	1-8
Sarac (1989)	?	CIRC SQR	?	37.5°, 48°	?	0.35-4
Bouazza & Finlay (1990)	Chamber	CIRC	37.5mm	33.8°, 39°, 43.7° Layered	?	2-5
Sakai & Tanaka (1998)	Chamber	CIRC	30mm - 200mm	?	?	1-3
Pearce (2000)	Chamber	CIRC	50mm - 125mm	Loose to very Dense	?	2-15
Ilamparuthi *et al* (2002)	Chamber	CIRC	100mm-400mm	Loose to Dense		0.85 - 11.97

* Plane strain friction angle

Subsequent variations upon these early theories have been proposed including that of Balla (1961) who determined the shape of slip surfaces for shallow horizontal anchors in dense sand and proposed a rational method for estimating the capacity of anchors based on the observed shapes of the slip surfaces. Baker and Kondner (1966) confirmed Balla's major findings regarding the behavioural difference of deep and shallow anchors in dense sand. Sutherland (1965) presented results for the pull-out of 150mm horizontal anchors in loose and dense sand, as well as large diameter shafts in medium dense to dense sands. It was concluded that the mode of failure varied with sand density and that Balla's analytical approach may give reasonable results only in sands of intermediate density. Kananyan (1966) presented results for horizontal circular plate anchors in loose to medium dense sand. He also performed a series of tests on inclined anchors and observed the failure surface, concluding that most of the soil particles above the anchor moved predominantly in a vertical direction. In these tests, the ultimate capacity increased with the inclination angle of the anchors.

Extensive chamber testing programs have been performed by Murray and Geddes (1987, 1989), who performed pull-out tests on horizontal strip, circular, and rectangular anchors in dense and medium dense sand with $\phi' = 43.6^{\circ}$and $\phi' = 36^{\circ}$ respectively. Anchors were typically 50.8mm in width/diameter and were tested at aspect ratios (L/B) of 1, 2, 5 and 10. Based on their observations, Murray and Geddes made several conclusions: (1) the uplift capacity of rectangular anchors in very dense sand increases with embedment ratio and with decreasing aspect ratio L/B ; (2) there is a significant difference between the capacity of horizontal anchors with rough surfaces compared to those with polished smooth surfaces (as much as 15%); (3) experimental results suggest that an anchor with an aspect ratio of $L/B = 10$ behaves like a strip and does not differ much from an anchor with $L/B = 5$, and; (4) the capacity of circular anchors in very dense sand is approximately 1.26 times the capacity of square anchors. Several of these conclusions confirm the findings of Rowe (1978). It is also of interest to note that for all the tests performed by Murray and Geddes, no critical embedment depth was observed.

More recently, Pearce (2000) performed a series of laboratory pullout tests on horizontal circular plate anchors pulled vertically in dense sand. These tests were conducted in a large calibration chamber, with dimensions one meter in height and one meter in diameter. Various parameters such as anchor diameter, pullout rate and elasticity of loading system have been investigated. The model anchors used for the pullout tests varied in diameter from 50-125mm and were constructed from 8mm mild steel. Large diameter anchors were chosen (compared with previous research) due to the recognised influence of scale effects on the break-out factor for anchors of diameters less than 50mm (Andreadis *et al*, 1981).

A similar study to that of Pearce (2000) was performed by Ilamparuthi K., Dickin E. A., and Muthukrisnaiah (2002) who conducted a series of laboratory pullout tests on horizontal circular plate anchors pulled vertically in loose to dense sand. A discussion of the observed failure mechanisms, load displacement response and critical embedment depth was also provided. A set of empirical equations were presented for estimating the break-out factors for circular anchors with any friction angle.

Although not as popular as chamber testing, centrifuge testing of anchors has been undertaken by a number of Authors (see Table 1). Dickin (1988) performed 41 tests on 25mm anchor plates with aspect ratios of $L/B = 1, 2, 5$ and 8 at embedment ratios H/B up to 8 in both loose and dense sand. A number of conventional gravity tests were also performed and compared to the centrifuge results. This comparison revealed a significant difference between the estimated anchor capacities, particularly for square anchors where the conventional test results gave anchor capacities up to twice that given by the centrifuge. Without explaining why, Dickin concluded that direct extrapolation of conventional chamber test data to field scale would provide over-optimistic predictions of the ultimate capacity for rectangular anchors in sand.

Tagaya *et al* (1988) also performed centrifuge testing on rectangular and circular anchors, although the study was limited in comparison to that of Dickin (1988) discussed above.

In contrast to the variety of experimental results already discussed, very few rigorous numerical analyses have been performed to determine the pullout capacity of anchors in sand. Whilst it is essential to verify theoretical solutions with experimental studies wherever possible, results obtained from laboratory testing alone are typically problem specific. This is particularly the case in geomechanics where we are dealing with a highly nonlinear material which often displays pronounced scale effects. As a result, it is often difficult to extend the findings from laboratory research to full scale problems with different material or geometric parameters. Since the cost of performing laboratory tests on each and every field problem combination is prohibitive, it is necessary to be able to model soil uplift resistance numerically for the purposes of design.

Existing numerical analyses generally assume a condition of plane strain for the case of a continuous strip anchor or axi-symmetry for the case of circular anchors. The Author is unaware of any three dimensional numerical analyses to ascertain the effect of anchor shape on the uplift capacity. A summary of previous studies for horizontal anchors is provided in Table 2.

An approximate semi-empirical theory for the uplift capacity of horizontal strip, circular, and rectangular anchors has been proposed by Meyerhof and Adams (1968). For a strip anchor, an expression for the ultimate capacity was obtained by considering the equilibrium of the block of soil directly above the anchor (i.e. contained within the zone created when vertical planes are extended from the anchor edges). The cohesive force was assumed to act along the vertical planes extending from the anchor edges, while the total passive earth pressure was assumed to act at some angle to these vertical planes. This angle was selected based on laboratory test results while the passive earth pressures were evaluated from the results of Caquot and Kerisel (1949). For shallow anchors where the failure surface extends to the soil surface, the ultimate capacity was determined by considering equilibrium of the material between the anchor and soil surface. For a deep anchor the equilibrium of a block of soil extending a vertical distance H above the anchor was considered, where H was less than the actual embedment depth of the anchor. The magnitude of H was determined from the observed extent of the failure surface from laboratory tests.

The analysis of strip footings was extended by Meyerhof and Adams to include circular anchors by using a semi-empirical shape factor to modify the passive earth pressure obtained for the plane strain case. The failure surface was assumed to be a vertical cylindrical surface through the anchor edge and extending to the soil surface. An approximate analysis for the capacity of rectangular anchors was obtained as for downward loads (Meyerhof 1951), by assuming the earth pressure along the circular perimeter of the two end portions of the failure surface is governed by the same shape factor adopted for circular anchors.

The paper by Meyerhof and Adams (1968) is widely referenced when considering the capacity of anchors. It is, however, based on two key assumptions; namely, the shape of the failure surface and the distribution of stress along the failure surface. Even so, the theory presented by Meyerhof and Adams (1968) has been found to give reasonable estimates for a wide range of anchor problems. It is one of only two methods available for estimating the capacity of rectangular anchors.

Table 2 Theoretical studies on horizontal anchors in cohesionless soil.

Author	Analysis Method	Anchor shape	Anchor Roughness	Friction Angles	H/B or H/D
Meyerhof & Adams (1968)	Limit Equilibrium - Semi-analytical	STRIP SQR/CIRC	?	-	-
Vesic (1971)	Cavity Expansion	STRIP/ CIRC	?	0 - 50°	0-5
Rowe & Davis (1982b)	Elastoplastic Finite Element	STRIP	Smooth	0 - 45°	1-8
Vemeer & Sutjiadi (1985)	Elastoplastic Finite Element/Upper bound	STRIP	?	All	1-8
Tagaya *et al* (1988) Tagaya *et al* (1983)	Elastoplastic Finite Element	CIRC/ RECT L/B=2	?	31.6°,35.1° 42°	0-30
Saeedy (1987)	Limit Equilibrium	CIRC	?	20-45	1-10
Murray & Geddes (1987)	Limit Analysis & Limit Equilibrium	STRIP RECT CIRC	?	All	All
Koutsabeloulis & Griffiths (1989)	Finite Element - Initial Stress Method	STRIP/ CIRC	?	20°,30°,40°	1-8
Sarac (1989)	Limit Equilibrium	CIRC/SQR	?	0-50°	1-4
Basudhar & Singh (1994)	Limit Analysis - Lower bound	STRIP	Rough/ Smooth	32°	1-8
Kanakapura *et al* (1994)	Method of Characteristics	STRIP	Smooth	5° - 50°	2-10
Ghaly & Hanna (1994)	Limit Equilibrium	CIRC	?	30°-46°	1-10
Smith (1998)	Limit Analysis - Lower bound	STRIP	Rough ?	25° - 50°	1-28
Sakai & Tanaka (1998)	Elastoplastic Finite Element	CIRC	?	Dense	1-3

The finite element method has also been used by Vemeer & Sutjiadi (1985), Tagaya *et al* (1983,1988), and Sakai and Tanaka (1998). Unfortunately, only limited results were presented in these studies.

Tagaya *et al* (1983,1988) conducted two-dimensional plane strain and axi-symmetric finite element analyses using the constitutive law of Lade and Duncan (1975). Scale effects for circular anchors in dense sand were investigated by Sakai and Tanaka (1998) using a constitutive model for a non-associated strain hardening-softening elasto-plastic material. The effect of shear band thickness was also introduced.

Koutsabeloulis and Griffiths (1989) investigated the trapdoor problem using the initial stress finite element method. Both plane strain and axi-symmetric studies were conducted. The Authors concluded that an associated flow rule has little effect on the collapse load for strip anchors but a significant effect (30%) for circular anchors. Large displacements were observed for circular anchors prior to collapse.

The remaining numerical studies shown in Table 2 estimate the anchor capacity using either the Limit Equilibrium Method (LEM) or method of Limit Analysis.

In the LEM, an arbitrary failure surface is assumed along with a distribution of stress along the assumed surface. Equilibrium conditions are then considered for the failing soil mass and an estimate of the collapse load is obtained. In the study of horizontal anchor capacity, the failure mechanism is generally assumed to be log spiral in shape (Saeedy (1987), Sarac (1989), Murray and Geddes (1987), Ghaly and Hanna (1994)) and the distribution of stress is obtained by using either Kotter's equation (Balla (1961)), or by making an assumption regarding the orientation of the resultant force acting on the failure plane.

Upper and lower bound limit analysis techniques have been used used by Murray and Geddes (1987,1989), Basudhar and Singh (1994) and Smith (1998) to estimate the capacity of horizontal and vertical strip anchors. Basudhar and Singh (1994) obtained estimates using a generalized lower bound procedure based on finite elements and non-linear programming similar to that of Sloan (1988). The solutions of Murray and Geddes (1987,1989) were obtained by manually constructing kinematically admissible failure mechanisms (upper bound), while Smith (1998) presented a novel rigorous limiting stress field (lower bound) solution for the trapdoor problem.

2. Results and Discussion

The popularity of helical screw anchors in civil engineering applications has provided the stimulus behind the large number of laboratory studies shown in Table 1. However, rigorous theoretical estimates of the capacity of circular or square anchors are scarce, as evidenced in Table 2. In this Section the results obtained for the capacity of circular and square anchors in cohesionless soil are presented. For the sake of brevity, these results are compared to only a selected number of available numerical and laboratory studies.

Estimates of the ultimate anchor pullout load have been obtained by using the three dimensional lower bound procedure developed by Lyamin (1999). This procedure can be used to obtain a lower bound collapse load for three dimensional geotechnical stability problems. Full details of the formulation can be found in Lyamin (1999) and Lyamin and Sloan (1997, 2000), and will not be repeated here. In addition, the displacement finite element formulation SNAC as presented by Abbo (1997) and Abbo and Sloan (2000), has been used to estimate the capacity of circular anchors using axi-symetrical elements. The research software SNAC (Solid Nonlinear Analysis Code), was developed with the aim of reducing the complexity of elasto-plastic analysis by using advanced solution algorithms with automatic error control. The resulting formulation greatly enhances the ability of the finite element technique to predict collapse loads accurately, and avoids many of the locking problems discussed by Toh and Sloan (1980) and Sloan and Randolph (1982). These break-out factors can then be compared to those obtained using the three dimensional finite element lower bound limit analysis.

2.1. Square anchors

Lower bound estimates of the anchor break-out factor N_γ (equation (1)) are shown in Figure 2 for various friction angles. The break-out factors increase in a nonlinear manner with increasing embedment ratio, with the greatest rate of increase occurring for medium to dense cohesionless soils where $\phi' \geq 30^\circ$.

Assuming a simple rigid block upper bound mechanism consisting of straight lines and circular arcs, Murray and Geddes (1987) proposed the following relationship for estimating the break-out of rectangular anchors.

$$N_\gamma = 1 + \frac{H}{B}\tan\phi\left(1 + \frac{B}{L} + \frac{\pi}{3}\frac{H}{L}\tan\phi\right) \tag{2}$$

Murray and Geddes compared the predictions given by Equation (2) to their laboratory findings for rectangular anchors with $L/B = 5$, and found that it overestimated the break-out factor. This relationship has been used to predict the break-out factors for square anchors ($L/B = 1$) and the results are shown in Figure 4(a). This Figure indicates that the break-out factors from (2) agree remarkably well with the numerical lower bounds for embedment ratios of $H/B \leq 5$. Above this embedment ratio, Equation (2) tends to overestimate the break-out factor.

The results obtained by Murray and Geddes (1987) from a series of uplift tests on small scale anchors are compared to the numerical lower bounds in Figure 4(b). Polished steel plates were used in these experiments with an interface friction angle of around 11°. The laboratory findings compare well with the numerical results over the range of embedment ratios shown.

Laboratory results obtained by Dickin (1988) from centrifuge testing and conventional gravity testing on square anchors in dense sand are also presented in Figure 4(b). The numerical lower bounds compare favourably with the conventional gravity test results of Dickin (1988), but overestimate the small scale centrifuge results for embedment ratios of $H/B > 4$.

The effect of anchor shape on the uplift resistance may be conveniently expressed as a dimensionless shape factor according to

$$S_F = \frac{N_{\gamma square}}{N_{\gamma strip}}$$

Figure 5(a) shows a plot of the numerical lower bound shape factors against embedment ratio. Also shown in this Figure are the experimental shape factors obtained by Murray and Geddes (1987). Although the experimental shape factors are around 20% below the numerical estimates, the trend observed in both sets of results are very similar.

2.2. Circular anchors

Lower bound and displacement finite element estimates of the anchor break-out factor N_γ are shown in Figure 3. As was the case for square anchors, the break-out factors increase in a nonlinear manner with increasing embedment ratio, with the greatest increase occurring for dense soils with high friction angles. As expected, the SNAC axi-symetrical displacement finite element results plot above the lower bound results by between 4-14%.

The effect of anchor shape can be expressed in terms of the dimensionless shape factor according to

$$S_F = \frac{N_{\gamma\, circle}}{N_{\gamma\, square}}$$

where and $N_{\gamma\, square}$ and $N_{\gamma\, circle}$ are obtained from Figure 2 and Figure 3 respectively. The lower bound shape factors are shown in Figure 5(b) for $\phi' = 20^\circ$, 30° and 40°. Considering equivalent areas of a circle and square, and ignoring stress concentration effects, we expect the shape factor to be close to 1.27 or $4/\pi$. Figure 5 suggests that the shape factor lies close to 1.27 for $H/B, H/D \geq 6$.

As highlighted in Table 2, most previous studies into circular anchor behaviour have been carried out using approximate techniques such as limit equilibrium or slip line methods. The Author is unaware of any rigorous three dimensional numerical studies to determine the behaviour of circular anchors in cohesionless soil. Nonetheless, the results obtained from a selected number of previous studies are reproduced for comparison purposes in Figure 6 and Figure 7.

As shown in Figure 6(a), the solutions of Murray and Geddes (1987), Balla (1961) and Meyerhof and Adams (1968), compare rather poorly with the numerical lower bounds for a loose soil with $\phi' = 20^\circ$. Indeed, the first two solutions overestimate or underestimate the break-out factor by up to 50%, while the Meyerhof and Adams (1968) predictions are of limited use for $H/D \geq 2$. In contrast, the limit equilibrium solutions of Sarac (1989) compare very well with the numerical lower bounds, but are only available for relatively shallow anchors where $H/D \leq 4$. For $\phi' \geq 20^\circ$, however, Sarac's solutions underestimate the lower bound break-out factors by up to 30% (Figure 6(b)).

For medium to dense soils with high friction angles, Figure 7 shows that the solutions of Murray and Geddes (1987) and Balla (1961) agree reasonably well with the numerical lower bounds, particularly when $\phi' = 30^\circ$. The solution of Meyerhof and Adams (1968) again significantly underestimates the break-out factor at larger embedment ratios, although for $\phi' = 40^\circ$ and $H/D \leq 4$ the solution is much improved.

Also shown in Figure 7 are the solutions obtained using the theories proposed by Saeedy (1987) and Ghaly and Hanna (1994). Both Authors use the limit equilibrium method as a basis for their analyses combined with assumptions regarding the distribution of stress on the failure plane. The predictions of Ghaly and Hanna (1994) are close to the numerical lower bounds for $\phi' = 40^\circ$, but become unconservative for looser soils where $\phi' \leq 30^\circ$. The solutions of Saeedy (1987) are very similar to those of Meyerhof and Adams (1968) and are grossly conservative for all but the smallest embedment ratios.

The disparity between the results shown highlights the problems inherent in using approximate methods such as limit equilibrium. These problems arise because they require significant assumptions regarding the shape of the failure mechanism and the distribution of the stresses throughout the failure zone.

Koutsabeloulis and Griffiths (1989) investigated the trapdoor problem using the finite element method with the initial stress method to implement soil plasticity. The bulk of their analyses were performed on trapdoors in a non-associated ($\psi' = 0^\circ$) material and, based on a limited number of analyses for associated soil, a correction to account for dilation was proposed. It is not entirely clear whether Koutsabeloulis and Griffiths (1989) had intended this correction factor to be used in the axi-symmetric case or not. The results of their axi-symmetric analyses are shown in Figure 8 for the associated (corrected) and non-associated ($\psi' = 0^\circ$) cases. For an associated material (Figure 8(a)) with $\phi' \leq 30^\circ$, the break-out factors of Koutsabeloulis and Griffiths (1989) are up to 100% above the numerical lower bounds, with the greatest discrepancy occurring for low friction angles. For $\phi' \geq 40^\circ$ the reverse is true, with the lower bounds lying above the trapdoor solutions. In the non-associated case with zero dilatancy, there is reasonable agreement between the lower bound and Koutsabeloulis and Griffiths (1989) predictions for $\phi' \leq 30^\circ$, but poor agreement for $\phi' \geq 40^\circ$.

Although comparisons between experimental results and theoretical results are difficult due to uncertainty regarding the soil properties and anchor roughness, a comparison of several experimental studies is presented Figure 9.

The break-out factors determined by Murray and Geddes (1987), Baker and Konder (1966), and Saeedy (1987) show encouraging agreement with the numerical lower bounds (Figure 9(a)). In particular, the results of Murray and Geddes (1987) are remarkably close to the lower bound result obtained for $\phi' = 40^\circ$.

The break-out factors recently determined by Pearce (2000) also show encouraging agreement up to embedment ratios of $H/D = 8 - 9$ (Figure 9(b)). Above this embedment ratio the experimental break-out factors plot below the lower bound results. Due to the close proximity of the anchor to the base of the chamber at embedment ratios greater than $H/D = 10$, Pearce (2000) concluded that the anchor behaviour may be influenced by boundary effects for these cases. This may in part explain the discrepancy between the numerical lower bounds and Pearce's results at larger embedment ratios. Also shown in (Figure 9(b)) are the chamber test results of Ilamparuthi, Dickin and Muthukrisnaiah (2002). These chamber tests results compare more favourably to the numerical results than those of Pearce (2000).

3. Conclusions

The effect of anchor shape on the pullout capacity of horizontal anchors has been analysed using a three dimensional finite element formulation of the lower bound theorem and axi-symetrical displacement finite element analysis. Rigorous solutions for the ultimate capacity of horizontal square and circular anchors in cohesionless soil have been presented.

The following key conclusions can be drawn from the results presented in this paper:

(1) The break-out factors for circular and square anchors increase nonlinearly with increasing embedment ratio. The rate of increase is greatest for medium to dense cohesionless soils where $\phi' \geq 30^\circ$. The capacity of both square and circular anchors is significantly greater than strip anchors at the same embedment ratio.

(2) The three dimension lower bound estimates of the collapse load for circular anchors compare well to the axi-symetrical displacement finite element results. The axi-symetrical results tend to be an upper bound to the collapse load and are between 4-14% above the lower bound results.

(3) The comparison with other theoretical solutions, which use a range of approximate theoretical techniques, was less favourable. This highlights the difficulties in using approximate methods, such as limit equilibrium, to predict the capacity of anchors.

(4) Allowing for the effects of dilatancy and roughness, the finite element lower bounds for both square and circular anchors compare favourably with the results from a number of recent experimental studies.

(5) The effect of anchor shape on the uplift resistance has been conveniently expressed as a dimensionless shape factor. Relative to a square anchor, the shape factor for a circular anchor is around 1.2.

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Figures

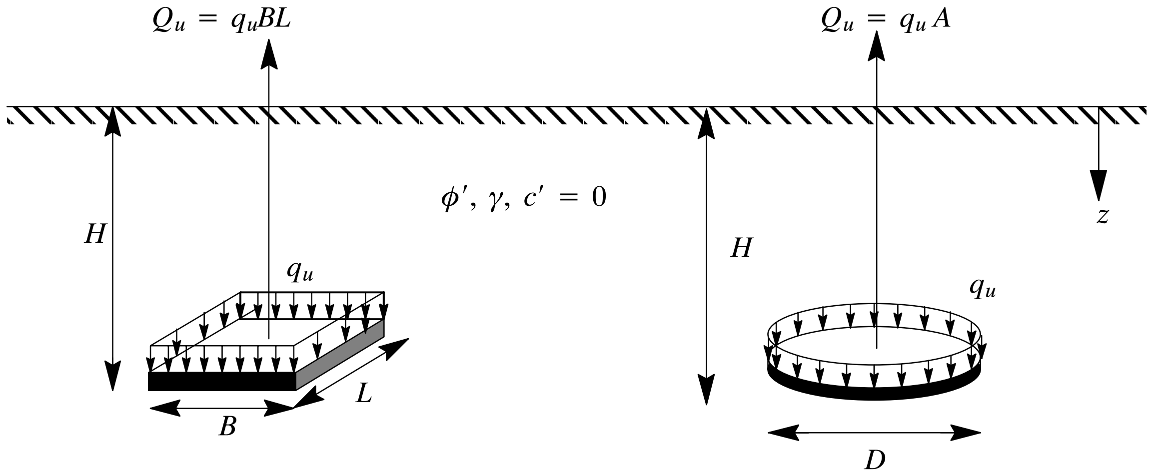


Figure 1. Problem Definition

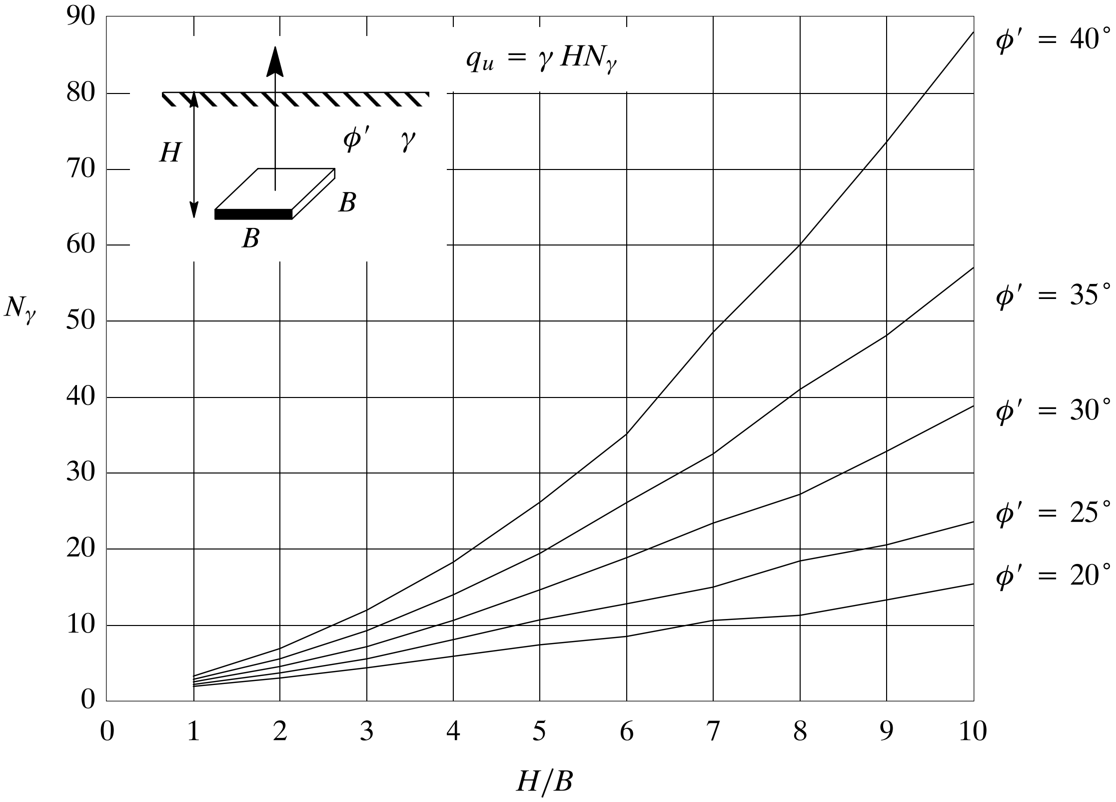


Figure 2 Break-out factors for square anchors in cohesionless soil.

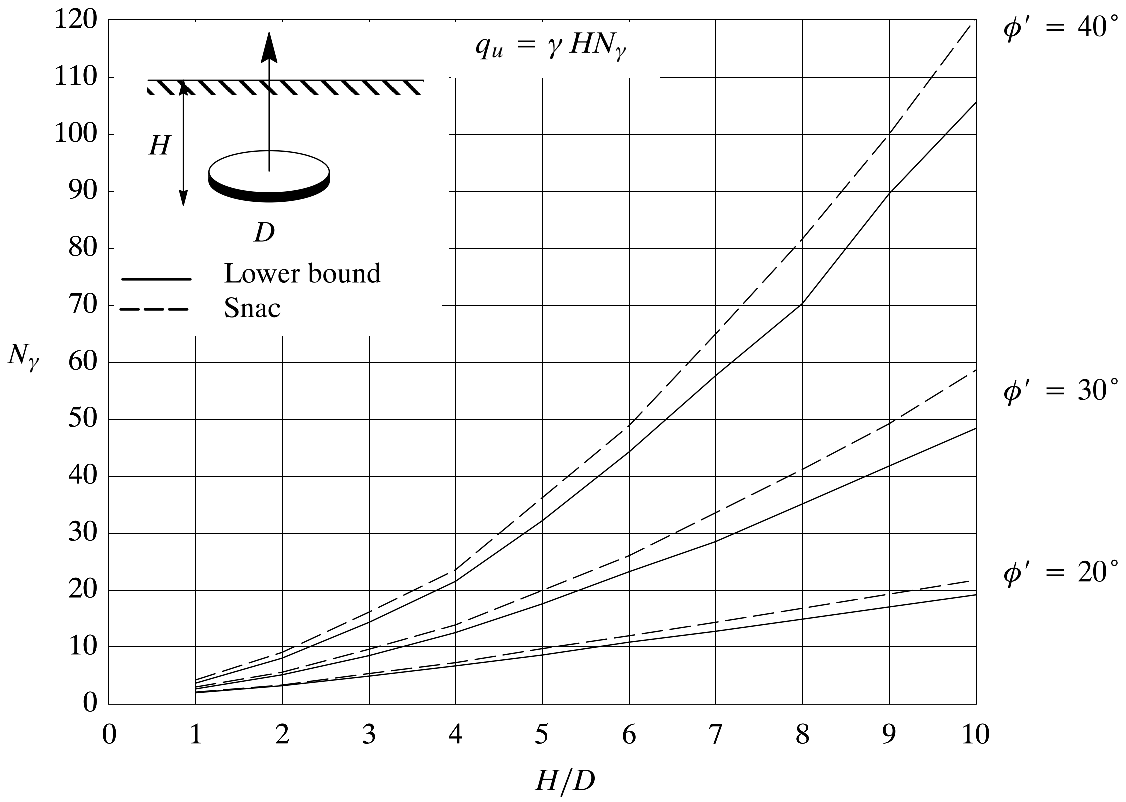


Figure 3 Break-out factors for circular anchors in cohesionless soil.

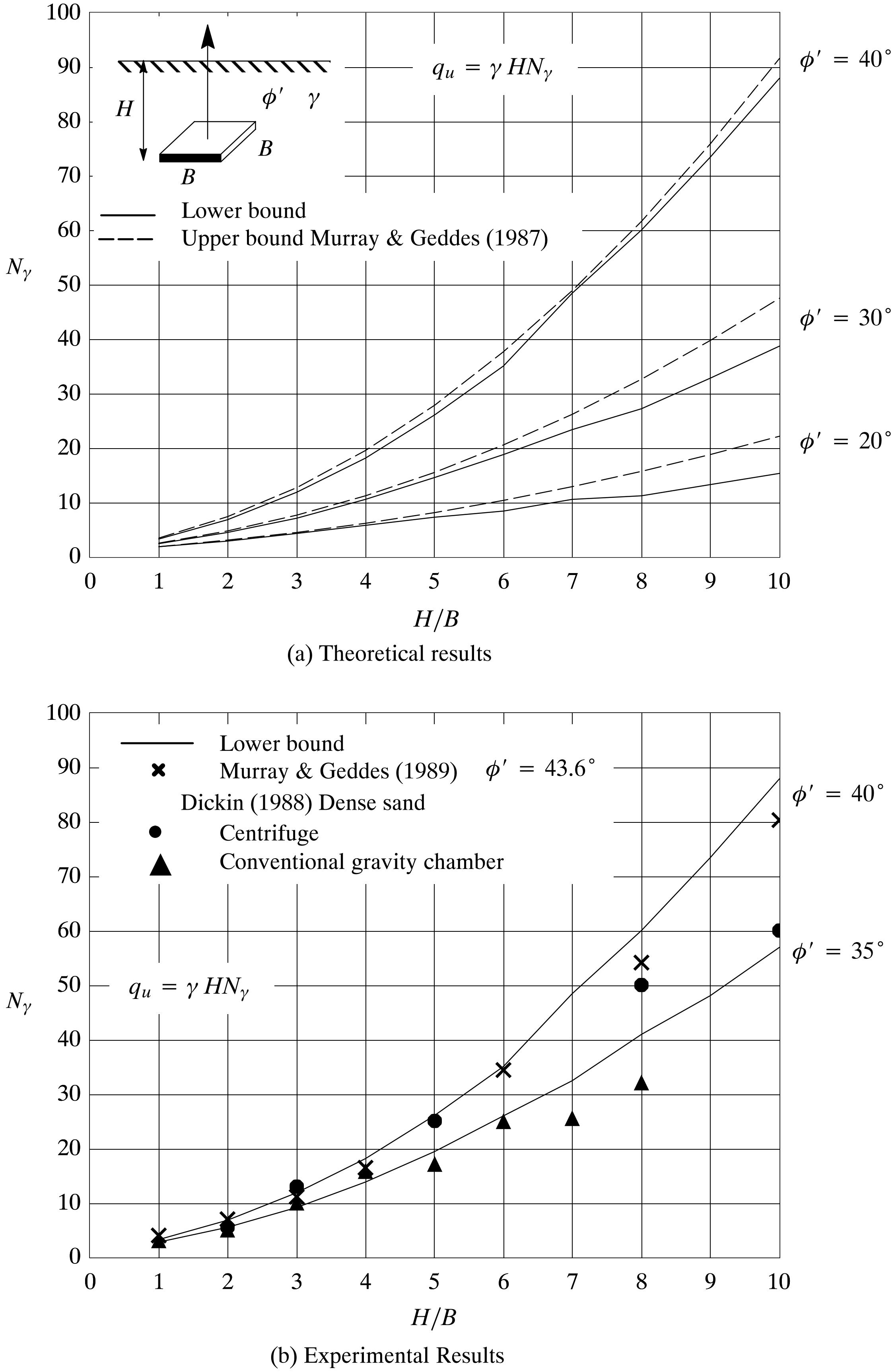


Figure 4 Comparison of break-out factors for square anchors in cohesionless soil.

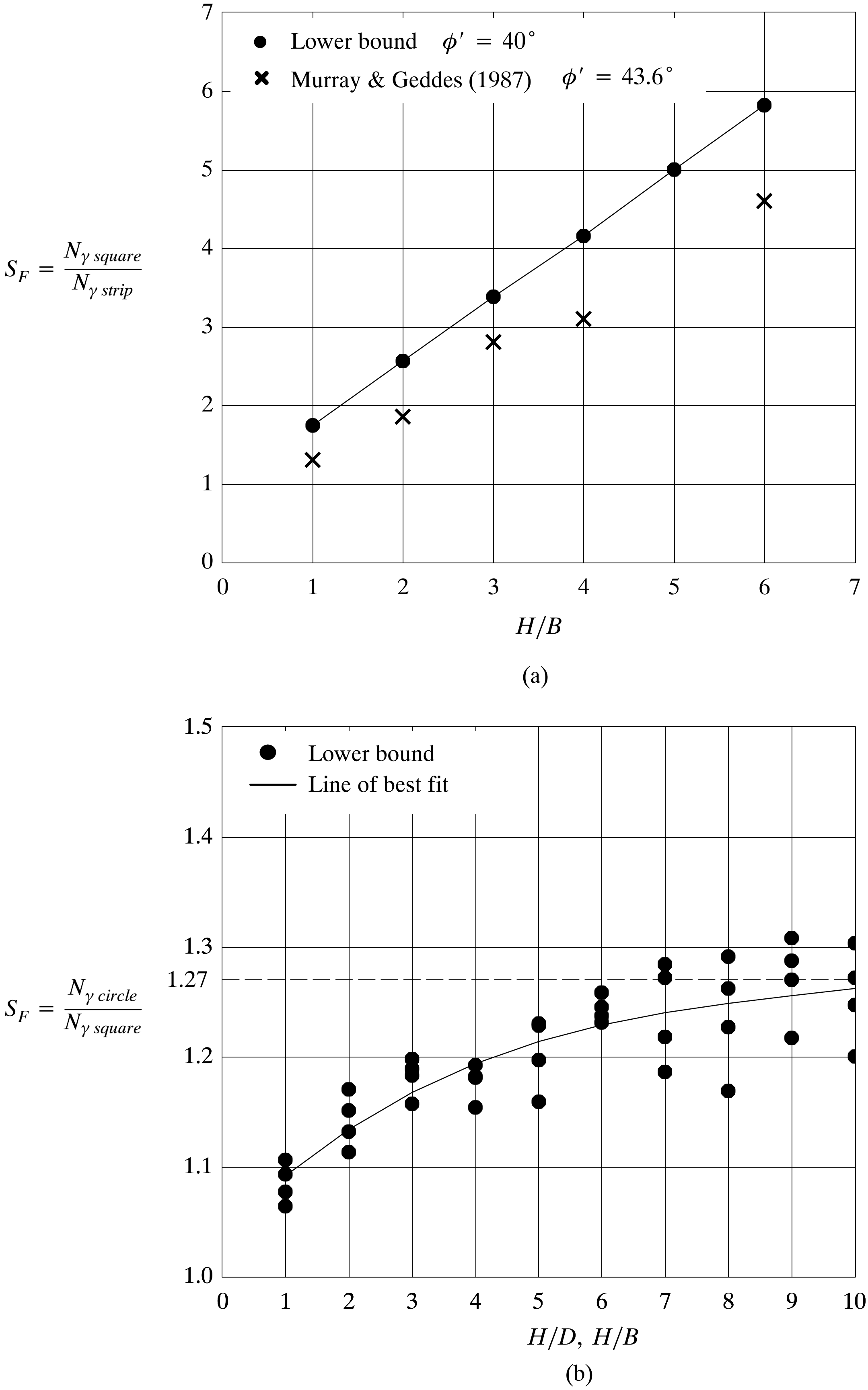


Figure 5 Shape factors for square and circular anchors in cohesionless soil.

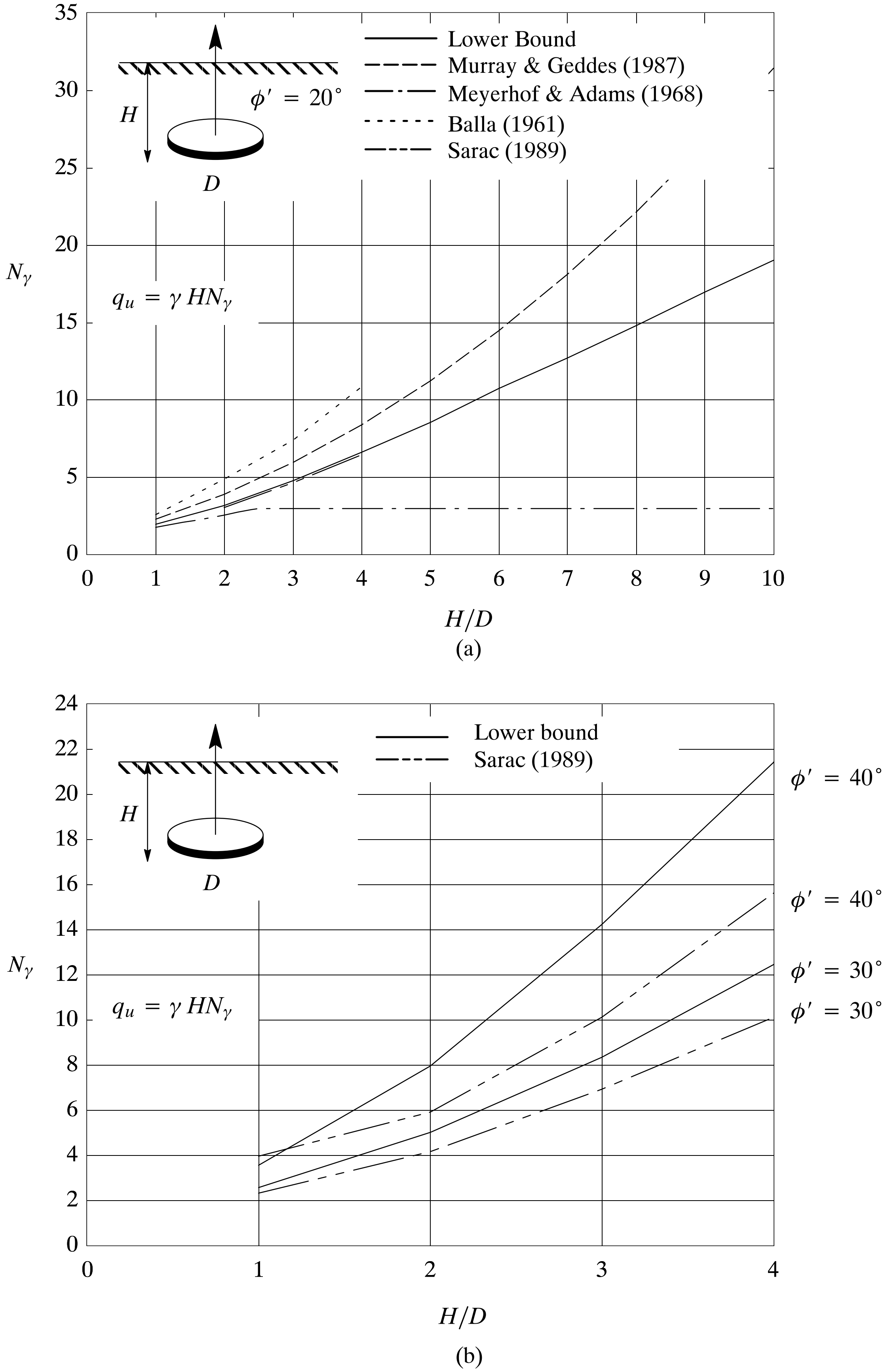


Figure 6 Comparison of theoretical break-out factors for circular anchors in cohesionless soil.

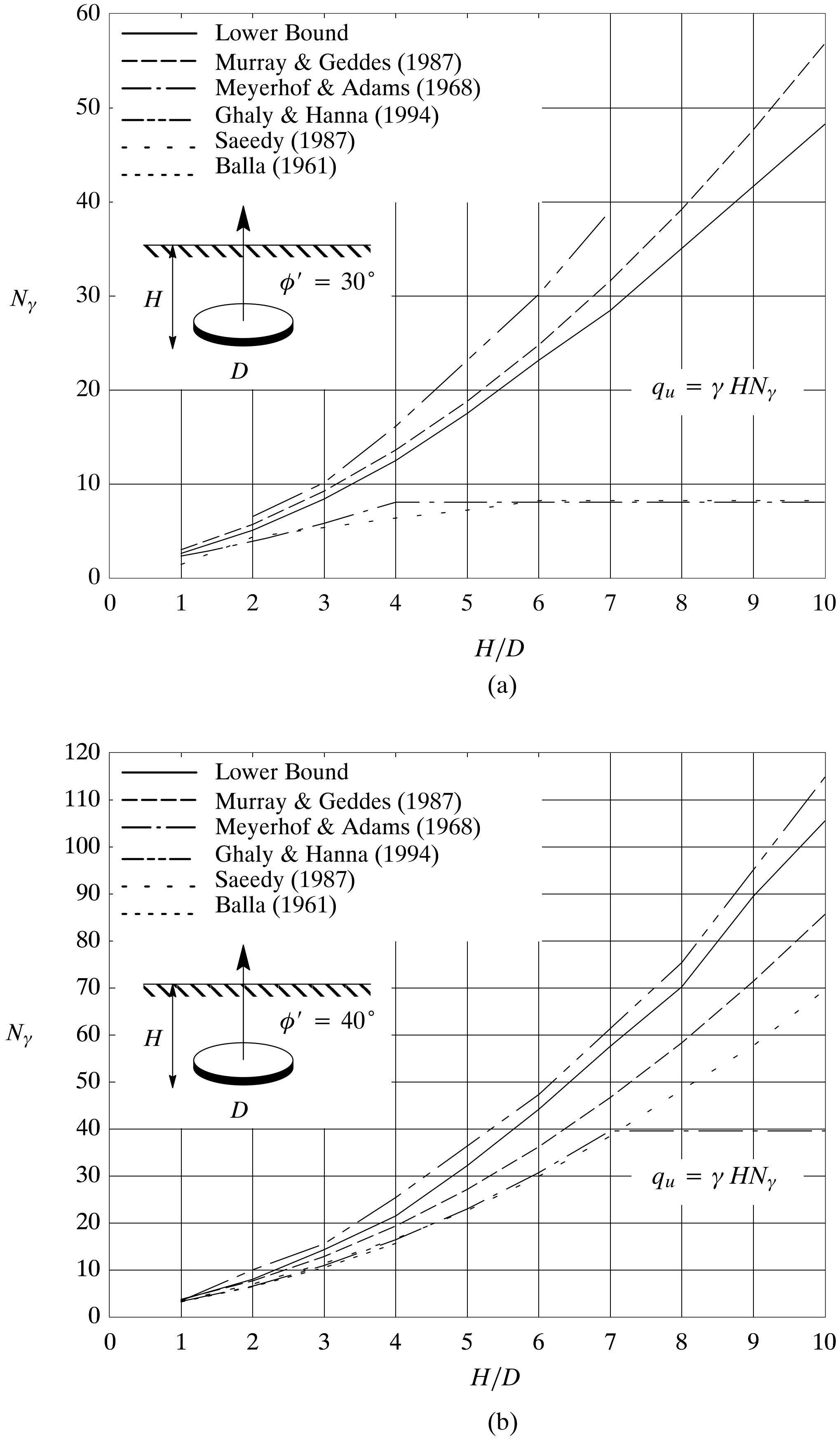


Figure 7 Comparison of theoretical break-out factors for circular anchors in cohesionless soil.

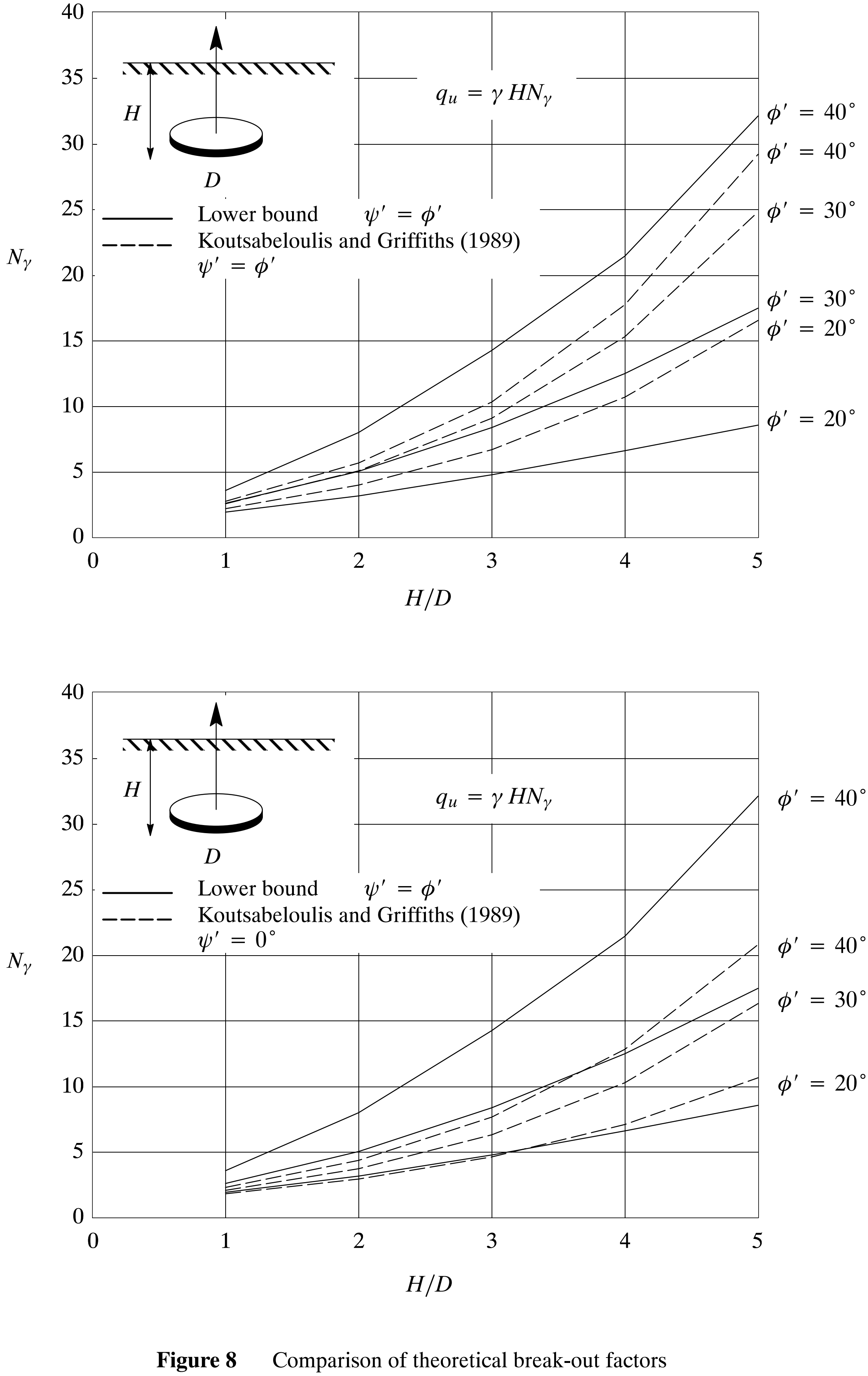
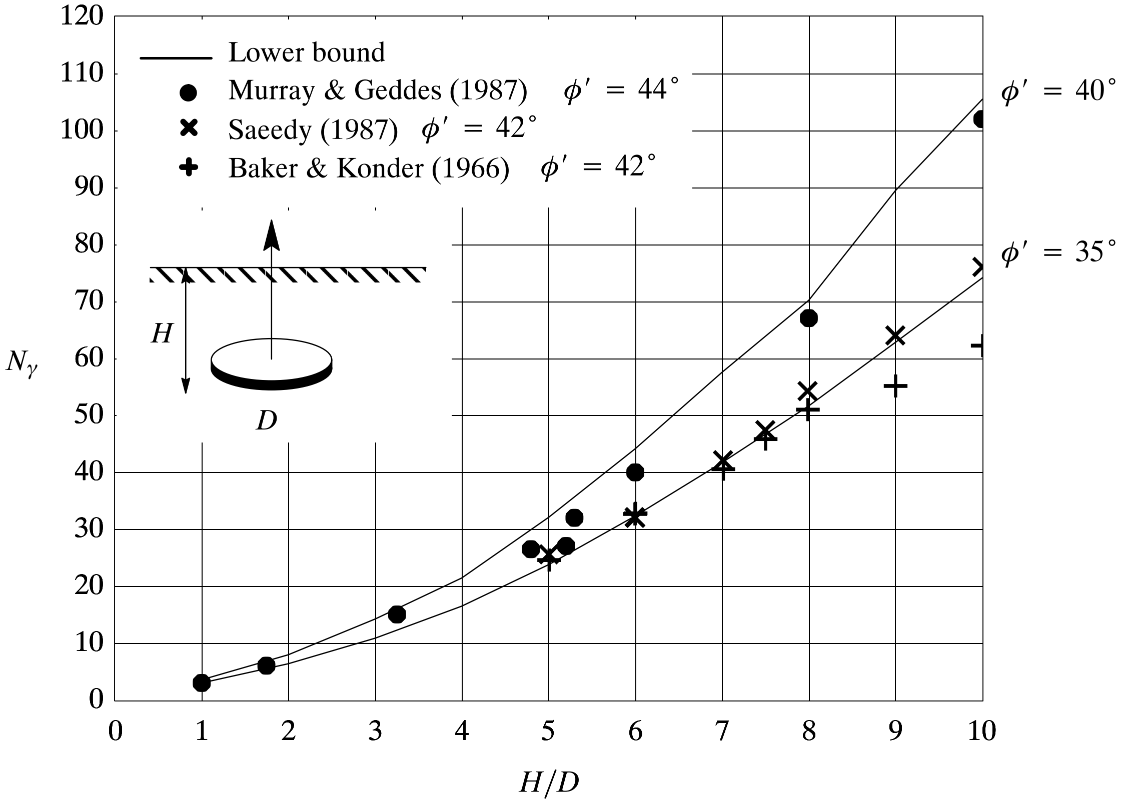
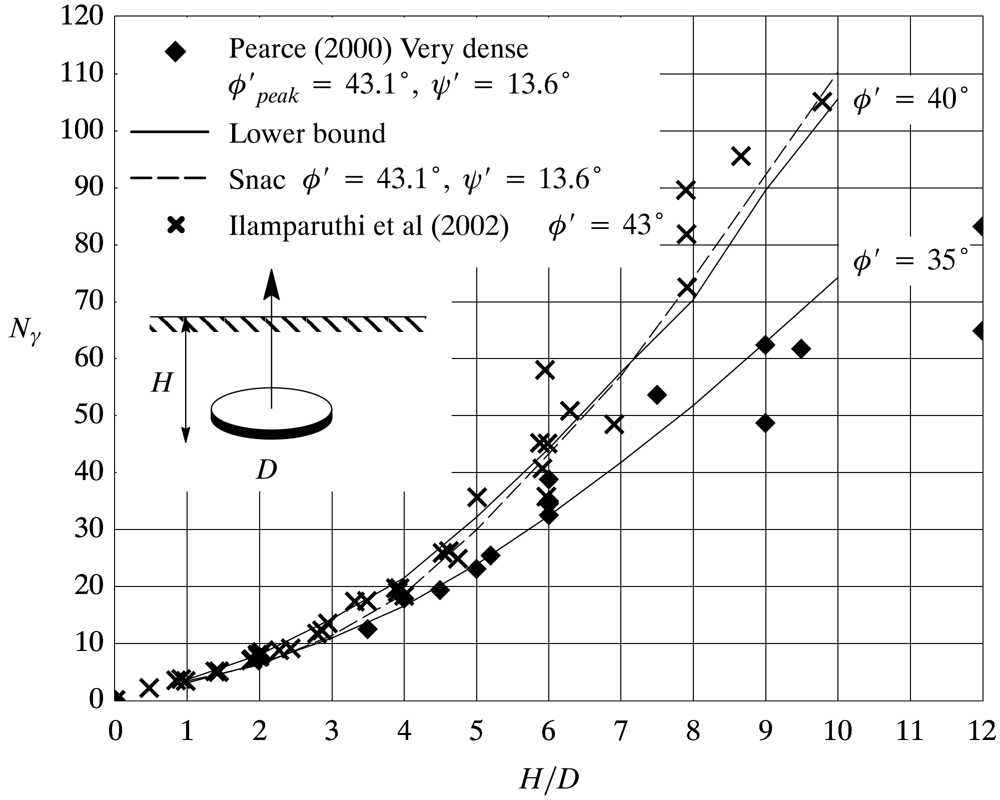


Figure 8 Comparison of theoretical break-out factors for circular anchors in cohesionless soil.



(a)



(b)

Figure 9 Comparison of experimental break-out factors for circular anchors in cohesionless soil (a) Experimental results reported in Saeedy (1987) (b) Experimental results of Pearce (2000) and Ilamparuthi et al (2002).