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PRELIMINARY TEST ESTIMATORS FOR THE MULTIVARIATE NORMAL MEAN BASED ON THE MODIFIED W, LR AND LM TESTS

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SUMMARY

In this paper we consider the preliminary test estimators (PTEs) of the mean vector of multivariate normal distribution under the modified Wald, likelihood ratio, and Lagrange multiplier tests. The properties of the estimators have been investigated under some popular statistical criteria. It has been observed that with respect to the quadratic bias the Wald test based PTE performs better than those based on the likelihood ratio and Lagrange multiplier tests. Whereas, with respect to the quadratic risk the Lagrange multiplier test based PTE performs better than those based on the likelihood ratio and Wald tests. The results of this study reveal that the use of the three modified tests in the formation of the PTEs significantly reduces the conflict among the PTEs as compared to the estimators based on the three original tests in terms of both quadratic bias and risk properties.

Keywords: Preliminary test estimator; Wald, likelihood ratio and Lagrange multiplier tests; quadratic bias; quadratic risk; relative efficiency and conflict.

AMS 2000 Subject Classification: Primary 62H12, Secondary 62F05.

1 Introduction

The multivariate normal distribution is appropriate to model many real life phenomenon, and hence in the literature there is a great deal of interest to improve the estimation of its parameters. Often, for a p-dimensional normal distribution, the interest is to estimate the

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mean vector $\boldsymbol{\mu}$ when the covariance matrix Σ is unknown. A common and popular estimator of $\boldsymbol{\mu}$ is the maximum likelihood estimator (MLE). Customarily, this MLE is based exclusively on the sample responses, and is known as the unrestricted estimator (UE). The sample information based UE of $\boldsymbol{\mu}$ is unbiased and uniformly minimum variance. This estimator obviously disregards any other kind of non-sample prior information on the parameter.

Often information on the value of $\boldsymbol{\mu}$ is available from expert knowledge or previous investigations. Inclusion of such prior information in the definition of estimator is likely to improve some statistical properties of the estimator. According to Fisher, this non-sample prior information can be expressed in the form of a null hypothesis $H_0 : \boldsymbol{\mu} = \boldsymbol{\mu}_0$ (cf. Ahmed and Saleh 1989). Usually, the researchers are not sure that the prior information is quite true, and hence there is an uncertainty in the validity of the null hypothesis. However, as suggested by Bancroft (1944), the uncertainty in the non-sample prior information can be removed by performing an appropriate statistical test on the null hypothesis. Based on the prior information, a restricted estimator (RE) of $\boldsymbol{\mu}$ is defined as $\widehat{\boldsymbol{\mu}} = \boldsymbol{\mu}_0$. Saleh (1973) applied this idea in the multivariate normal case with diagonal covariance matrix.

The UE and RE fail to use both sample and non-sample prior information. Therefore, it is desirable to develop an improved estimator by combining the non-sample prior information as well as the sample information. This is done by using the preliminary test estimator (PTE) which is a function of the UE, RE and an appropriate statistical test for testing the null hypothesis.

Many authors have used the likelihood ratio (LR) test or equivalent F-test to test H_0. In the literature there are two other competing tests, namely, the Wald (W) and Lagrange multiplier (LM) tests to test the same H_0. The W test was introduced by Wald (1943), and the LM test by Aitchison and Silvey (1958), and Silvey (1959). Engle (1984) has proved that the LM test is the same as the score test of Rao (1947). Instead of the LR test, these two tests can also be used to define the PTE (cf. Billah and Saleh 1998, 2000). Recently, Kibria (2002) uses the three original W, LR and LM tests in the definition of PTEs and compares their performances. Although it does not engage in the investigation of the conflict (difference between the largest and smallest relative efficiency), it is evident that the performances of the PTEs under different tests are different, and hence there is a great deal of conflict among the three PTEs based on the three original tests.

The exact sampling distributions of the W, LR and LM test statistics are complicated. In practice, the critical regions of the tests are considered based on the same asymptotic approximate distribution of the statistics. It has been proved (cf. Engle 1984) that under the null hypothesis the three test statistics are asymptotically equivalent, and distributed as a central chi-square variable with the same degrees of freedom. Evans and Savin (1982) have shown that these tests based on the approximate chi-square critical value differ with respect to their size and power, and hence likely to result in conflicting conclusions.

Evans and Savin (1982) have investigated the effects of some modifications on the three tests. In the context of linear multiple regression model they use the correction factors those derived from the degrees of freedom correction to the estimate of the error variance (c.f Gallant, 1975) and those derived from second order Edgeworth approximations to the exact distributions of the test statistics (Rothenberg, 1977). Their study shows that in case of the modified tests the conflict among the size and power properties of the three tests is reduced. In this study, we apply the Gallent's (1975) degrees of freedom correction to the estimate of the covariance matrix, and Rothenberg's (1977) correction to the exact distributions of the test statistics. We conjecture that the use of the modified tests in the definition of PTE will reduce the conflict among the properties of the PTEs under the three modified tests. The main objective of this study is to formulate the PTE of $\boldsymbol{\mu}$ based on the three modified tests, and investigate the small sample properties of the PTEs. Furthermore, this study compares the conflict among the performances of PTEs under the three modified tests with that of the performances of the PTEs under the three original tests.

The organization of this paper is as follows. In Section 2, we outline the test statistics and the PTEs based on different test statistics. The bias functions of the PTEs have been stated and analyzed in Section 3. Section 4 provides expressions for the quadratic risks and their analyses. Finally, some concluding remarks are given in section 5.

2 The Tests and the Estimators

Consider a random sample $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_n$ of size n from a p-dimensional multivariate normal distribution with unknown mean vector $\boldsymbol{\mu}$ and covariance matrix $\Sigma = \sigma^2 I_p$, where I_p is an indentity matrix of order p. Based on the sample information, the UE of $\boldsymbol{\mu}$ is defined as

$$\tilde{\boldsymbol{\mu}} = \bar{\boldsymbol{X}} = \frac{1}{n}\sum_{j=1}^{n} \boldsymbol{X}_j. \tag{2.1}$$

Let the non-sample prior information about $\boldsymbol{\mu}$ be expressed by the null hypothesis,

$$H_0 : \boldsymbol{\mu} = \boldsymbol{\mu}_0. \tag{2.2}$$

To test the null hypothesis in (2.2) the commonly used LR test based F statistic is defined as

$$F = \frac{\chi_p^2(\Delta)/p}{\chi_{n-p}^2/(n-p)} \tag{2.3}$$

where $\chi_p^2(\Delta)$ is a non-central chi-square variable having p degrees of freedom (d.f.) and non-centrality parameter $\Delta = \boldsymbol{\delta}'\boldsymbol{\delta}$ with $\boldsymbol{\delta} = \sqrt{n}\Sigma^{-\frac{1}{2}}\boldsymbol{\delta}^*$ in which $\boldsymbol{\delta}^* = \boldsymbol{\mu} - \boldsymbol{\mu}_0$, and χ_{n-p}^2 is a central chi-square variable with $(n-p)$ d.f. To test the same hypothesis the test statistics under the original W, LR and LM tests can be expressed as

$$\mathcal{T}_{\mathrm{W}} = \frac{np}{m}F_{p,m}(\Delta) \tag{2.4}$$

$$\mathcal{T}_{\mathrm{LR}} = \ln\left\{1 + \frac{p}{m}F_{p,m}(\Delta)\right\}^n \tag{2.5}$$

$$\mathcal{T}_{\mathrm{LM}} = \frac{npF_{p,m}(\Delta)}{m + pF_{p,m}(\Delta)} \tag{2.6}$$

where $m = n - p$, respectively. Under the null hypothesis these three test statistics are equivalent, and their asymptotic distribution is chi-square with p d.f. As suggested by Evans and Savin (1982), the application of the degrees of freedom correction to the estimate of the covariance matrix (Gallant, 1975) for W and LM tests, and second order Edgeworth approximations to the exact distributions of the test statistic (Rothenberg, 1977) for the LR test gives the following modified forms of the test statistics.

$$\mathcal{T}_{\mathrm{W}*} = pF_{p,m}(\Delta) \tag{2.7}$$

$$\mathcal{T}_{\mathrm{LR}*} = (m + \frac{p}{2} - 1)\ln\left(1 + \frac{pF_{p,m}(\Delta)}{m}\right) \tag{2.8}$$

$$\mathcal{T}_{\mathrm{LM}*} = \frac{(m+p)pF_{p,m}(\Delta)}{m + pF_{p,m}(\Delta)}. \tag{2.9}$$

This correction to the LR statistic ensures that the LR test has the correct significance level to order $1/m$ (see Anderson, 1958, p. 208).

Based on the sample information, non-sample prior information and an appropriate test for testing the hypothesis in (2.2), the preliminary test estimator of the mean vector $\boldsymbol{\mu}$ is defined as

$$\widehat{\boldsymbol{\mu}}_{\mathrm{G}}^{\mathrm{PTE}} = \widetilde{\boldsymbol{\mu}} - (\widetilde{\boldsymbol{\mu}} - \widehat{\boldsymbol{\mu}})\, I(\mathcal{T}_{\mathrm{G}} < \chi^2_\alpha) \tag{2.10}$$

where $\mathcal{T}_{\mathrm{G}}$ is any appropriate test for testing H_0, and $I(\cdot)$ is an indicator function which assumes value unity when the inequality in the argument holds and 0 otherwise. Therefore, when $I(\cdot) = 1$ the PTE becomes the RE, otherwise it is the UE. Replacing $\mathcal{T}_{\mathrm{G}}$ by any appropriate test statistic different PTEs of $\boldsymbol{\mu}$ can be obtained.

3 The Bias Function

In this section the bias functions of the PTEs of $\boldsymbol{\mu}$ under the three modified tests are stated. The bias function of an estimator of the parameter vector is also a vector. Therefore, any direct comparison among the biases is not meaningful. To facilitate the comparison the quadratic bias (QB) functions of the PTEs under the three modified tests are derived, and analysed both graphically and numerically. Moreover, the conflict among the QBs which is the difference of the maximum and minimum values of the QBs of the three PTEs, has been computed and analysed. For the expression of the bias functions of the PTEs under the three original tests readers may see Kibria (2002). The bias functions of the PTE under the three modified tests are stated in the following theorem.

Theorem 3.1: *The bias functions of the PTEs of $\boldsymbol{\mu}$ under the modified W, LR and LM tests are respectively*

$$\begin{aligned} B(\widehat{\boldsymbol{\mu}}_{\mathrm{W}*}^{\mathrm{SPTE}}; \boldsymbol{\mu}) &= -\delta^* G_{p+2,m}\left(h_1^{\mathrm{W}*}; \Delta\right) \\ B(\widehat{\boldsymbol{\mu}}_{\mathrm{LR}*}^{\mathrm{SPTE}}; \boldsymbol{\mu}) &= -\delta^* G_{p+2,m}\left(h_{1(E)}^{\mathrm{LR}*}; \Delta\right) \\ B(\widehat{\boldsymbol{\mu}}_{\mathrm{LM}*}^{\mathrm{SPTE}}; \boldsymbol{\mu}) &= -\delta^* G_{p+2,m}\left(h_1^{\mathrm{LM}*}; \Delta\right) \end{aligned}$$

where $h_i^{\mathrm{W}*} = \frac{\chi^2_\alpha}{(p+2i)}$, $h_{i(E)}^{\mathrm{LR}*} = \frac{m}{(p+2i)}\left(e^{\chi^2_\alpha/(m+\frac{p}{2}-1)} - 1\right)$, $h_i^{\mathrm{LM}*} = \frac{m\chi^2_\alpha}{(p+2i)\left(m+p-\chi^2_\alpha\right)}$, $i = 1, 2$; *and* $G_{a,b}(h;\Delta)$ *is the distribution function of the non-central F-distribution with* (a, b) *d.f. and non-centrality parameter* Δ *evaluated at* h.

The proof of the theorem is straightforward.

3.1 Analysis of Quadratic Bias

From Theorem 3.1 the following forms of the quadratic biases are obtained.

$$\begin{aligned}
QB[\widehat{\boldsymbol{\mu}}^{\mathrm{PTE}}_{\mathrm{W}*};\boldsymbol{\mu}] &= \boldsymbol{\delta}^{*\prime}\boldsymbol{\delta}^{*}\left\{G_{p+2,m}\left(h_1^{\mathrm{W}*};\Delta\right)\right\}^2 = Q_1^* \\
QB[\widehat{\boldsymbol{\mu}}^{\mathrm{PTE}}_{\mathrm{LR}*};\boldsymbol{\mu}] &= \boldsymbol{\delta}^{*\prime}\boldsymbol{\delta}^{*}\left\{G_{p+2,m}\left(h_{1(E)}^{\mathrm{LR}*};\Delta\right)\right\}^2 = Q_2^* \\
QB[\widehat{\boldsymbol{\mu}}^{\mathrm{PTE}}_{\mathrm{LM}*};\boldsymbol{\mu}] &= \boldsymbol{\delta}^{*\prime}\boldsymbol{\delta}^{*}\left\{G_{p+2,m}\left(h_1^{\mathrm{LM}*};\Delta\right)\right\}^2 = Q_3^* \quad \text{(say)}.
\end{aligned}$$

From Table 3.1 and Figure 1 & 2 it is observed that when $\Delta = 0$ (under the null hypothesis) the quadratic biases of the PTE under the three original as well as the three modified tests are 0. In both cases, as Δ deviates from 0, the quadratic biases start growing larger. From some moderate values of Δ, the quadratic biases start decreasing, and approach to 0 from some large value of Δ. Interestingly, the inequality

$$Q_1^* \leq Q_2^* \leq Q_3^* \tag{3.1}$$

exists among the quadratic biases of PTEs under the three original as well as the three modified tests. The above inequality relation of the QBs is the reverse of the inequality relation that exists among the original test statistics for arbitrary α. For any fixed Δ, as the sample size increases the conflict among the quadratic biases of the PTEs under the original and the modified tests decreases. For any fixed sample size and $\Delta = 0$, there is no conflict among the quadratic biases of the PTEs regardless of the use of the original and modified tests. As Δ deviates from zero, the conflict among the quadratic biases increases. From some moderate value of Δ (say, Δ_m) the conflict decreases, and it approaches to zero from some large value of Δ (say, Δ_l). The performances of the PTEs vary for varying α. In this study we use $\alpha = 0.05$ for the computation of the QBs of the PTEs. For the selection of an optimum α readers may see Chiou and Saleh (2002). The Table 3.1 reveals that the conflict among the QBs of the PTEs under the three original tests is much more higher than that of QBs of the PTEs under the three modified tests, unless $\Delta = 0$ or, $\Delta \geq \Delta_l$. For example, when $n = 50$ and $\Delta = 6$, the conflict is 0.943 for the three original tests, whereas it is 0.609 for the three modified tests.

Table 3.1: Quadratic bias of PTE for original and modified W, LR and LM tests for $p = 5$, $\alpha = 0.05$ and selected values of n, (here *Conft* stands for conflict).

	$n = 20$				$n = 30$				$n = 50$			
Δ	Q_1	Q_2	Q_3	*Conft*	Q_1	Q_2	Q_3	*Conft*	Q_1	Q_2	Q_3	*Conft*
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0.306	0.517	0.846	0.540	0.404	0.561	0.765	0.361	0.491	0.591	0.706	0.215
2	0.460	0.854	1.581	1.121	0.624	0.925	1.369	0.745	0.775	0.973	1.220	0.445
3	0.511	1.043	2.192	1.681	0.709	1.125	1.808	1.099	0.899	1.177	1.549	0.650
4	0.499	1.117	2.676	2.177	0.705	1.195	2.091	1.386	0.909	1.242	1.716	0.807
5	0.451	1.108	3.033	2.582	0.648	1.173	2.235	1.587	0.847	1.207	1.752	0.905
6	0.388	1.043	3.271	2.883	0.564	1.089	2.264	1.700	0.746	1.107	1.689	0.943
7	0.321	0.944	3.399	3.078	0.471	0.971	2.201	1.730	0.629	0.973	1.559	0.930
8	0.257	0.829	3.431	3.174	0.381	0.838	2.071	1.690	0.513	0.826	1.389	0.876
9	0.201	0.711	3.381	3.180	0.300	0.704	1.897	1.597	0.406	0.681	1.203	0.797
10	0.155	0.596	3.266	3.111	0.232	0.578	1.698	1.466	0.314	0.548	1.015	0.701
12	0.087	0.398	2.898	2.811	0.130	0.367	1.284	1.154	0.177	0.333	0.679	0.502
15	0.033	0.196	2.191	2.158	0.049	0.166	0.750	0.701	0.066	0.139	0.327	0.261
20	0.005	0.049	1.126	1.121	0.008	0.035	0.241	0.233	0.010	0.025	0.074	0.064
25	0.001	0.010	0.480	0.379	0.001	0.006	0.062	0.061	0.001	0.004	0.013	0.012
30	0.000	0.002	0.178	0.178	0.000	0.001	0.014	0.041	0.000	0.000	0.002	0.002
	Q_1^*	Q_2^*	Q_3^*	*Conft*	Q_1^*	Q_2^*	Q_3^*	*Conft*	Q_1^*	Q_2^*	Q_3^*	*Conft*
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0.516	0.703	0.846	0.330	0.554	0.680	0.765	0.211	0.585	0.661	0.706	0.121
2	0.852	1.245	1.581	0.729	0.912	1.177	1.369	0.457	0.960	1.120	1.220	0.260
3	1.039	1.632	2.192	1.153	1.106	1.501	1.808	0.702	1.158	1.394	1.549	0.391
4	1.112	1.878	2.676	1.564	1.172	1.675	2.091	0.919	1.218	1.515	1.716	0.498
5	1.102	2.004	3.033	1.931	1.147	1.727	2.235	1.088	1.180	1.516	1.752	0.572
6	1.037	2.030	3.271	2.234	1.063	1.685	2.264	1.201	1.080	1.432	1.689	0.609
7	0.939	1.980	3.399	2.460	0.945	1.578	2.201	1.256	0.947	1.296	1.559	0.612
8	0.824	1.873	3.431	2.607	0.813	1.430	2.071	1.258	0.802	1.132	1.389	0.587
9	0.706	1.729	3.381	2.675	0.681	1.262	1.897	1.216	0.659	0.961	1.203	0.544
10	0.592	1.563	3.266	2.674	0.558	1.087	1.698	1.140	0.529	0.796	1.015	0.486
12	0.395	1.212	2.898	2.503	0.353	0.762	1.284	0.931	0.320	0.512	0.679	0.359
15	0.194	0.746	2.191	1.997	0.158	0.397	0.750	0.592	0.133	0.233	0.327	0.194
20	0.036	0.216	0.962	0.926	0.024	0.079	0.187	0.163	0.016	0.034	0.054	0.038
25	0.007	0.063	0.398	0.391	0.004	0.016	0.046	0.042	0.002	0.005	0.009	0.007
30	0.002	0.021	0.178	0.176	0.001	0.004	0.014	0.013	0.000	0.001	0.002	0.002

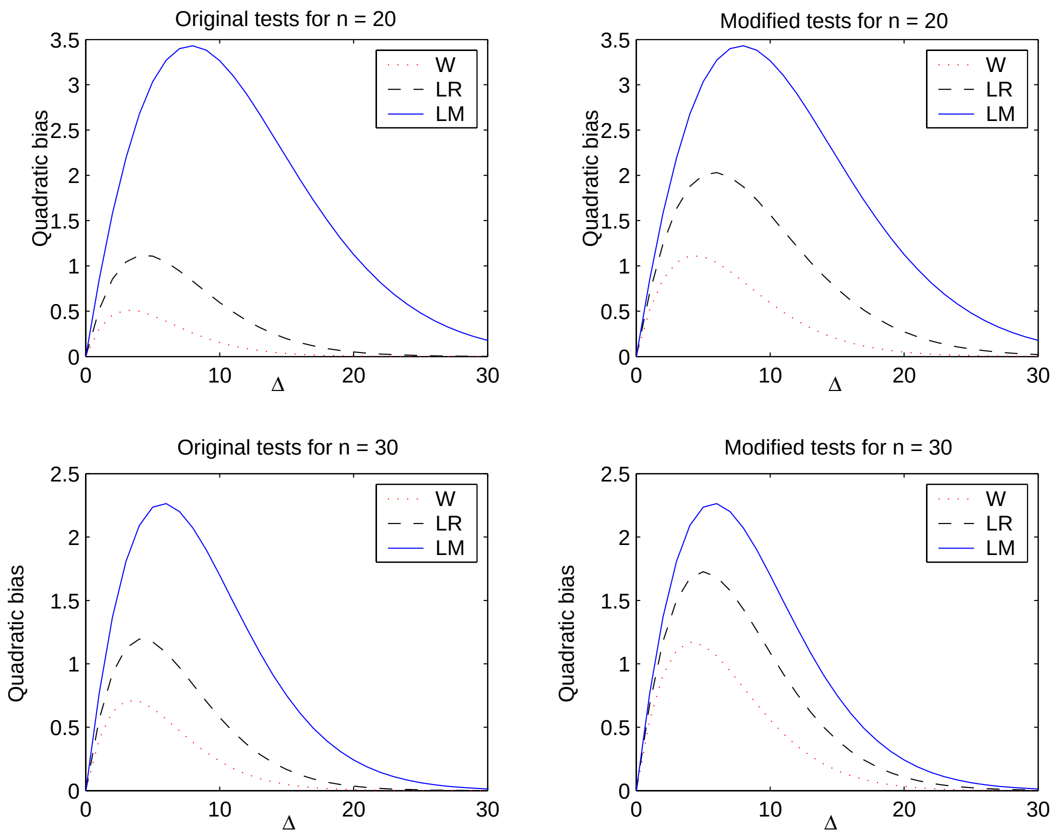

Figure 1: Quadratic biases of the PTE based on the W, LR and LM tests for $p = 5$ and $\alpha = 0.05$

4 The Quadratic Risk Function

In the literature, the quadratic risk (QR) has been widely used as a statistical criterion to judge the performance of the estimators. An estimator with smaller risk is always preferred over the other that with larger risk. Researchers are interested to find an estimator with minimum risk. In this section the quadratic risk functions of the PTEs based on the three modified W, LR and LM tests are stated and analysed both graphically and numerically. Moreover, the conflict (the difference of the maximum and minimum values of the relative efficiencies) among the relative efficiencies relative to the UE of the three PTEs under the three original as well as the three modified tests have been obtained and compared. For the expressions of the QR functions of the PTEs under the three original tests, and an analytical comparison among themselves readers may see Kibria (2002).

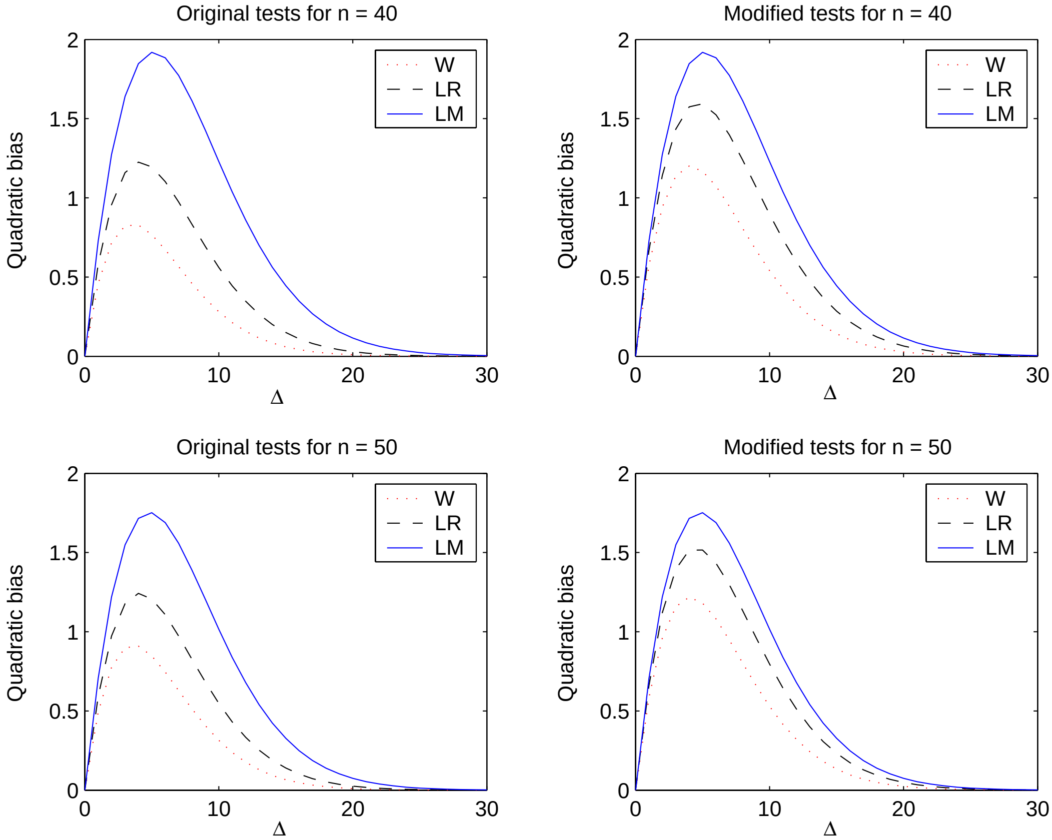

Figure 2: Quadratic biases of the PTE based on the W, LR and LM tests for $p = 5$ and $\alpha = 0.05$

Theorem 4.1: *The quadratic risk functions of the PTEs of the mean vector $\boldsymbol{\mu}$ of multivariate normal distribution can be stated as*

$$\begin{aligned} R_{W*} &= p - pG_{p+2,m}(h_1^{W*};\Delta) + \Delta\left\{2G_{p+2,m}(h_1^{W*};\Delta) - G_{p+4,m}(h_2^{W*};\Delta)\right\} \\ R_{LR*} &= p - pG_{p+2,m}(h_1^{LR*};\Delta) + \Delta\left\{2G_{p+2,m}(h_1^{LR*};\Delta) - G_{p+4,m}(h_2^{LR*};\Delta)\right\} \\ R_{LM*} &= p - pG_{p+2,m}(h_1^{LM*};\Delta) + \Delta\left\{2G_{p+2,m}(h_1^{LM*};\Delta) - G_{p+4,m}(h_2^{LM*};\Delta)\right\}, \end{aligned}$$

where $R_W* = QR\left[\widehat{\boldsymbol{\mu}}^{\mathrm{PTE}}_{\mathrm{W}*};\boldsymbol{\mu}\right]$, $R_{LR*} = QR\left[\widehat{\boldsymbol{\mu}}^{\mathrm{PTE}}_{\mathrm{LR}*};\boldsymbol{\mu}\right]$, $R_{LM*} = QR\left[\widehat{\boldsymbol{\mu}}^{\mathrm{PTE}}_{\mathrm{LM}*};\boldsymbol{\mu}\right]$; $h_i^{\mathrm{W}*}$, $h_{i(E)}^{\mathrm{LRr}*}$ *and* $h_i^{\mathrm{LM}*}$, *for* $i = 1, 2$ *are defined earlier in Theorem 3.1* .
The proof of the theorem is straightforward.

4.1 Analysis of Quadratic Risk

To analyse the quadratic risk functions of the PTEs based on the three modified tests, the relative efficiencies of the PTEs relative to the UE are computed and investigated.

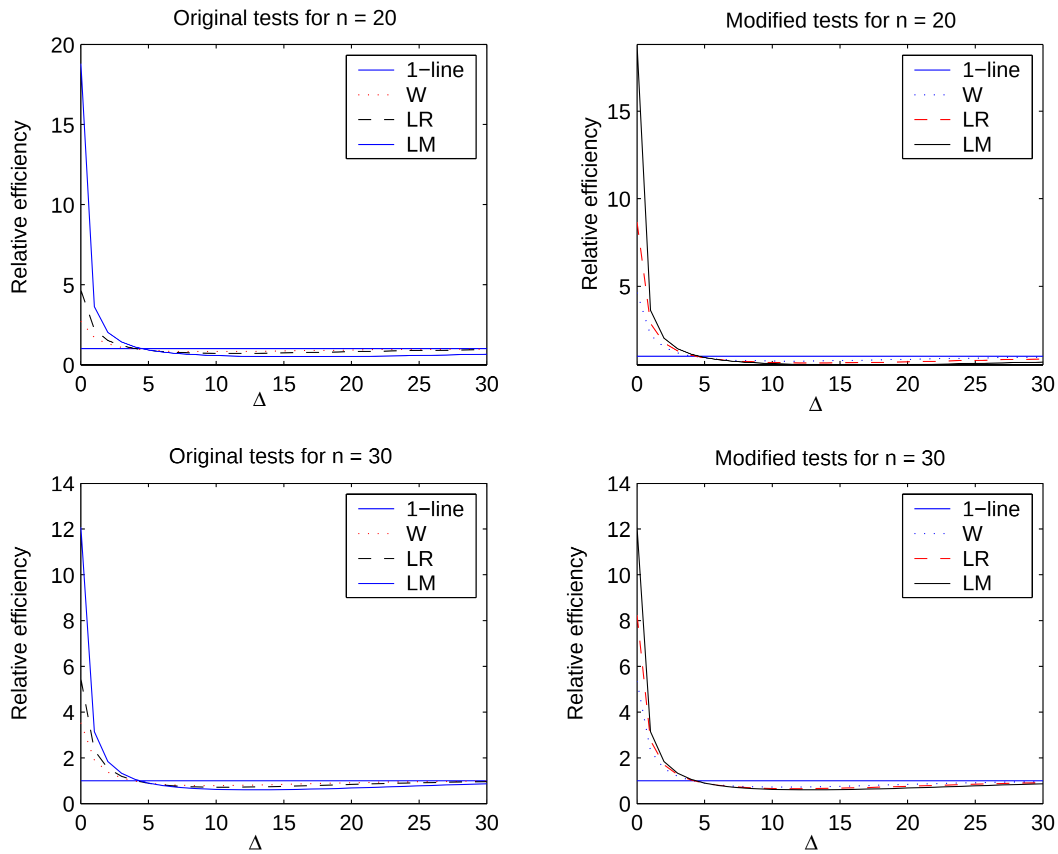

Figure 3: Relative efficiencies of the PTE based on the W, LR and LM tests for $p = 5$ and $\alpha = 0.05$

From Figure 3 & 4 and Table 4.1 it is observed that the performance of the PTEs with respect to the UE is the best at $\Delta = 0$. As Δ departs from zero, the relative efficiencies decrease and cross the 1-line at some small value of Δ. For any fixed sample size, and from 0 to some small value of Δ (say, Δ_0) the relative efficiency of the PTE under the modified LM test is the highest followed by those of the PTEs under the modified LR and W tests respectively.

Therefore, for $0 \leq \Delta \leq \Delta_0$ the following inequality relation holds

$$R_{W*} \leq R_{LR*} \leq R_{LM*}. \tag{4.1}$$

But from Δ_0 to some large values of Δ (say, Δ_1) the relative efficiency of the PTE based on the modified W test is the highest followed by those of the PTEs under the modified LR and LM tests respectively. Therefore, for $\Delta_0 \leq \Delta \leq \Delta_1$ the following inequality relation holds

$$R_{LM*} \leq R_{LR*} \leq R_{W*}. \tag{4.2}$$

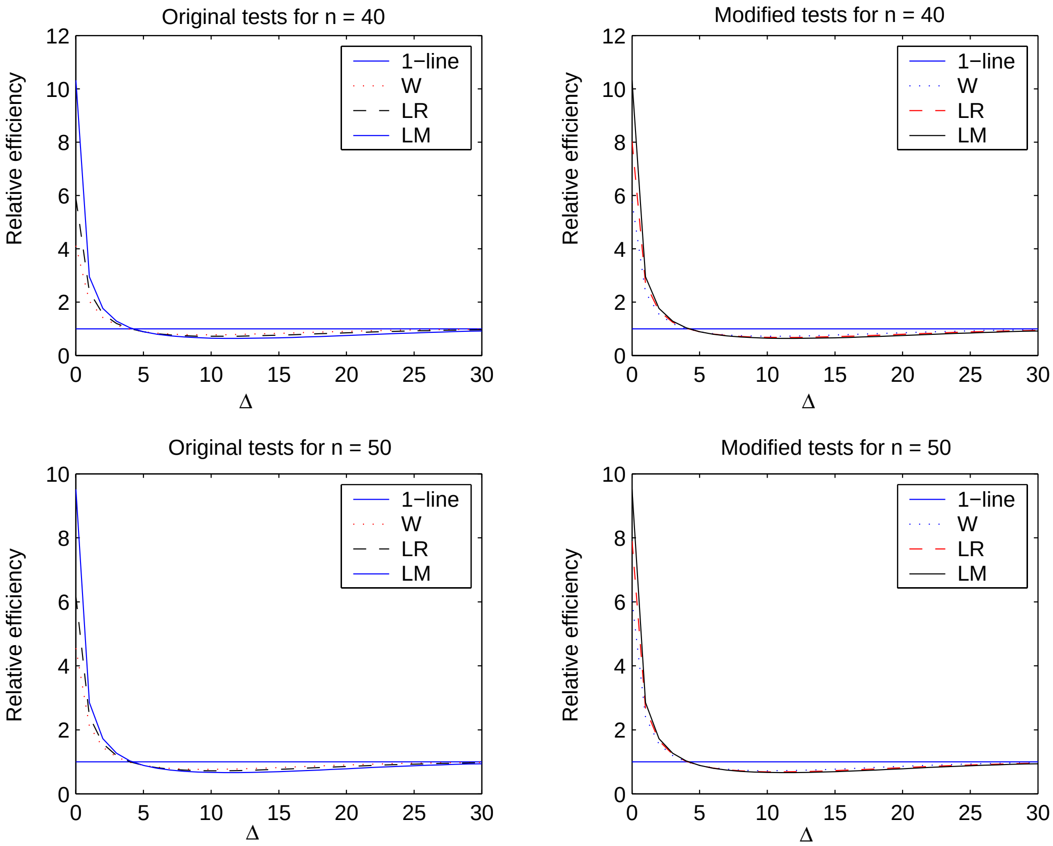

Figure 4: Relative efficiencies of the PTE based on the W, LR and LM tests assuming $p = 5$ and $\alpha = 0.05$

However, when Δ is greater than Δ_1, the relative efficiencies of the PTEs based on all three tests tend to be the same as that of the UE. Therefore, with respect to the quadratic risk there is no uniform domination of the PTE based on any particular test over those based on the other tests for all values of Δ and for any fixed α. From Table 4.1 it is observed that the amount of conflict decreases as Δ grows larger up to some moderate value (say, Δ_*). As from some large value of Δ the relative efficiencies of the PTE under the three tests approaches to that of the UE, the conflict among the relative efficiencies approaches to 0. Table 4.1, and Figures 3 & 4 reveal the fact that the use of the three modified tests in the definition of the PTE reduces the conflict among their QRs as compared to that based on the three original tests. For example, when $n = 50$ and $\Delta = 1$ the amount of conflict among the QR of the PTE under the three original tests is 0.927, which is 0.453 under the three modified tests. Hence there is a substantial gain in reducing conflict by using the modified tests instead of the original tests used by Kibria (2002).

Table 4.1: Relative efficiencies of PTE under the original and modified W, LR and LM tests for $p = 5$, $\alpha = 0.05$ and few values of n, (here *Conft* stands for conflict).

	$n = 20$				$n = 30$				$n = 50$			
Δ	R_W	R_{LR}	R_{LM}	*Conft*	R_W	R_{LR}	R_{LM}	*Conft*	R_W	R_{LR}	R_{LM}	*Conft*
0	2.724	4.658	18.81	16.09	3.533	5.456	12.08	8.542	4.551	6.170	9.516	4.956
1	1.700	2.217	3.627	1.927	1.912	2.335	3.141	1.229	2.124	2.421	2.853	0.927
2	1.301	1.516	2.030	0.729	1.379	1.546	1.841	0.462	1.453	1.566	1.723	0.270
3	1.098	1.191	1.427	0.329	1.125	1.197	1.329	0.204	1.151	1.199	1.268	0.117
4	0.981	1.011	1.114	0.133	0.983	1.007	1.062	0.079	0.987	1.003	1.030	0.043
5	0.910	0.900	0.926	0.026	0.898	0.892	0.902	0.001	0.890	0.887	0.889	0.003
6	0.866	0.828	0.802	0.064	0.846	0.820	0.798	0.048	0.830	0.814	0.800	0.030
7	0.840	0.781	0.716	0.124	0.815	0.774	0.729	0.086	0.794	0.769	0.742	0.052
8	0.826	0.751	0.655	0.171	0.798	0.745	0.681	0.117	0.774	0.741	0.704	0.070
9	0.821	0.733	0.610	0.211	0.791	0.728	0.649	0.142	0.766	0.726	0.681	0.085
10	0.822	0.723	0.576	0.246	0.791	0.720	0.627	0.164	0.765	0.721	0.668	0.097
12	0.835	0.721	0.534	0.301	0.805	0.724	0.607	0.198	0.780	0.728	0.664	0.116
15	0.869	0.747	0.508	0.361	0.844	0.757	0.615	0.229	0.823	0.768	0.693	0.130
20	0.928	0.821	0.525	0.306	0.914	0.840	0.684	0.230	0.903	0.858	0.785	0.118
25	0.967	0.894	0.585	0.382	0.962	0.914	0.780	0.182	0.957	0.930	0.878	0.079
30	0.987	0.945	0.668	0.319	0.985	0.961	0.869	0.116	0.984	0.971	0.942	0.042
	R_{W*}	R_{LR*}	R_{LM*}	*Conft*	R_{W*}	R_{LR*}	R_{LM*}	*Conft*	R_{W*}	R_{LR*}	R_{LM*}	*Conft*
0	4.640	8.666	18.81	14.17	5.349	8.252	12.08	6.726	6.040	7.901	9.516	3.476
1	2.213	2.868	3.627	1.414	2.315	2.758	3.141	0.826	2.400	2.667	2.853	0.453
2	1.514	1.763	2.030	0.516	1.538	1.704	1.841	0.303	1.558	1.656	1.723	0.165
3	1.191	1.303	1.427	0.236	1.194	1.267	1.329	0.135	1.196	1.239	1.268	0.072
4	1.010	1.056	1.114	0.104	1.006	1.034	1.062	0.056	1.002	1.018	1.030	0.028
5	0.900	0.906	0.926	0.026	0.892	0.895	0.902	0.010	0.887	0.887	0.889	0.002
6	0.828	0.808	0.802	0.026	0.821	0.805	0.798	0.023	0.815	0.805	0.800	0.015
7	0.782	0.741	0.716	0.066	0.775	0.746	0.729	0.046	0.770	0.752	0.742	0.028
8	0.752	0.694	0.655	0.097	0.747	0.707	0.681	0.066	0.743	0.719	0.704	0.039
9	0.733	0.662	0.610	0.123	0.731	0.682	0.649	0.082	0.729	0.699	0.681	0.048
10	0.724	0.640	0.576	0.148	0.723	0.666	0.627	0.096	0.723	0.689	0.668	0.055
12	0.722	0.618	0.534	0.188	0.727	0.657	0.607	0.120	0.732	0.690	0.664	0.068
15	0.748	0.621	0.508	0.240	0.761	0.679	0.615	0.146	0.772	0.725	0.693	0.079
20	0.822	0.677	0.525	0.297	0.844	0.759	0.684	0.160	0.861	0.817	0.785	0.076
25	0.895	0.762	0.585	0.310	0.917	0.850	0.780	0.137	0.932	0.902	0.878	0.054
30	0.946	0.845	0.668	0.278	0.962	0.921	0.869	0.093	0.972	0.956	0.942	0.030

5 Concluding Remarks

LM test rather than the original LM test From the foregoing analyses it is observed that for both the original and modified tests, with respect to the quadratic bias the performance of the W test based preliminary test estimator is the best followed by those based on the LR and LM tests respectively. In general, the conflict among the QBs of the PTEs based on the modified tests is less than that of the PTEs based on the original tests. With respect to the quadratic risk, there is no uniform domination of one PTE based on a particular test over the others for all values of Δ. This is true for all PTEs under the three original tests as well as under the three modified tests. As the non-sample prior information is obtained from experts knowledge or previous studies, Δ is likely to be not too far from 0, and hence our interest centers around the value of Δ near 0. Since the relative efficiency of the PTE based on the LM test is the highest near $\Delta = 0$, the practitioners may use the LM test based PTE to minimize the quadratic risk. However, as the use of the modified tests in the definition of the PTE reduces the conflict among the estimators it is preferable to use the modified LM test rather than the original LM test in the formation of the PTE.

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